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Study on Numerical Models & Symplectic Manifolds for Integral Equations

Dipti Dhingra[1]* Dr. Ashwani Kumar[2]

[1] Research Scholar of OPJS University, Churu, Rajasthan

[2] Associate Professor, OPJS University, Churu, Rajasthan

***Abstract** – A symplectic two m-dimensional manifold/or bifold(Ő)represented by a closed form ҭwhere, ҭmdiminishes transversally as well as ҭ is confined maximally non-degenerate hyper methods H. This H is also known as folding hyper-methods. This is the way of introducing folded-symplectic form which is nothing but the conjunction of more than one symplectic manifolds. A Numerical, folded symplectic or bifold can be said a folded-symplectic manifold (Ő2m, ҭ) equipped with an effective, Hamiltonian action of a torus (T) with dimension m. This whole complex system is nothing but the generalizations of Numerical as well as symplectic or bifolds with deep sense of hypermethodss.*

The Analysis of these symplectic orbifolds is a connection among Numerical symplectic geometry & singularity theory. The aim of these two is complementary to each other as one provide smooth functioning to degenerate while other's degeneracies are far away.

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INTRODUCTION

The evolution of this folded-symplectic geometry happened in 1969 by Martinet. However, on a local four-orbifold the author would like to demonstrate a two form consisting a singularity of type $\Sigma_{2,0}$ & represented in coordinate system asydy∧dz+dx∧dt to show folded-symplectic form. It can also be denoted as ω_{R^4} with the help of fold map $\varphi(y,z,x,w)=(\frac{y^2}{2},z,x,w)$ to validate the name fold symplectic.

Cannas da Silva has startedto unveil the folded-symplectic geometry in late nineties to emphasize on the existence of spin-c structures to the existence of symplectic orbifolds. In literature researchers illustrate an unfolding concept in which a folded-symplectic orbifold can be divided into symplectic pieces. Guillemin, Cannas and Piresrevealed compact Numerical origami thus the folding hypermethods fibrates over a compact base by a locally free circle action. It shows that Numerical origami orbifolds as well as templates of unimodular polytopes in the dual of the Lie Integral of the torus, T^*. This is nothing but the generalization of Delzant's theorem which illustrates one-to-one correspondence among compact symplectic orbifolds & polytopes in T^*. Furthermore, the above polytope is delzant which is simple, smooth rather rational. The labelled polytopes & symplectic Numerical manifolds, it can be observed thatpolytopes have combinatorial properties to achieve some geometric information regarding Numerical manifolds.

The main aim of this work is to broad these fundamental concepts to Numerical, foldedsymplectic orbifolds as well as non-compact. Lee proposed in literature about isomorphismamong Numerical, folded symplectic 4-orbifolds.

NUMERICAL ORBIFOLD

Numerical symplectic orbifolds have at most no. of commuting Hamiltonian relations. Its differentials arehaving linear curve at the folding hypermethods. So, the orbifold can be seen asdegenerate system which is completely integrable. It can be illustrated that the orbifolds with corners as well as without corners are defined separately.We can also say that the orbit space of a Numerical orbifold with foldedhypermethods is known as orbifold with corners. To provide the authenticity of above statements, we reveal some fact in a manner such that the stabilizers in a folded-symplectic orbifold are Numerical and then debate on that the specified orbifolds are local. Further, it can be focus that the moment map fall downwards and represents unimodular map with folds.

Definition 1:

Numerical orbifold can be denoted as ($Ő^{2m}$, ҭ) if equipped with an effective, Hamiltonian action of a torus (T) with dimension m which is half the dimension of the orbifold.

$$\varphi : \ddot{O}^{2m} \rightarrow R^m$$

An integral system states that a symplectic orbifold $Ő^{2m}$ equipped either with m linearly independent Poisson functions p1, p2,p3. . . , pnor Hamiltonian vector fields.Thus, a Numerical orbifold is an integrable system for which these above functions taken in a manner that the Hamiltonian vector of the Poisson functions are unit periodic. The moment map φ(Ő) is represented as convex polyhedron or Newton polytope of Ő.

All polytopes emerging from folded Numerical orbifolds fulfil the below specified:

Proposition:

1. For every vertex vexists there are m edges leaving it.
2. These edges having relation v + tw$_i$ (i = 1,.., m) where w$_i$∈ $(Q^m)^*$ (= Λ^{Weight})).
3. These weights w_1,.., w_m form a basis of the weight lattice Λ^{Weight}, for every vertex v.

Remark: Ő is a folded Numerical manifold if and only if the above mentioned condition 1 and 2 are satisfied.

Theorem 1:

Numerical orbifolds (according to Delzant theorem) are arrangedin terms of moment polytopes, or we can call it as, if $Ő_1$, $Ő_2$ are two folded Numerical orbifolds with moment maps φ($Ő_1$) and φ($Ő_2$) and φ($Ő_1$) =φ($Ő_2$), then it is said that there exist a T^nfolded symplectic equivariant diffeomorphism among$Ő_1$ and$Ő_2$.

If a polytope P fullfill above mentioned 1-3 proposition one can say a folded Numerical symplectic orbifold Ő with φ(Ő)=P.

Thus, $P = \cap_{i=1}^{r} \{a \in R^n :< a, d_i > \leq \eta_j\}$ for d$_i$є R^n, η$_i$є R^n.

Definition 2:

If the polytope P is illustrated as an n-dimensional polyhedron in R^n, then (a) ithdimentional face (M$_i$)of polytope Pif and only if M$_i$ is an i-simplex (b) IntM$_i$ is nothing but congruent to the interior of the mentioned i-simplex. (c) All point in given polytope P is in the interior of exactly single face.

Definition 3:

A facet is defined for an m-dimensional polytope P is nothing but an (m − 1)-dimensional face. The count of desired facets in polytope P is f: the facets indexing is given by k, and normals having relation η_k∈ R^n. Although, as$\eta_k$cannot represent by an integer multiple of another element, thus is said to be primitive.

The exact vector spaces V sequence can be represented as:

$$0 \rightarrow r \xrightarrow{l} R^p \xrightarrow{E} R^n \rightarrow 0$$

Where, $E : \pi_k \mapsto \eta_k$ and thus η_kєΛ^{Weight} = Hom (R^n , 2πR), this resultsinto

$$1 \rightarrow \Re \xrightarrow{l} A(1)^p \xrightarrow{E} A(1)^n \rightarrow 1$$

And it shows $\Re = Ker(E)$ is a torus.

The well-known fact for the moment map for the action of $A(1)^p$ on C^p is shown below:

$$G : (r_1, \ldots\ldots, r_p) \mapsto -\frac{1}{2}(|r_1|^2, \ldots\ldots, |r_p|^2) + a.$$

However, $a = \eta_1, \ldots\ldots, \eta_p$.

Thus,

for the action of given $\Re$ overC^p the moment map is expressed by φ*(Ő) where

$$0 \rightarrow R^p \xrightarrow{E^*} R^p \xrightarrow{r^*} r^* \rightarrow 0.$$

Example:

The moment polytope is demonstrated with the help of the right triangle with three vertices (0, 1), (0, 0) & (1, 0). Assume$\eta_k$can be represented for ith face normal vector. η1 = (0, −1), η2 = (−1, 0) and η3 = 1/2 (1, 1).

$$r \rightarrow R^3 \xrightarrow{E} R^2$$

$$E : \pi_k \mapsto \eta_k$$

$$(0,0,1) \mapsto (1,1)$$
$$(0,1,0) \mapsto (-1,0)$$
$$(1,0,0) \mapsto (0,-1)$$

$\chi_1=\chi_2=0$ and $\chi_3=1/2$

$$E = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

r = R(1, 1, 1) ⊂ R^3 = Ker(E) and $\Re \cong A(1)$ thus, C^3 can be minimized w.r.t. action of $\Re$ & demonstrated by:

$$j : (1,1,1) \mapsto R^3$$
$$j^* : R^3 \to R$$
$$j^*(\upsilon) = <\upsilon, (1,1,1)>$$
$$\ddot{O}(r_1, r_2, r_3) = -\frac{1}{2}(|r_1|^2, |r_2|^2, |r_3|^2) + (\chi_1, \chi_2, \chi_3)$$
$$\varphi^* \{\ddot{O}(r_1, r_2, r_3)\} = -\frac{1}{2}\sum_i |r_i|^2 + (\chi_1 + \chi_2 + \chi_3)$$
$$= -\frac{1}{2}\sum_i |r_i|^2 + \frac{1}{2}$$
$$(\varphi^* . \ddot{O})^{-1}(0) / E = CP^2$$

NON COMPACT SYMPLECTIC NUMERICAL ORBIFOLD

Theorem 1: Let us assume (Ő, т, φ :Ő → T*) be a Numerical, folded-symplectic orbifold with moment map φ : Ő → T*, where T* is the Lie Integral of the torus T acting on Ő. Let us say the fold f⊆ Ő is co-orientable. Thus,

a. Ő/T is generally represented as a orbifold with corners

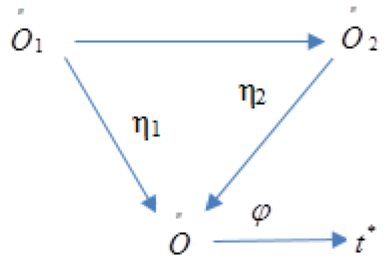
b. The moment map φfall downwards to a smooth map $\bar{\varphi}: \ddot{O}/T \to T^*$, which described as aunimodular map with folds.

Definition: Let us say corners C and assume unimodular map with orbifold represented by $\varphi: C \to t^*$. Where t is nothing but Lie Integral of torus T. Thus, $\ddot{O}_{\varphi}(C)$ can be defined as a category having triple objects.

(Ő, т, η : Ő → C)

Hence, Numerical folded symplectic orbifold described by (Ő, т, φ.η). where, η is quotient map &φ.η is moment map with torus T.

However, a morphism among objects (Őj, тj, η_j : Ő → C) in case of two j=1,2. So the commutative diagram for $\varphi: \ddot{O}_1 \to \ddot{O}_2$



andit is also seen that φ *τ_2 = τ_1. Whereas, φ is elaborated as an equi-variant symplectic morphism that preserves moment maps. By the given definition, all morphism is invertible, thus it can be said $\ddot{O}_{\varphi}(C)$ is a groupoid.

Theorem 2: Isomorphism family of objects ingroupoid $\ddot{O}_{\varphi}(C)$ having mapping both one-to-one with L^2(Ő; I_T × R), where I_T = ker(exp: t → T) is nothing but the integral lattice form of the torus orbifold T whichworks on various objects of $\ddot{O}_{\varphi}(C)$.

MOMENT POLYTOPE

The main focus behind this topic to understand a Numerical orbifold from its moment polytope. After studying the fundamentalsof Morse theory, it can be determinedthe following: firstly the homology of symplectic Numerical orbifolds; Secondlyto find suitable Morse functionality given by a moment map w.r.t. a fit subgroup of circle. In this section, we emphasis on basic surgery constructions lied over symplectic reduction, which clasp in the family of symplectic Numerical orbifolds.

Darboux Theorem

The below mentioned two theorems illustrate principle neighbourhoods of fixed points. The proofs of these theorems depend on the equivariant class of the Moser trick.

Theorem 1:

Assume thatorbifold pair (Ő, т) is 2m-dimensional symplectic orbifold provided witha symplectic action of a compact Lie group L, and suppose f be a fixed point. So, the Darboux chart for orbifold is a chart (D, a1, a2….a_n, b_1, b_2,…..b_n) centred at f and L-equivariant w.r.t. a linear action of Lover R^{2m} such that т

$$\tau |_D = \sum_{i=1}^{m} da_i \wedge db_i$$

Though, there exists a Darboux chart centred at every point of a symplectic orbifold by the above Darboux theorem.

Thus, a worthy linear action over R^{2m} is equivalent to the persuade action of Lie Integral on (T_fŐ). In specific, if Lrepresents a torus, this linear action is distinguished by the weights over(T_fŐ). So any specified symplectic action is known to be Hamiltonian.

In order to develop the mathematical calculation of the Betti numbers for symplectic Numerical orbifold with the help of moment map as defined in Morse functionality, further identify the general image of moment map around a fixed point f of a Hamiltonian torus action.

Theorem 2:

Let us suppose that $(\ddot{O}^{2m},\tau,T^m,\psi)$ canbe defined as a Hamiltonian T^m-space, where f is said to be a fixed point. Then there occurs a chart (D, a1, a2….a_n, b_1, b_2,…..b_n) centred at f as well as weights $\xi^1,\dots\dots,\xi^m$ belongs to Z^m. It can be illustrated by

- $$\tau\,|_D=\sum_{i=1}^{m} da_i\wedge db_i$$

and

- $$\psi\,|_D=\psi(f)-\frac{1}{2}\sum_{i=1}^{m}\xi^i(a_i^2+b_i^2)$$

This theorem promises the phenomenon of a Darboux chart centred at any fixed point f where the moment map seems to be the moment map for a linear action on R^{2m}.

Morse Fundamentals

SupposeŐrepresents m-dimensional orbifold and Morsedefine a smooth function over orbifold □: Ő → R if and only if all critical points of this function are non-degenerate.

However, the index of a bilinear function having maximal dimension of a subspace of R with this relation B: $R^n\times R^n \rightarrow R$ for non-positive values of B.The invalidity of B is nothing but the dimension of its nullspace, such that, the subspace containing of every η belongs to R^n to demonstrate the equality B(η,τ) = 0 for each τ element ofR^n. Thus, a critical point r of fixed point f i.e. □: Ő → R is non-degenerate when the Hessian relation satisfy following:

H_r: $R^n\times R^n \rightarrow R$ consist invalidity equal to zero.

Although r is nothing but a non-degenerate critical point for orbifold □: Ő → R. The index of this smooth function at point r is known as the index of the hessian relation mentioned above. It is well known fact that the index term does not dependent on the possibility of local coordinates. Furthermore, the Morse concept results that there occurs a chart (D, a1, a2….a_n, b_1, b_2,…..b_n) centred at r in a manner that

$$S\,|_D=S(r)-(a_1)^2-\dots\dots-(a_\xi)^2+(a_{\xi+1})^2+\dots.+(a_n)^2$$

Where ξ represents the index of □ at point r. So, it can be said that non-degenerate critical points are mandatory isolated.

Suppose □ defines a Morse basis over orbifold Ő. For x∈ R, we consider the equality shown below:

$$(\ddot{O})^x=S^{-1}(-\infty,x]=\{r\in O\,|\,S(r)\le x\}$$

Theorem: (Morse Findings)

A. $$z_\xi(\ddot{O})\le C_\xi$$

B. $$\sum_\xi(-1)^\xi z_\xi(\ddot{O})=\sum_\xi(-1)^\xi C_\xi$$

C. $$y_\xi(\ddot{O})-y_{\xi-1}(\ddot{O})+\dots\dots\pm y_0(\ddot{O})\le C_\xi-C_{\xi-1}+\dots\dots\pm C_0.$$

Thus, after these findings it is quite easy to say that a Morse basis over orbifold is known to be a Morse function for which the above mentioned inequalities are to be equalities.

Homology of Numerical Orbifold

Let us suppose that $(\ddot{O},\tau,T,\psi)$ canbe defined as a 2m-dimensional symplectic Numerical orbifold. Now, choose a vector V whose elements do not depends on U by selecting a proper generic direction in R^m. This outcome ensures followings:

- The single-dimensional subset, T^V belongs to T^m, caused by the vector V is opaque in T^m
- Vector V of the moment polytope is having sequence to the facets represented by $\gamma:=\psi(\ddot{O})$
- All the vertices of Ɣ have various projections along vector V.

Symplectic Blow-up

Let us suppose that Gis known to be the tautological line bundle on Q^{m-1}, it means:

$G=\{([Q],c)\,|\,q\in C^m\setminus\{0\},c=\eta_q$ for some η ϵ C} with estimate to Q^{m-1} represented by ([Q], c) → [Q]. The fiber of G over the point [Q] belongs to $Q^{m-1}$is nothing but the complex line in C^m introduced by that point.

Definition:

The symplectic orbifold blow-up of C^m at the genesis is known as the total space of the bundle G. The respectivesymplectic orbifold blow-down map is represent the map σ : G → C^m illustrated by following:

$$\sigma([Q], c) = c.$$

It canalso be observed that the total space of G may be putrefiedin a manner that nothing is common in two groups, $U:=\{([Q],0)\,|\,Q\in C^m\setminus\{0\}\}$ and $U:=\{([Q],c)\,|\,q\in C^m\setminus\{0\},c=\eta_q$ for some η ϵ C^*}.

The above set U is known as the exceptional diffeo-morphic divisor to Q^{m-1} and finds mapped to the creation by σ. Furthermore, the limitation of σ to the complementary group ∩ is said to be a diffeomorphism over C^m\{0}. Thus, the conclusive remarks is that Gis procured from C^m by replacing with the image of Q^{m-1}.

CONCLUSION

A symplectic m-dimensional manifold/orbifold(Ő)is illustrated with the help of a closed second form тwhere, $т^m$diminishes transversally as well as т is confined maximally non-degenerate hyper methods H.This is the way of introducing folded-symplectic form which is nothing but the conjunction of more than one symplectic manifolds.A Numerical, folded symplectic orbifold can be proved that a folded-symplectic manifold pair ($Ő^m$, т) equipped with an effective, Hamiltonian action of a torus (T) with dimension m. In this chapter classification of Numerical orbifold has been discussed in terms of Delzant theorem, symplectic reduction as well as Morse fundamentals. Furthermore, moment polytope also has been illustrated using Darboux theorem.

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Corresponding Author

Dipti Dhingra*

Research Scholar of OPJS University, Churu, Rajasthan