Operations on Intuitionistic Fuzzy Directed Graphs

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Abstract- ***The motivation for introducing IFGs is due to The first definition and concept of Intuitionistic Fuzzy Graph (IFG) was introduced. Karunambigai analyzed the properties of minmax IFGs in Shannon and Atanassov defined intuitionistic fuzzy graphs using five types of Cartesian products. In this paper IFGs so defined are named. Some isomorphic properties on IFGs are discussed. et al., discussed the properties of isomorphism on fuzzy graphs and properties of isomorphism on strong fuzzy graphs in which motivated us to develop the same on IFGs and on strong IFGs. The main aim of this study is to build basic definitions of an IFG which will be useful for the researchers for their future study in IFGs. Since the title of the paper is given so. For graph theoretical definitions and throughout this paper all the properties are analyzed for simple minmax IFG.***

Keywords: Intuitionistic, Fuzzy Graphs, Fuzzy Directed Graph

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INTRODUCTION

Graph theory is a very beneficial device in fixing combinatorial problems in exceptional areas consisting of geometry, algebra, wide variety principle, topology, operations studies, optimization, laptop technological know-how, engineering, and bodily, organic, and social structures. Point-to-point interconnection networks for parallel and allotted systems are normally modeled by means of directed graphs (or digraphs). A digraph is a graph whose edges have instructions and are referred to as arcs (edges). Arrows on the arcs are used to encode the directional information: an arc from vertex (node) x to vertex y suggests that one may circulate from x to y but no longer from y to x. Presently, technology and era are featured with complicated processes and phenomena for which entire information isn't always continually to be had. For such instances, mathematical fashions are advanced to handle styles of structures containing elements of uncertainty. A large quantity of those models are based totally on an extension of the ordinary set principle, namely, fuzzy units. The notion of fuzzy units turned into brought through Zadeh [1] as a way of representing uncertainty and vagueness. Since then, the theory of fuzzy units has end up a vigorous place of studies in specific disciplines, such as clinical and life sciences, management sciences, social sciences, engineering, data, graph theory, synthetic intelligence, sign processing, multiagent systems, pattern reputation, robotics, laptop networks, professional systems, selection making, and automata concept.

Fuzzy graph idea is locating an increasing number of packages in modeling actual time systems where the extent of statistics inherent in the gadget varies with unique stages of precision. Fuzzy models are getting useful because of their aim of reducing the variations between the conventional numerical fashions used in engineering and sciences and the symbolic fashions used in expert systems.

Kauffman's initial definition of a fuzzy graph was based totally on Zadeh's fuzzy family members. Rosenfeld brought the fuzzy analogue of numerous fundamental graph-theoretic principles and Bhattacharya gave a few comments on fuzzy graphs. Mordeson and Nair [6] defined the idea of complement of fuzzy graph and studied some operations on fuzzy graphs. In , the definition of supplement of a fuzzy graph turned into modified in order that the supplement of the supplement is the unique fuzzy graph, which is of the same opinion with the crisp graph case. Atanassov introduced the concept of intuitionistic fuzzy members of the family and intuitionistic fuzzy graphs. Akram et al. added many new concepts, inclusive of sturdy intuitionistic fuzzy graphs, intuitionistic fuzzy hyper-graphs, intuitionistic fuzzy cycles, and intuitionistic fuzzy trees. Wu mentioned fuzzy digraphs. In this paper, the intuitionistic fuzzy organizational neural network models, intuitionistic fuzzy neurons in medical diagnosis, intuitionistic fuzzy digraphs in vulnerability evaluation of gasoline pipeline networks, and intuitionistic fuzzy digraphs in tour time are supplied as examples of intuitionistic fuzzy digraphs in selection assist systems.

Algorithms of those selection support structures also are designed and applied.

OBJECTIVES

1. To investigate edge dominance in fuzzy graphs and intuitionistic fuzzy graphs in a safe and fair manner.

METHOD ANALYSIS

Consider the two IFDGs $G_1 = [V_1, E_1, \mu_{i,j}, \nu_{i,j}]$ and $G_2 = [V_2, E_2, \mu_{p,q}, \nu_{p,q}]$, where V_1 and V_2 are the vertex sets and, E_1 and E_2 are the edge sets of G_1 and G_2 respectively.

Definition 1.

The addition of two IFDGs G_1 and G_2, denoted by $G = G_1 \oplus G_2$, is defined by

$$G_1 \oplus G_2 = \{V_1 \cup V_2,\ V_1 \cup V_2, \{\langle \mu_r,\ \nu_r \rangle\}\}, [V_1 \cup V_2, V_1 \cup V_2, \{\langle \mu_{i,j},\ \nu_{i,j} \rangle\}]$$

, where

$$\{\langle \mu_r, \nu_r \rangle\} = \begin{cases} \langle \mu_i, \nu_i \rangle & if\ \ \nu_r \in V_1 - V_2 \\ \langle \mu_p, \nu_p \rangle & if\ \ \nu_r \in V_2 - V_1 \\ \langle \max(\mu_i, \mu_p), \min(\nu_i, \nu_p) \rangle & if\ \ \nu_r \in V_1 \cap V_2 \\ \langle 0,1 \rangle & otherwise \end{cases}$$

and

$$\{\langle \mu_{r,s}, \nu_{r,s} \rangle\} = \begin{cases} \langle \mu_{i,j}, \nu_{i,j} \rangle & if\ \ \nu_r = \nu_i \in V_1\ \ and\ \ \nu_s = \nu_j \in V_1 - V_2 \\ & or\ \ \nu_r = \nu_i \in V_1 - V_2\ \ and\ \ \nu_s = \nu_j \in V_1 \\ \langle \mu_{p,q}, \nu_{p,q} \rangle & if\ \ \nu_r = \nu_p \in V_2\ \ and\ \ \nu_s = \nu_q \in V_2 - V_1 \\ & or\ \ \nu_r = \nu_p \in V_2 - V_1\ \ and\ \ \nu_s = \nu_q \in V_2 \\ \langle \max(\mu_{i,j}, \mu_{p,q}), \min(\nu_{i,j}, \nu_{p,q}) \rangle & \\ \quad if\ \nu_r = \nu_i = \nu_p \in V_1 \cap V_2\ \ and\ \nu_s = \nu_j = \nu_q \in V_1 \cap V_2 & \\ \langle 0,1 \rangle & otherwise \end{cases}$$

Example 2.

Consider the IFDGs G_1 and G_2 as in Figure 2.

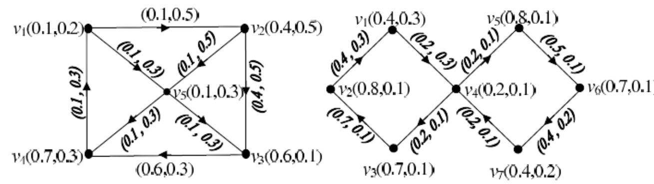

Figure 2.: G_1 and G_2

The index matrix of G_1 is $G_1 = \{V_1, V_1\{\langle \mu_{i,j},\ \nu_{i,j} \rangle\}\}$, where $V_1 = \{v_1, v_2, v_3, v_4, v_5\}$

and

$\{\langle \mu_{i,j}, \nu_{i,j} \rangle\} \equiv$

	v_1	v_2	v_3	v_4	v_5
v_1	$\langle 0,1 \rangle$	$\langle 0.1, 0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.1, 0.3 \rangle$
v_2	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.4, 0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0.1, 0.5 \rangle$
v_3	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.6, 0.3 \rangle$	$\langle 0,1 \rangle$
v_4	$\langle 0.1, 0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_5	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.1, 0.3 \rangle$	$\langle 0.1, 0.3 \rangle$	$\langle 0,1 \rangle$

The index matrix of G_2 is

$G_2 = \{V_2,\ V_2, \{\langle \mu_{p,q},\ \nu_{p,q} \rangle\}\}$,where $V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$\{\langle \mu_{p,q}, \nu_{p,q} \rangle\} \equiv$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_2	$\langle 0.4, 0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_3	$\langle 0,1 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_4	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_5	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.5, 0.1 \rangle$	$\langle 0,1 \rangle$
v_6	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.4, 0.2 \rangle$
v_7	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2, 0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$

The index matrix of $G_1 \oplus G_2$ is $[V_1 \cup V_2,\ V_1 \cup V_2, \{\langle \mu_{r,s},\ \nu_{r,s} \rangle\}]$,

Where, $V_1 \cup V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and

$\{\langle \mu_{r,s}, \nu_{r,s} \rangle\} \equiv$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	$\langle 0,1 \rangle$	$\langle 0.1,0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2,0.3 \rangle$	$\langle 0.1,0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_2	$\langle 0.4,0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0.4,0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0.1,0.5 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_3	$\langle 0,1 \rangle$	$\langle 0.7,0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.6,0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_4	$\langle 0.1,0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2,0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2,0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_5	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.1,0.3 \rangle$	$\langle 0.1,0.3 \rangle$	$\langle 0,1 \rangle$	$\langle 0.5,0.1 \rangle$	$\langle 0,1 \rangle$
v_6	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.4,0.2 \rangle$
v_7	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0.2,0.1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$

The graph of $G_1 \oplus G_2$ is shown in Figure 3.

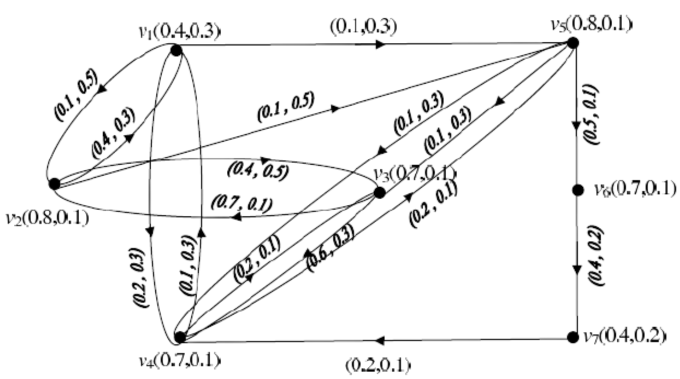

Figure 3: $G_1 \oplus G_2$

Definition 3

The vertex wise multiplication of two IFDGs G_1 and G_2, denoted by $G_1 \otimes G_2$, is defined by $G_1 \otimes G_2 = \langle [V_1 \cap V_2, V_1 \cap V_2, \{\langle \mu_r, \nu_r \rangle\}], [V_1 \cap V_2, V_1 \cap V_2, \{\langle \mu_{r,s}, \nu_{r,s} \rangle\}] \rangle$

,

Where,
$\{\langle \mu_r, \nu_r \rangle\} = \langle \min(\mu_i, \mu_p), \max(\nu_i, \nu_p) \rangle$ *if* $v_r \in V_1 \cap V_2$;

$\{\langle \mu_{r,s}, \nu_{r,s} \rangle\} = \langle \min(\mu_{i,j}, \mu_{p,q}), \max(\nu_{i,j}, \nu_{p,q}) \rangle$ if $v_r \in V_1 \cap V_2$; and v_s $v_j = v_q \in V_1 \cap V_2$.

Example 4

The index matrix of $G_1 \otimes G_2$ is $[V_1 \cap V_2, V_1 \cap V_2, \{\langle \mu_{r,s}, \nu_{r,s} \rangle\}]$,

Where, $V_1 \cap V_2 = \{v_1, v_2, v_3, v_4, v_5\}$ and

$\{\langle \mu_{r,s}, \nu_{r,s} \rangle\} \equiv$

	v_1	v_2	v_3	v_4	v_5
v_1	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_2	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_3	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_4	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$
v_5	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$	$\langle 0,1 \rangle$

The graph of $G_1 \otimes G_2$, a null IFDG, is displayed in Figure 5.

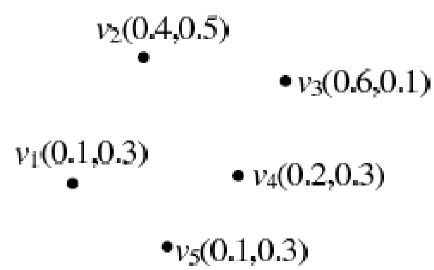

Figure 5 : $G_1 \otimes G_2$

Example 5

Consider the two IFDGs G_1 and G_2 as shown in Figure 6.

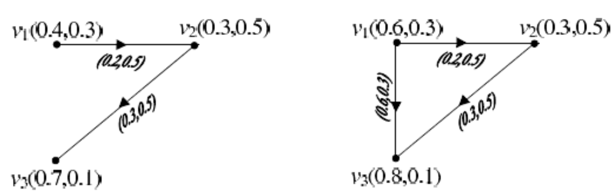

Figure 6: G_1 and G_2

Figure 6 depicts $G_1 \otimes G_2$, which is not a null IFDG.

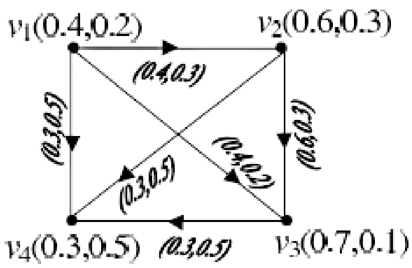

Figure 7: $G_1 \otimes G_2$

Definition 6

G_1 G_2 is the structural subtraction of two IFDGs G_1 and G_2, indicated by G_1 G_2.

$G_1 \ominus G_2 = [V_1 - V_2,\ \{\langle \mu_r,\ v_r \rangle\},\ \{\langle \mu_{r,s},\ v_{r,s} \rangle\}]$, where $'-'$ is the set theoretic difference operation and

$$\{\langle \mu_r, v_r \rangle\} = \begin{cases} \langle \mu_i, v_i \rangle & if \quad v_r \in V_1 \\ \langle \mu_p, v_p \rangle & if \quad v_r \in V_2 \\ \langle 0,1 \rangle & otherwise \end{cases}$$

$\{\langle \mu_{r,s},\ v_{r,s} \rangle\} = \{\langle \mu_{i,j},\ v_{i,j} \rangle\}$, for $v_r = v_i \in V_1 - V_2$ and $v_s = v_j \in V_1 - V_2$, if $V_1 - V_2 = \phi$, then graph of $G_1 \ominus G_2$ is also empty. Figure 7 shows the structural subtraction of the IFDGs G_1 and G_2 given in Figure 2.

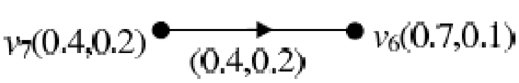

Figure 8: $G_1 \ominus G_2$

CONCLUSION

On this paper, the definition of complement of an IFG is given and a few properties of self - complementary IFGs are studied. Also, we bear in mind the operations union, be part of, Cartesian product and composition of IFGs and proved that the supplement of the join of two IFGs is the union of their enhances. Also, we've proved that composition of two strong IFGs is also sturdy. Much greater work can be executed to investigate the shape of IFGs which could have programs in Communication networks, Information Technology, Pattern Clustering, Image Retrieval and so on. The authors proposed to similarly extend the principles of fuzzy graphs into IFGs.

REFERENCES

1. L. A. Zadeh, Toward a generalized theory of uncertainty (GTU) - an outline, Information Sciences, 172 (1 - 2), 2005, 1 - 40.
2. Ayyaswamy. S,K and Natarajan. C, “Strong (weak) domination in Fuzzy Graphs”, International Journal of Mathematical and Computational Sciences ,Vol 4,No.7,2010,1035-1037.
3. Berge C, “Graphs and Hyper-graphs”, North-Holland, Amsterdam, 1976.
4. C. R. Bector and S. Chandra, Fuzzy mathematical programming and fuzzy matrix games, Springer-Verlag, Vol. 169, 2005.
5. D. Dubois and H. Prade, Fuzzy sets and Systems: Theory and Applications, New York: Academic Press, Vol. 144, 1980.
6. J. N. Mordeson and P. S. Nair, Fuzzy graphs and fuzzy hyper-graphs, Physica verlag, Heidelberg 1998, Second Edition, 2001.
7. Rosenfeld, Fuzzy graphs, in: L. A. Zadeh, K.S.Fu, M. Shimura (Eds.), Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975, 77 - 95.
8. G. Ausiello, Alessandro D’ Atri, and DomenicoSacca, Strongly equivalent directed hypergraps, Analysis and Design of Algorithms for Combinatorial Problems, Annals of Discrete Mathematics, 25, 1985, 1 - 25.
9. K. T. Atanassov, G. Pasi and R. Yager, Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making, International Journal of Systems Science, 36 (14), 2005, 859 - 868.
10. M. R. Berthold and K. P. Huber, Neural network based construction of fuzzy graphs, In Proceedings of the Fourth Annual Conference on Fuzzy Theory and Technology, North Carolina, 1995, 170 - 173.
11. P. Bhattacharya, Some remarks on fuzzy graphs, Patteren Recognition Let-ters, 6 (5), 1987, 297 - 302.
12. Sunil Mathew and SunithaM.S, ”Node Connectivity and arc connectivity of a fuzzy graph”, Information Sciences,180(2010)519-531.

13. S. M. Baas and H. Kwakernaak, Rating and ranking of multiple-aspect alter-natives using fuzzy sets, Automatica, 13 (1), 1977, 47 - 58.
14. Thilagavathi S, Parvathi R. and Karunambigai M.G, "Intuitionistic fuzzy hyper-graphs", J. Cybernetics and Information Technologies, 9(2), (2009), p. 46-53 Sofia.
15. Wen-Liang Hung, Min-ShenYang,"On Similarity Measures Between Intuitionistic Fuzzy Sets", International Journal of Intelligent System, Vol 23,364-383.
16. Yeh R.T and Bang S.Y, "Fuzzy relations, fuzzy graphs, and their applications to clustering analysis", Fuzzy Sets and their Applications to Cognitive and Decision Processes, pp 125-149,1975.
17. K. R. Bhutani, On auto-morphisms of fuzzy graphs, Pattern recognition letters, 9 (3), 1989, 159 - 162.
18. S. M. Baas and H. Kwakernaak, Rating and ranking of multiple-aspect alter-natives using fuzzy sets, Automatica, 13 (1), 1977, 47 - 58.
19. S. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17 (2), 1985, 113 - 129.
20. P. Bhattacharya, Some remarks on fuzzy graphs, Patteren Recognition Let-ters, 6 (5), 1987, 297 - 302.
21. P. Bhattacharya and F. Suraweera, An Algorithm to compute the max-min powers and a property of fuzzy graphs, Patteren Recognition Letters, 12 (7), 1991, 413 - 420.

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