

Stud on Pre A* - Algebras and Rings in Logic Gates and Circuits

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Abstract – Boolean was a mathematician and scholar who created methods for communicating sensible procedures utilizing arithmetical images, in this way making a part of science known as representative rationale, or Boolean variable based math. It wasn't until years after the fact that Boolean polynomial math was connected to processing by John Vincent Atanasoff. He was endeavoring to manufacture a machine dependent on a similar innovation utilized by Pascal and Babbage, and needed to utilize this machine to settle direct arithmetical conditions. In the wake of battling with rehashed disappointments, Atanasoff was so baffled he chosen to take a drive. He was living in Ames, Iowa, at the time, however gotten himself 200 miles away in Illinois before he all of a sudden acknowledged how far he had driven. Atanasoff had not planned to drive that far, yet since he was in Illinois where he could legitimately purchase a beverage in a bar, he sat down, requested a whiskey, and acknowledged he had driven a significant separation to get a beverage (Atanasoff consoled the creator that it was not the beverage that driven him to the accompanying disclosures—truth be told, he left the beverage immaculate on the table.) Exercising his material science and arithmetic foundations and concentrating on the disappointments of his past processing machine, he made four basic achievements important in the machine's new structure.

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INTRODUCTION

Boolean algebra relies upon two component rationale. Pre A*-algebra is a regular augmentation of Boolean rationale to three truth values, where the third truth value stands for an indistinct truth value. In this chapter we investigate the algebraic structures of Boolean algebra, Pre A*-algebra.

This Chapter deals with the idea of Boolean algebras and Pre A*-algebras. This Chapter begins with the idea of Boolean algebra and some basic fundamental consequences of Boolean algebra. It also incorporates the valuable properties of Boolean algebra. We present the idea of Pre A*-algebra and obtain the valuable characterizations. We obtain the various strategies for generation of Pre A*-algebras from Boolean algebra.

This chapter comprises of two areas. In the primary area we concentrate on Boolean algebras, another arrangement of least number of axioms for Boolean algebras utilizing Huntington's hypothesis. In the second segment, we examine the algebraic structure of Pre A* - Algebra which is generated by a Boolean algebra and the strategies for generating Pre A*-algebras from Boolean algebras. First we start with the idea of Boolean algebras.

INTRODUCTION TO LOGIC REASONING AND DEDUCTION

The essential fixing in the examination of method of reasoning is the guidelines and system used to perceive disputes that are considerable and those that are assuredly not. Method of reasoning oversees considering and the ability to determine or achieve some reasonable goals. In normal day by day presence we consider what will happen dependent on past experiences; "No doubt it will rain" we state suggesting that it may rain today. In case we stick around adequately long, by then it may rain. This is an instance of inductive reasoning. In science we can discover paying little mind to whether a hypothesis is directly by checking if our choices can be finished up from results certainly known. This is called deductive reasoning.

The starting phase of method of reasoning is a declaration. A declaration in the specific sense is definitive and is either substantial or false, anyway can't be both in the meantime.

In reason it is insignificant whether a declaration is substantial or false, strangely, it should be unquestionably either. Reason declarations must be either substantial or false.

A Statement: is a definitive sentence which is either valid or false.

Instances of definitive proclamations:

- (a) New Haven is a city in Connecticut.
- (b) The significant lot of June has thirty days.
- (c) The moon is made of red cheddar.
- (d) Tomorrow is Saturday.

PRE A*-ALGEBRAS AND RINGS

A ring is a set R equipped with two binary operations $+: R \times R \rightarrow R$ and $\cdot: R \times R \rightarrow R$ (where \times denotes the cartesian product), called addition and multiplication. To qualify as a ring, the set and two operations, $(R, +, \cdot)$, must satisfy the following requirements known as the ring axioms. (i) $(R, +)$ is an abelian group under addition:

1. Closure under addition. For all a, b in R , the result of the operation $a + b$ is also in R .

Associativity of addition. For all a, b, c in R , the equation $(a + b) + c = a + (b + c)$ holds.

Existence of additive identity. There exists an element 0 in R , such that for all elements a in R , the equation $0 + a = a + 0 = a$ holds.

Existence of additive inverse. For each a in R , there exists an element b in R such that $a + b = b + a = 0$. Commutativity of addition. For all a, b in R , the equation $a + b = b + a$ holds.

- (ii) (R, \cdot) is a semi group under multiplication:

1. Closure multiplication. under For all a, b in R , the result of the operation $a \cdot b$ is also in R .

Associativity of multiplication. For all a, b, c in R , the equation $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ holds.

- (iii) The distributive laws:

1. For all a, b and c in R , the equation $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ holds.
2. For all a, b and c in R , the equation $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$ holds.

This definition assumes that a binary operation on R is a capacity characterized on $R \times R$ with values in R . In this manner, for any a and b in R , the addition $a + b$ and the item $a \cdot b$ are components of R .

This Chapter deals with the idea of ring on a Pre A*-algebra. In this chapter we characterize a ring on Pre A* - algebra. This chapter comprises of four

segments. In the principal area we demonstrate some basic hypotheses on Pre A* - algebra. In the second area, we characterize a ring on a Pre A* - algebra and its properties. We establish the personalities to demonstrate Pre A* - algebra as a ring and ring as a Pre A* - algebra. In the third action we characterize a Boolean ring on a Pre A* - algebra and we present the hypothesis Pre A* - algebra as a Boolean ring and Boolean ring as a Pre A* - algebra. . In the fourth segment we characterize a p-ring, 3-ring and we demonstrate Pre A* - algebra as a 3-ring and we demonstrate 3-ring as a Pre A* - algebra.

In this chapter we characterize a ring on Pre A* - algebra. This chapter comprises of four areas. In the principal segment we demonstrate some basic hypotheses on Pre A* - algebra. In the second area, we characterize a ring on a Pre A* - algebra and its properties. We establish the personalities to demonstrate Pre A* - algebra as a ring and ring as a Pre A* - algebra. In the third segment we characterize a Boolean ring in a Pre A* - algebra and we demonstrate Pre A* - algebra as a Boolean ring, Boolean ring as a Pre A* - algebra. . In the fourth area we characterize a p-ring, 3-ring and we demonstrate Pre A* - algebra as a 3-ring and we demonstrate 3-ring as a Pre A* - algebra.

5.1 Basic Theorems on Pre A* - algebra:

5.1.1 Theorem 1: De-Morgan laws :

Let $(A, \wedge, (-)^\sim, 1)$ be a Pre A* - algebra. Then,

- (i) $(a \wedge b)^\sim = a^\sim \vee b^\sim$
- (ii) $(a \vee b)^\sim = a^\sim \wedge b^\sim$

Proof: By the definition [1.1(d)] of Pre A* - algebra we have

- (i) $(a \wedge b)^\sim = a^\sim \vee b^\sim$
- (ii) By note 1.2.2, we have
 $(a \vee b)^\sim = a^\sim \wedge b^\sim$

5.1.2 Lemma 1: Uniqueness of identity in a Pre A* - algebra:

Let $(A, \wedge, (-)^\sim, 1)$ be a Pre A* - algebra and $a \in B(A)$ be an identity for \wedge , then a^\sim is an identity for \vee , a is unique if it exists, denoted by 1 and a^\sim by 0 where $B(A) = \{x/x \vee x^\sim = 1\}$ i.e., $(a) 1 \wedge x = x, \forall x \in A$.

- (b) $0 \vee x = x, \forall x \in A$.

Proof : Suppose $a \in B(A)$ is an identity for \wedge .

$$\Rightarrow a \wedge x = x, \forall x \in A \rightarrow (i)$$

To prove that $a \sim \in A$ is an identity for \vee :

Consider $a \sim \vee x = (a \wedge x \sim) \sim$

$= (x \sim) \sim$ [Since by (i)]

$= x$ [Since by definition 1.2.1 (a)]

Therefore $a \sim \vee x = x, \forall x \in A$.

Thus $a \sim$ is an identity for \vee .

Uniqueness : Suppose a and b are two identities for \wedge .

$\Rightarrow a \wedge x = x, \forall x \in A$ and

$b \wedge x = x, \forall x \in A$

Therefore $a \wedge b = b$ and $b \wedge a =$

Now $a = b \wedge a$

$= a \wedge b$ [Since by 1.2.1(c)] $= b$

Therefore $a = b$

Hence a if it exists is unique.

ie, $1 \wedge x = x, \forall x \in A$

$0 \vee x = x, \forall x \in A$

ie, 0 is identity for \vee

1 is identity for \wedge

5.1.3 Lemma 2: Let A be a Pre A^* - algebra with 1 and 0 and let $x, y \in A$.

(i) If $x \vee y = 0$, then $x = y = 0$

(ii) If $x \vee y = 1$, then $x \vee x \sim = 1$

Proof :(i) Suppose $x \vee y = 0 \rightarrow (A)$

Consider $x = 0 \vee x$

$= (x \vee y) \vee x$ [By (A)]

$= x \vee (y \vee x)$ [By 12.1 (e)~]

$= x \vee (x \vee y)$ [By 1.2.1 (c)~]

$= (x \vee x) \vee y$ [By 1.2.1 (e)~]

$= (x \vee y)$ [By 1.2.1 (b)~]

$= 0$ [By (A)]

Therefore $x = 0$

Similarly we can prove that $y = 0$

(ii) Suppose $1 = x \vee y \rightarrow (B)$

$= x \vee (x \sim \wedge y)$ [By 1.2.1 (g) ~]

$= (x \vee x \sim) \wedge (x \vee y)$ [By 1.2.1 (f) ~]

$= (x \vee x \sim) \wedge 1$ [By (B)]

$= x \vee x \sim$ [By Lemma 1]

$x \vee x \sim = 1$

5.1.4 Theorem 2

Let A , be a Pre A^* - algebra with 1 and $x, y \in A$.

If $x \wedge y = 0, x \vee y = 1$, then $y = x \sim$

Proof: If $x \vee y = 1$, then $x \vee x \sim = 1$ [By Lemma (2)]

$\Rightarrow x \sim \wedge x = 0$ (By the duality)

Now $y = 1 \wedge y$

$= (x \vee x \sim) \wedge y$

$= (x \wedge y) \vee (x \sim \wedge y)$ [By 1.2.1 (f)]

$= 0 \vee (x \sim \wedge y)$

$(x \sim \wedge x) \vee (x \sim \wedge y)$

$= x \sim \wedge (x \vee y)$ [By 1.2.1(f)]

$= x \sim \wedge 1$

$= x \sim$

Thus $y = x \sim$

5.1.5 Theorem 3

Let $(A, \wedge, (-) \sim, 1)$ be a Pre A^* - algebra.

Then we have the following

(i) Involution law:

$(a \sim) \sim = a, \forall a \in A$

(ii) $0 \sim = 1, 1 \sim = 0$

Proof: By 1.1 (a) we have (i),

(iii) Since we have

$$0 \wedge 1 = 0, 0 \vee 1 = 1$$

$$1 \wedge 0 = 0, 1 \vee 0 = 1$$

and By theorem 2, we have $0 \sim = 1, 1 \sim = 0$

CONCLUSION

The word algebra in the title of this part should caution you that more science is coming. Most likely, some of you are tingling to continue ahead with advanced plan instead of handling more math. Be that as it may, as your involvement in designing and science has shown you, arithmetic is a fundamental necessity for all fields in these zones. Similarly as intuition requires information of a language in which ideas can be planned, so any field of building or science requires learning of certain numerical subjects as far as which ideas in the field can be communicated and comprehended?

The numerical reason for computerized frameworks is Boolean algebra. We will presently set up various outcomes (hypotheses, tenets, or laws) that pursue from Huntington's hypothesizes and from the duality rule. The verifications will be done well ordered, with express legitimization for each progression given by alluding to the proper hypothesize or recently demonstrated hypothesis. Two of the general strategies for verification utilized in science are

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