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A Research on Different Basic Mathematical Approaches of Elasticity: Some Solutions

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Abstract – The studies inspected in this section were looked over a huge number distributed during the most recent five years that are applicable to the twin issues of how problem solving is found out and how it very well may be instructed. The mathematical theory of elasticity is busy with an endeavour to diminish to calculation the state of strain, or relative displacement, within a solid body which is liable to the activity of an equilibrating arrangement of forces. Numerical values of the amounts that can be associated with handy problems may serve to demonstrate the diminutiveness of the strains that happen in structures which are observed to be sheltered.

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INTRODUCTION

The mathematical theory of elasticity is busy with an endeavor to lessen to estimation the condition of strain, or relative removal, inside a solid body which is liable to the activity of an equilibrating arrangement of forces, or is in a condition of slight interior relative motion, and with undertakings to get results which will be essentially significant in applications to design, building, and all other helpful expressions where the material of development is solid.

In the historical backdrop of the theory of elasticity the two incredible milestones, are the disclosure of Hooke's law and the detailing of the general differential equations by Navier. In 1822 Cauchy had acquired his stress-strain relationship for isotropic materials by methods for two presumptions, viz.: l) that the relations being referred to are linear, 2) that the chief planes of stress are typical to the chief tomahawks of strain. Sometime in the future Cauchy stretched out his theory to the instance of crystalline bodies and to the anisotropic bodies moreover. Poisson's commitment to a similar subject is astounding for its various applications to exceptional issues. Theory of elasticity built up by Cauchy and Poisson was connected by them to various issues of vibrations and statical elasticity. Later Green had built up the theory from the strain vitality work. From there on Boussines had made huge commitments towards the precise advancement of the mathematical theory of elasticity. Point by point history in such manner in consecutive request might be found in the books.

The mathematical theory of Elasticity is busy with an endeavor to lessen to computation the condition of strain, or relative removal, inside a solid body which is liable to the activity of an equilibrating arrangement of forces, or is in a condition of little inward relative motion, by the guide of trial information and physical axioms expected ahead of time, and with undertakings to acquire results which will be for all intents and purposes significant in applications to design, building, and all other helpful expressions wherein the material of development is solid. Its history should grasp that of the progreBB of our exploratory learning of the conduct of strained bodies, so far as it has been exemplified in the mathematical theory, of the improvement of our originations concerning the physical axioms important to frame an establishment for theory, of the development of that part of mathematical examination where the proceBB of the computations comprises, and of the continuous procurement of handy guidelines by the elucidation of expository outcomes. We propose to give a sketch of such a history, so far as to incorporate the topic of the present volume, barring the exceptional issues of the equilibrium and vibrations of flimsy wires and plates, and the related hypotheses of effect and elastic security. In a subject in a perfect world worked out, the advancement which we ought to have the option to follow would be, in different points of interest, one from less to additional, however, we may state, that concerning the expected physical axioms, progreBB comprises in going from additional to less. Alike in the trial learning acquired, and in the scientific techniques and results, nothing that has once been found ever loses its worth, or must be disposed of; however the physical axioms come to be decreased to less and increasingly general standards, so the theory is carried more into accord with that of other physical subjects, a similar general dynamical standards being at last imperative and adequate to fill in as a reason for them all. What's more, in spite of the fact that, in our subject, we find visit retrogressions with respect to the experimentalist, and blunders with respect to the mathematician, predominantly in receiving hypotheses not obviously settled or effectively ruined, in pushing to boundaries techniques only inexact, in rushed generalizations, and in errors of physical standards, yet we watch a consistent and persistent advancement in every one of the regards referenced when we overview the historical backdrop of our subject from the principal enquiries of Galilei to the last works of SaintVenant and Sir William Thomson.

The primary mathematician to consider the idea of the obstruction of solids to crack was Galileil. In spite of the fact that he regarded solids as inelastic, not being in control of any law interfacing the relocations delivered with the power creating them, or of any physical speculation fit for yielding such a law, yet his enquiries provided the guidance which was accordingly trailed by numerous agents. He tried to decide the obstruction of a shaft, one finish of which is incorporated with a divider, when the inclination to break it emerges from its own or a connected weight, and he inferred that the pillar will in general turn about a hub opposite to its length, and in the plane of the divider. This issue, and, specifically, the assurance of this pivot is known as Galilei's concern.

During the only remaining century the theory of elasticity has discovered extensive applications in the arrangement of designing issues. There are numerous cases wherein the basic techniques for quality of materials are insufficient to outfit tasteful data with respect to stress distribution in building structures, and plan of action must be made to the more dominant strategies for the theory of elasticity. The basic theory is inadequate to give data with respect to nearby stresses close to the heaps and close to the backings of beams. It bombs additionally in the cases when the stress distribution in bodies, every one of the components of which are of a similar request, must be researched. The stresses in rollers and in chunks of bearing can be discovered distinctly by utilizing the theory of elasticity. The basic theory gives no methods for examining stresses in areas of sharp variety in cross-segments of beams or shafts. It is realized that at re-contestant corners a high-stress focus happens and because of this breaks are probably going to begin at such corners, particularly if the structure is submitted to an inversion of stresses. Most of breaks of machine parts in administration can be ascribed to such splits. During late years' thought advancement has been made in tackling such for all intents and purposes significant issues. In cases where a thorough arrangement can't be promptly acquired, surmised strategies have been created. Now and again, arrangements have been acquired by utilizing test techniques. The strategy for direct assurance of stresses and the utilization of the similarity equations regarding stress parts have been connected for taking care of different issues and as a rule, the vitality technique for arrangement of elasticity issues has been utilized.

Elasticity Theory: When a power is connected to a material, it twists. This implies the particles of the material are uprooted from their unique positions. Given the power does not surpass a basic value, the removals are reversible; the particles of the material come back to their unique positions when the power is evacuated, and no changeless distortion results. This is called elastic conduct.

The traditional uncoupled theory of thermo elasticity predicts two wonders not perfect with physical perceptions. In the first place, the condition of warmth conduction of this theory does not contain any elastic terms in opposition to the way that elastic changes produce warmth impacts. Second, the warmth condition is of parabolic kind foreseeing unending rates of proliferation for heatwaves. Despite the fact that Biot's theory of coupled thermo elasticity takes out the primary Catch 22 of old style theory, the two theories share the second inadequacy since the warmth condition for the coupled theory is additionally parabolic.

Endeavors to kill the mystery of boundless heat propagation speed are being made for over a century. As right on time as in 1867, Maxwell hypothesized the event of wave type warmth stream presently called second sound, while building up a kinetic theory of gases, and proposed an alteration of the Fourier's law. In 1917 Nernst guessed the likelihood of the event of temperature waves in great warm conductors, at low temperatures. In 1940, Tisza anticipated the likelihood of very little warmth propagation rates in fluid helium. In his examinations on excessively liquid helium, Landau depicted the second sound as the propagation of phonon thickness unsettling influence, and anticipated that its speed ought to be at OK, where vp is the speed of the normal sound (first sound). Experimentally, the subsequent sound was first recognized in fluid helium by Peshkov who observed its speed to be equivalent to 19m/s at 1.4K. Tisza's and Landau's expectations were checked experimentally by Maurer and Herlin.

During the most recent five decades, non-classical thermo elasticity theories including hyperbolic sort warmth transport equations conceding limited speeds for warm flag have been formulated. As per these theories heat propagation is to be seen as a wave marvel as opposed to a dissemination wonder. A wave-like the warm unsettling influence is alluded to as second sound, the principal sound being the standard sound (wave). These non-classical theories are alluded to as generalized thermo elasticity theories, thermo elasticity theories with limited speeds or thermo elasticity theories with second sound. A wide assortment of issues uncovering fascinating marvel describing these theories have been examined. These theories are persuaded by investigations showing the genuine event of second sound at low temperatures and for little intervals of time.

Adequate confirmations are accessible in the literature to demonstrate that warm unsettling influences do engender with limited speeds. Experimental investigations directed on different solids by Ackerman and that's only the tip of the iceberg, for instance have demonstrated that warmth heartbeats do engender with limited speeds. These experimental investigations infer that coupled thermo elasticity, with all benefits shockingly, has its very own downside. So as to conquer this Catch, endeavours were made to change coupled thermo elasticity, on various grounds, to acquire a wave-type heat conduction equation by Kaliski.

The idea of generalized thermo elasticity covers a wide scope of expansions of classical dynamical coupled thermo elasticity that incorporates following understood thermo elasticity theories. Thermo elasticity theory proposed by Lord and Shulman (LS model), in which, in contrast with the classical theory, the Fourier law of Heat conduction is supplanted by the Maxwell-Cattaneo law that generalizes the Fourier law and brings a solitary unwinding time into thought. It concedes limited speeds of propagation of warmth and elastic waves. The warmth equation related with this theory is a hyperbolic one and subsequently consequently takes out the oddity of unending speeds of propagation inalienable in both the uncoupled and coupled theories of thermo elasticity. Uniqueness of the answer for this theory was demonstrated under various conditions by Ignaczak.

Low-temperature thermo elasticity, presented by Hetnarski and Ignaczakv (HI model), in which, in contrast with the classical theory, both the free energy and the warmth transition depend not just on the temperature and the strain tensor yet in addition on an elastic warmth stream that fulfills a nonlinear development equation.

Dual-phase lag thermo elasticity, proposed by Chandrasekharaiah and Tzou (CT model), in which, the Fourier law is supplanted by an estimate to an adjustment of the Fourier law with two distinctive time interpretations for the heat flux and the temperature gradient. Tzou considered microstructural impacts into the deferred reaction in time in the plainly visible formulation by considering that the expansion of the grid temperature is postponed due to phonon-electron associations on the naturally visible level. Tzou acquainted two-phase-lags with both the heat flux vector and the temperature gradient. As indicated by this model, classical Fourier's law $\vec{q} = -K\vec{\nabla}T$ has been supplanted by $\vec{q}(P, t+\tau_q) = -K\vec{\nabla}T(P, t+\tau_T)$, where the temperature gradient $\vec{\nabla}T$ at a point P of the material at time $t+\tau_T$ compares to the heat flux vector q at a similar point at time $t+\tau_q$. Here K is the thermal conductivity of the material. The postpone time tt is translated as that brought about by the microstructural cooperation and is known as the phase-lag of the temperature gradient. The other postpone time τ_q is deciphered as the unwinding time because of the quick transient impacts of thermal inactivity and is known as the phase-lag of the heat flux. For $\tau_q = \tau_T = 0$, this is indistinguishable with classical Fourier's law. In the event that $\tau_q = \tau$ and $\tau_T = 0$, Tzou alludes to the model as single-phase-lag model. Roy choudhuri examined one-dimensional thermo-elastic wave propagation in an elastic half-space with regards to the dual-phase-lag model.

BASIC EQUATION

In this area, the overseeing equations of classical elasticity, classical thermo elasticity, classical coupled thermo elasticity, and generalized thermo elasticity are introduced. Give V and S a chance to be individually the volume and surface of the body under thought and T_0 be the reference temperature (estimated as far as supreme unit) in its undeformed state. Let (x_i) (i=l,2,3) signify the co-ordinates of any of its focuses alluded to a fixed rectangular Cartesian co-ordinate framework.

I. Classical Elasticity

Strain-displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), i, j = 1, 2, 3,$$

Where e_{ij} is the strain tensor and U_i is the displacement component.

Strain compatibility equations

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0, \; i, j, k, l = 1, 2, 3$$

Stress-strain relations

$$\tau_{ij} = C_{ijkl}e_{kl}, \; i, j, k, l = 1, 2, 3,$$

or

$$e_{ij} = S_{ijkl}\tau_{kl}, \; i, j, k, l = 1, 2, 3,$$

Where

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij},$$
$$S_{ijkl} = S_{jikl} = S_{ijlk} = S_{klij},$$

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$$S_{ijkl} = (C_{ijkl})^{-1};$$

and $Tij, Cijkl, Sijkl$ are stress tensor, elasticity tensor and elastic compliance tensor respectively

Equations of motion

$$\tau_{ij,j} + \rho F_i = \rho \ddot{u}_i, \; i, j = 1, 2, 3,$$

where Fi is the component of body force per unit mass and p (> 0) the mass density.

II. Classical Thermo elasticity (CTE)

Stress-strain temperature relations

$$\tau_{ij} = C_{ijkl}e_{kl} - \beta_{ij}(T - T_0), \; i, j, k, l = 1, 2, 3,$$

or

$$e_{ij} = S_{ijkl}\tau_{kl} + \alpha_{ij}(T - T_0), \; i, j, k, l = 1, 2, 3,$$

where $\beta_{ij} = C_{ijkl}\alpha_{kl}, \; \alpha_{ij} = S_{ijkl}\beta_{kl}$; and β_{ij}, α_{ij} are thermal moduli, thermal expansion tensor respectively, and T is the absolute temperature.

Fourier law of heat conduction

$$q_i = -K_{ij}T_{,j}, \; i, j = 1, 2, 3,$$

Where ς_i is the component of the heat flux vector and K_{ij} is the thermal conductivity tensor.

Energy equation

$$-q_{i,i} + \rho Q = \rho C_v \dot{T}, \; i = 1, 2, 3,$$

where Q is the heat source acting per unit mass per second and Cv is the specific heat at constant strain.

Heat equation

Equations together give the parabolic type heat transport equation

$$(K_{ij}T_{,j})_{,i} + \rho Q = \rho C_v \dot{T}, \; i, j = 1, 2, 3.$$

Equations above together constitute the complete mathematical model of the theory of classical thermo elasticity (CTE).

III. Classical Coupled Thermo elasticity (CCTE)

Energy equation

$$-q_{i,i} + \rho Q = \rho C_v \dot{T} + T_0 \beta_{ij} \dot{e}_{ij}, \; i, j = 1, 2, 3,$$

where the term $T_0 \beta_{ij} \dot{e}_{ij}$ brings about coupling between temperature and strain field.

Heat equation

Equations (II.2) and (III.l), on eliminaion of g*, lead to the parabolic type heat transport equation

$$(K_{ij}T_{,j})_{,i} + \rho Q = \rho C_v \dot{T} + T_0 \beta_{ij} \dot{e}_{ij}, \; i, j = 1, 2, 3.$$

Equations together constitute the complete mathematical model of the theory of classical coupled thermo elasticity (CCTE).

IV. Lord-Shulman Model (ETE)

Modified Fourier law of heat conduction

$$q_i + \tau_0 \dot{q}_i = -K_{ij}T_{,j}, \; i, j = 1, 2, 3,$$

Heat equation

Elimination of qi from equations results in generalized heat transport equation

$$(K_{ij}T_{,j})_{,i} + \rho(\dot{Q} + \tau_0 \ddot{Q}) = \rho C_v(\dot{T} + \tau_0 \ddot{T}) + T_0 \beta_{ij}(\dot{e}_{ij} + \tau_0 \ddot{e}_{ij}), \; i, j = 1, 2, 3.$$

This model gives hyperbolic sort heat transport equation with limited speed of thermal wave. This theory is otherwise called broadened thermo elasticity (ETE). As $\tau_0 \to 0$,, Lord Shulman model (ETE) lessens to classical coupled thermo elasticity (CCTE). r0 is called unwinding time, which is the time required to keep up enduring state heat conduction in a component of volume of an elastic body when an unexpected temperature gradient is forced on that volume component.

Equation together with other equations comprise the field equations of Lord-Shulman model.

V. Green-Lindsay Model (TRDTE)

Stress-strain temperature relations

$$\tau_{ij} = C_{ijkl}e_{kl} - \beta_{ij}[(T - T_0) + t_1 \dot{T}], \; i, j = 1, 2, 3.$$

Fourier law of heat conduction

$$q_i = -(C_i \dot{T} + K_{ij}T_{,j}), \; i, j = 1, 2, 3,$$

with $K_{ij} = K_{ji}$

Energy equation

$$-q_{i,i} + \rho Q = \rho C_v(\dot{T} + t_2 \ddot{T}) - C_i \dot{T}_{,i} + T_0 \beta_{ij} \dot{e}_{ij}, \; i, j = 1, 2, 3,$$

where t1, t2, Ci are new material constants for GL model

Heat equation

Elimination of q_i from last two equations gives rise to the generalized heat transport equation

$$(K_{ij}T_j)_{,i} + \rho Q = \rho C_v(\dot{T} + t_2\ddot{T}) - 2C_i\dot{T}_{,i} + T_0\beta_{ij}\dot{e}_{ij},\ i,j = 1,2,3,$$

Here ti and t2 $(ti > t2 > 0)$ are two constitutive constants having the element of time. This theory gives hyperbolic sort heat transport equation with limited speed of thermal wave. This theory is frequently alluded to as temperature-rate subordinate thermo elasticity (TRDTE). For a material having a focal point of symmetry at each point, equation (V.2) decreases to classical Fourier's law. In like manner, GL theory concedes the second sound without damaging the classical Fourier's law.

The arrangement of equations comprises the total mathematical model for Green-Lindsay theory.

VI. Green-Naghdi Model II (Thermo elasticity of type ii)

Law of heat conduction

$$q_i = -K^{\star}_{ij}\nu_j \text{ where } \dot{\nu} = T,\ i,j = 1,2,3.$$

Here v is the thermal displacement and K*j is the tensor of additional material constant.

Heat equation

Equations together gives the generalized heat transport equation

$$(K^{\star}_{ij}T_{,j})_{,i} + \rho\dot{Q} = \rho C_v\ddot{T} + T_0\beta_{ij}\dddot{e}_{ij},\ i,j = 1,2,3.$$

Equations constitute complete mathematical model of the Green-Naghdi model II.

Reevaluation of equations fcr the GN theory type II uncovers that no damping term shows up in the arrangement of equations and consequently the GN theory type II is known as the thermo elasticity without energy dissipation (TEWOED)

VII. Green-Naghdi Model III (Thermo elasticity of type III)

Law of heat conduction

$$q_i = -(K_{ij}T_j + K^{\star}_{ij}\nu_j) \text{ where } \dot{\nu} = T,\ i,j = 1,2,3$$

Heat equation

Equation together gives the generalized heat transport equation

$$(K_{ij}\dot{T}_j)_{,i} + (K^{\star}_{ij}T_j)_{,i} + \rho\dot{Q} = \rho C_v\ddot{T} + T_0\beta_{ij}\ddot{e}_{ij},\ i,j = 1,2,3$$

Equations constitute complete mathematical model of the Green-Naghdi model III. This theory is also known as thermo elasticity with energy dissipation (TEWED).

Two exceptional cases of the GN theory, in particular kind II and I, might be gotten from the equations of the GN theory type III. To acquire the GN theory type II, from the equations of the GN theory type III, we set $K_{ij} \to 0$, where 0 is the zero tensor. Whenever $K^{*}_{ij} \to 0$ the equations of the GN theory type III lessen to the GN theory type I, which is indistinguishable with the classical theory of thermo elasticity.

PROBLEM BASED ON CLASSICAL ELASTICITY

Stresses and displacements in a transversely isotropic thick plate-A numerical study

The problem of symmetric deformation and comparing stress distribution in a semiinfinite isotropic elastic solid because of weight disseminated symmetrically as for the focal point of a hover on the free plane boundary was created. Sneddon has stretched out the theory to a thick roundabout plate of limited range. The torsion problem of semi-vast solid and of an enormous thick plate of the isotropic body was explained by Yi-Yuan-Yu where the torsional shearing force is recommended over the plane boundary of semi-unbounded body and for the plate it is endorsed over both the plane limits. The recommended shearing force is communicated as far as Fourier-Bessel integral. Sengupta has determined the stresses in a sandwiched plate of an isotropic elastic material by applying a comparable methodology.

The distribution of stresses in an isotropic elastic half space affected by symmetrical normal burden on the jumping plane is displayed by Lekhnitskii, the normal loads being communicated as far as Fourier-Bessel integral. They have talked about the stress distribution in a thick plate of transversely isotropic solid because of symmetrical normal loads connected on the plane essences of the plate.

In the present study we have contemplated displacements and stress distribution in a thick transversely isotropic elastic plate because of the application of normal loads of various sizes on the jumping planes. These normal loads are taken as elements of outspread separation and communicated as far as Fourier-Bessel integrals. The numerical evaluations for the relating displacements and stresses have been acquired and delineated graphically. The numerical codes arranged for this problem are legitimate for an arbitrary decision of normal loads. At the point when the sizes of the normal loads are equivalent, the sizes of displacements and stresses are nearly the equivalent at various profundities for fixed outspread separation. WTien loads of inconsistent extents are connected, noteworthy consequences for displacements and stresses are watched. The investigation of this sort of problems is of impressive significance in perspective on their broad applications in mechanical designing, and so forth.

Formulation of the problem

Give us a chance to consider a homogeneous transversely isotropic elastic strip consuming the space $|z| \leq h$ exposed to normal loads on its plane faces symmetrically disseminated as for the z-hub. Give the body a chance to be alluded to a barrel shaped co-ordinate framework r, θ, z. z-pivot being brought the hub of elastic symmetry of the transversely isotropic elastic material. The displacement is taken to be symmetrical as for z-hub and is given by

$$u_\theta = 0,$$

$$\varepsilon_z = a_{13}(\sigma_r + \sigma_\theta) + a_{33}\sigma_z,$$

$$u_z = w(r, z).$$

Only non-vanishing stress-strain relations are given by

$$\varepsilon_r = a_{11}\sigma_r + a_{12}\sigma_\theta + a_{13}\sigma_z,$$

$$\varepsilon_\theta = a_{12}\sigma_r + a_{11}\sigma_\theta + a_{13}\sigma_z,$$

$$\varepsilon_z = a_{13}(\sigma_r + \sigma_\theta) + a_{33}\sigma_z,$$

$$\gamma_{rz} = a_{44}\tau_{rz},$$

where a_{ij}'s are elastic constants of the transversely isotropic material.

The stress field satisfies the following equations of equilibrium

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0,$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0,$$

together with the compatibility conditions

$$a_{11}\sigma_r + a_{12}\sigma_\theta + a_{13}\sigma_z - \frac{\partial}{\partial r}[r(a_{12}\sigma_r + a_{12}\sigma_\theta + a_{13}\sigma_z)] = 0,$$

$$\frac{\partial^2}{\partial z^2}(a_{11}\sigma_r + a_{12}\sigma_\theta + a_{13}\sigma_z) + \frac{\partial^2}{\partial r^2}(a_{12}\sigma_r + a_{11}\sigma_\theta + a_{13}\sigma_z) - a_{44}\frac{\partial^2 \tau_{rz}}{\partial r \partial z} = 0.$$

The boundary conditions are given by

$$\sigma_z = -p_1(r); \tau_{rz} = 0 \text{ on } z = h,$$

$$\sigma_z = -p_2(r); \tau_{rz} = 0 \text{ on } z = -h,$$

where Pi(r)(i = 1,2) being suitably chosen functions of r.

Method of solution

The expressions for stresses in terms of a single stress function ϕ(r, z) for the transversely isotropic body are given by

$$\sigma_r = -\frac{\partial}{\partial z}\left[\frac{\partial^2 \phi}{\partial r^2} + \frac{b}{r}\frac{\partial \phi}{\partial r} + a\frac{\partial^2 \phi}{\partial z^2}\right]$$

$$\sigma_\theta = -\frac{\partial}{\partial z}\left[b\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + a\frac{\partial^2 \phi}{\partial z^2}\right]$$

$$\sigma_z = \frac{\partial}{\partial z}\left[c\frac{\partial^2 \phi}{\partial r^2} + \frac{c}{r}\frac{\partial \phi}{\partial r} + d\frac{\partial^2 \phi}{\partial z^2}\right]$$

$$\tau_{rz} = \frac{\partial}{\partial r}\left[\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + a\frac{\partial^2 \phi}{\partial z^2}\right]$$

Where

$$a = \frac{a_{13}(a_{11} - a_{12})}{a_{11}a_{33} - a_{13}^2}, \qquad b = \frac{a_{13}(a_{13} + a_{44}) - a_{12}a_{33}}{a_{11}a_{33} - a_{13}^2},$$

$$c = \frac{a_{13}(a_{11} - a_{12}) + a_{11}a_{14}}{a_{11}a_{33} - a_{13}^2}, \quad d = \frac{a_{11}^2 - a_{12}^2}{a_{11}a_{33} - a_{13}^2}.$$

Prom the second equilibrium equation we obtain the following equation for determination of the stress function ϕ(r, z)

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + a\frac{\partial^2 \phi}{\partial z^2}\right) + \frac{\partial^2}{\partial z^2}\left(c\frac{\partial^2 \phi}{\partial r^2} + \frac{c}{r}\frac{\partial \phi}{\partial r} + d\frac{\partial^2 \phi}{\partial z^2}\right) = 0$$

The solution of above equation is given by

$$\phi(r, z) = \int_0^\infty [A(t)e^{s_1 tz} + B(t)e^{s_2 tz} + C(t)e^{-s_1 tz} + D(t)e^{-s_2 tz}]J_0(tr)dt$$

with

$$s_1 = \sqrt{\frac{(a+c)+\sqrt{(a+c)^2-4d}}{2d}},$$

$$s_2 = \sqrt{\frac{(a+c)-\sqrt{(a+c)^2-4d}}{2d}},$$

and A(t), B(t), C(t), D(t) are unknown parameters to be determined. Substituting the stress function ϕ(r, z) in the equation we obtain the stress components as

$$\sigma_r = \int_0^\infty [A(t)s_1(1-as_1^2)e^{s_1tz} + B(t)s_2(1-as_2^2)e^{s_2tz}$$

$$-C(t)s_1(1-as_1^2)e^{-s_1tz} - D(t)s_2(1-as_2^2)e^{-s_2tz}]t^3J_0(tr)dt$$

$$+\frac{b-1}{r}\int_0^\infty [A(t)s_1e^{s_1tz} + B(t)s_2e^{s_2tz} - C(t)s_1e^{-s_1tz} - D(t)s_2e^{-s_2tz}]t^2J_1(tr)dt,$$

$$\sigma_\theta = \int_0^\infty [A(t)s_1(b-as_1^2)e^{s_1tz} + B(t)s_2(b-as_2^2)e^{s_2tz}$$

$$-C(t)s_1(b-as_1^2)e^{-s_1tz} - D(t)s_2(b-as_2^2)e^{-s_2tz}]t^3J_0(tr)dt$$

$$-\frac{b-1}{r}\int_0^\infty [A(t)s_1e^{s_1tz} + B(t)s_2e^{s_2tz} - C(t)s_1e^{-s_1tz} - D(t)s_2e^{-s_2tz}]t^2J_1(tr)dt$$

$$\sigma_z = \int_0^\infty [A(t)s_1(ds_1^2-c)e^{s_1tz} + B(t)s_2(ds_2^2-c)e^{s_2tz}$$

$$-C(t)s_1(ds_1^2-c)e^{-s_1tz} - D(t)s_2(ds_2^2-c)e^{-s_2tz}]t^3J_0(tr)dt,$$

$$\tau_{rz} = -\int_0^\infty [A(t)(as_1^2-1)e^{s_1tz} + B(t)(as_2^2-1)e^{s_2tz}$$

$$+C(t)(as_1^2-1)e^{-s_1tz} + D(t)(as_2^2-1)e^{-s_2tz}]t^3J_1(tr)dt.$$

Now the components of displacement $(u, 0, w)$ can be found from the relations

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = a_{44}\tau_{rz},$$

$$\frac{\partial w}{\partial z} = a_{13}(\sigma_r + \sigma_\theta) + a_{33}\sigma_z.$$

Substituting the stress components from the other equations in the above equation we obtain on integration

$$u = \int_0^\infty [A(t)E_1(e^{s_1tz}-1) + B(t)E_2(e^{s_2tz}-1)$$

And

$$w = \int_0^\infty [A(t)F_1(e^{s_1tz}-1) + B(t)F_2(e^{s_2tz}-1)$$
$$+C(t)F_1(e^{-s_1tz}-1) + D(t)F_2(e^{-s_2tz}-1)]t^2J_0(tr)dt,$$

Where

$$E_1 = \frac{1}{s_1}[(1-as_1^2)(a_{13}+a_{44}) + (b-as_1^2)a_{13} + (ds_1^2-c)a_{33}],$$

$$E_2 = \frac{1}{s_2}[(1-as_2^2)(a_{13}+a_{44}) + (b-as_2^2)a_{13} + (ds_2^2-c)a_{33}],$$

$$F_1 = [a_{13}(1-as_1^2) + a_{13}(b-as_1^2) + a_{33}(ds_1^2-c)a_{33}],$$

$$F_2 = [a_{13}(1-as_2^2) + a_{13}(b-as_2^2) + a_{33}(ds_2^2-c)a_{33}].$$

Now we have

$$p_i(r) = \int_0^\infty tJ_0(tr)dt\int_0^\infty p_i(y)yJ_0(ty)dy$$

$$= \int_0^\infty t\psi_i(t)J_0(tr)dt, \quad i = 1, 2,$$

Where

$$\psi_i(t) = \int_0^\infty p_i(y)yJ_0(ty)dy.$$

Using the relations the boundary conditions reduce to

$$(as_1^2-1)e^{s_1th}A(t) + (as_2^2-1)e^{s_2th}B(t)$$

$$+(as_1^2-1)e^{-s_1th}C(t) + (as_2^2-1)e^{-s_2th}D(t) = 0,$$

$$(as_1^2-1)e^{-s_1th}A(t) + (as_2^2-1)e^{-s_2th}B(t)$$

$$+(as_1^2-1)e^{s_1th}C(t) + (as_2^2-1)e^{s_2th}D(t) = 0,$$

$$s_1(ds_1^2-c)e^{s_1th}A(t) + s_2(ds_2^2-c)e^{s_2th}B(t)$$

$$-s_1(ds_1^2-c)e^{-s_1th}C(t) - s_2(ds_2^2-c)e^{-s_2th}D(t) = -\frac{\psi_1(t)}{t^2},$$

$$s_1(ds_1^2-c)e^{-s_1th}A(t) + s_2(ds_2^2-c)e^{-s_2th}B(t)$$

$$-s_1(ds_1^2-c)e^{s_1th}C(t) - s_2(ds_2^2-c)e^{s_2th}D(t) = -\frac{\psi_2(t)}{t^2}.$$

NUMERICAL RESULTS AND DISCUSSIONS

We have taken the material of the plate as sapphire for which the values of elastic constants are as follows

$$A_{11} = 4.968 \times 10^{11} Nm^{-2},$$

$$A_{12} = 1.636 \times 10^{11} Nm^{-2},$$

$$A_{13} = 1.109 \times 10^{11} Nm^{-2},$$

$$A_{33} = 4.981 \times 10^{11} Nm^{-2},$$

$$A_{44} = 1.474 \times 10^{11} Nm^{-2}.$$

Also we take normal loads on the bounding planes in the following form

$$p_1(r) = \frac{k_1}{(k_1^2 + r^2)^{\frac{3}{2}}}, k_1 > 0 \text{ on } z = h$$

and

$$p_2(r) = \frac{k_2}{(k_2^2 + r^2)^{\frac{1}{2}}}, k_2 > 0 \text{ on } z = -h$$

so that we have

$$\psi_1(t) = e^{-k_1 t},$$

$$\psi_2(t) = \frac{e^{-k_2 t}}{t}$$

Gauss Quadrature in the interval (0,1] and Simpson's rule in the interval [1, ∞)have been used to evaluate the integrals on the right hand side of the expressions for the relevant stress components and displacement components.

In Fig.1 the radial stress oy is plotted against radial distance for z = 0,1,2,3 when two different loads pi(r) and p2(r) are acting on the planes z = 3 and z = -3, while Fig. 2 is for the identical loads. It is observed that in Fig 1 the magnitude of the stress oT increases as z-increases for particular value of r and approaches zero for large value of r and in Fig. 2 the magnitudes of σr in different planes almost coincide and are very small which is quite plausible. Fig. 3 and Fig.4 depict hoop stress ae against r for z = 0,1,2,3 for different and identical loads respectively. Here also the qualitative behaviour of the curves are the same as in Fig.1 and Fig.2.

In Fig.5 the normal stress σx is plotted against the radial distance r [for different loads acting on the bounding planes] and Fig.3b [for identical loads] for z = 0,1,2,3. Here the qualitative behaviour of the curves are different Gauss Quadrature in the interim (0,1] and Simpson's standard in the interim [1, ∞)have been utilized to assess the integrals on the correct hand side of the expressions for the significant stress components and displacement components.

In Fig.la the spiral stress oy is plotted against outspread separation for z = 0,1,2,3 when two unique loads pi(r) and p2(r) are following up on the planes z = 3 and z = - 3, while Fig.lb is for the indistinguishable loads. It is seen that in Fig.1 the extent of the stress σr increments as z-increments for specific value of r and methodologies zero for huge value of r and in Fig.lb the sizes of ar in various planes nearly correspond and are little which is very conceivable. Fig.2a and Fig.2b portray loop stress ae against r for z = 0,1,2,3 for various and indistinguishable loads separately. Here likewise the subjective conduct of the curves are equivalent to in Fig.1 and Fig.2.

In Fig.5 the normal stress σx is plotted against the outspread separation r [for various loads following up on the jumping planes] and Fig.6 [for indistinguishable loads] for z = 0,1,2,3. Here the subjective conduct of the curves is not quite the same as those of Fig.1 and Fig.3. The greatness of the stress diminishes as the value of z increments and is very nearly zero on the bouncing plane z = 3 and this happens in genuine circumstances. For indistinguishable loads the extent of normal stress is little and it Coincides for various planes. In Fig.7 [for diverse loads] shearing stress T_{TZ} is plotted against spiral separation and for indistinguishable loads extent of the stress is insignificantly little. It is seen from Fig.4 that the extent of the shearing stress T_{TZ} increments as z-diminishes for specific value of r and methodologies zero for huge value of T.

Fig.8 [for distinctive loads] depictsthe outspread displacement ur againstrforz = 1,2 and 3. It is seen from Fig that when r = 0, the extent of the displacement for various planes are zero and increments as r increments and for specific value of r, uT increments as z increments. For same burden the greatness of displacement is irrelevantly little. Fig.9 [different loads] portrays the normal displacement uz against r for z = 1,2 and 3. Here the greatness of displacement methodologies zero as r increments in various planes and for indistinguishable loads the extent of the displacement is extremely little .from those of Figures The magnitude of the stress decreases as the value of z increases and is almost zero on the bounding plane z = 3 and this occurs in real situations. For identical loads the magnitude of normal stress is small and it Coincides for different planes. In Fig.7 [for different loads] shearing stress T_{TZ} is plotted against radial distance and for identical loads magnitude of the stress is negligibly small. It is observed from Fig.7 that the magnitude of the shearing stress T_{TZ} increases as z-decreases for particular value of r and approaches zero for large value of r.

Fig.9 [for different loads] depicts the radial displacement ur against for z = 1,2 and 3. It is observed from Fig.5 that when r = 0, the magnitude of the displacement for different planes are zero and increases as r increases and for particular value ofr, uT increases as z increases. For same load the magnitude of displacement is negligibly small. Fig. [different loads] depicts the normal displacement uz against r for z = 1,2 and 3. Here the magnitude of displacement approaches zero as r increases in different planes and for identical loads [Fig.6b] the magnitude of the displacement is very small.

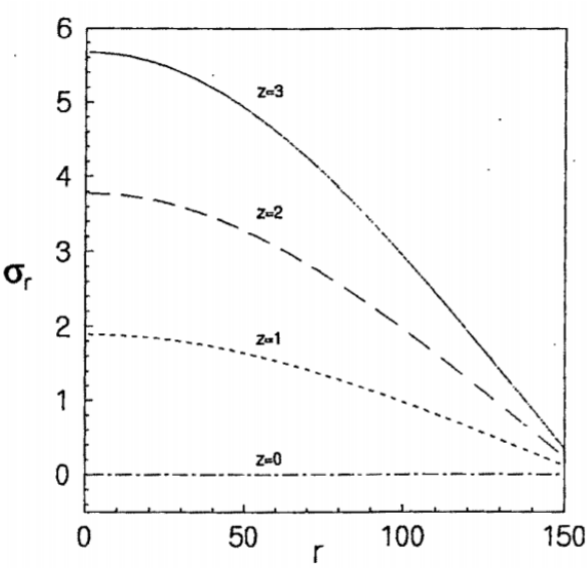

Figure 1: Radial stress vs r (different loads)

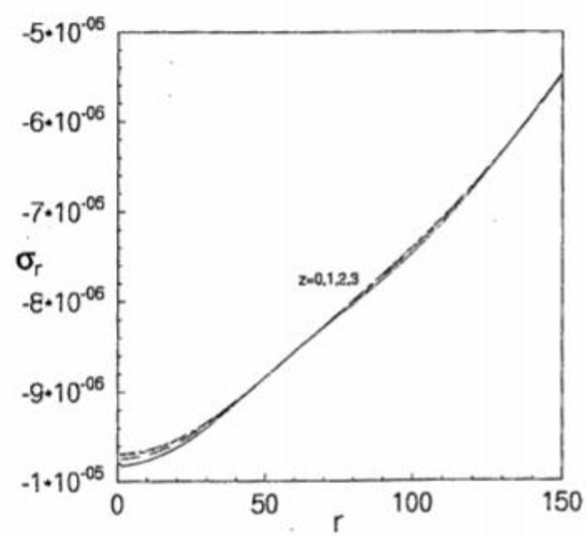

Figure 2: Radial stress vs r (identical loads)

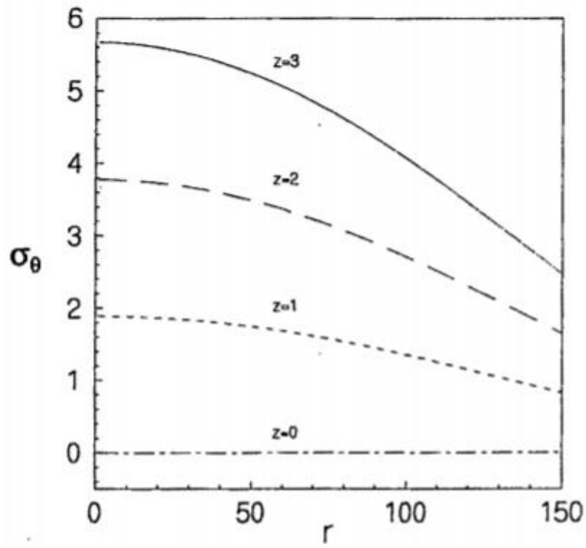

Figure 3: Hoop stress vs r (different loads)

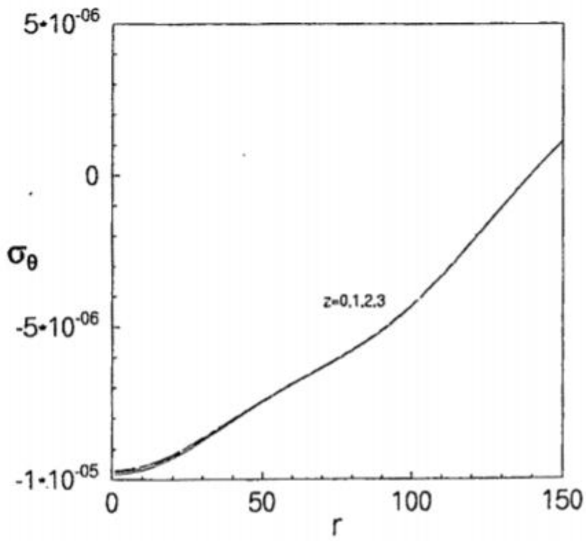

Figure 4: Hoop stress vs r (identical loads)

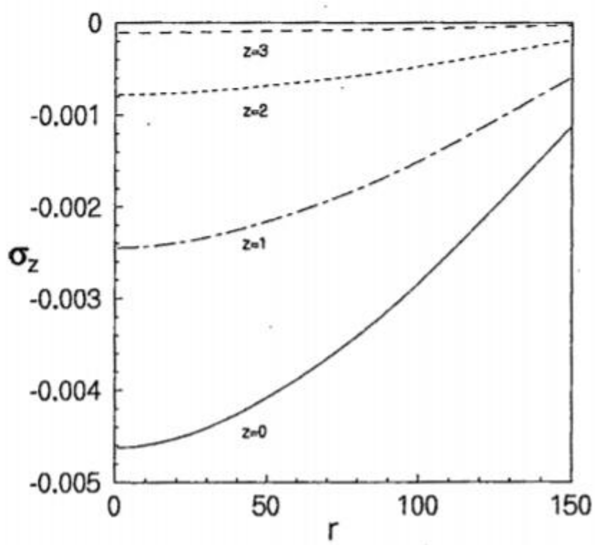

Figure 5 Normal stress vs r (different loads)

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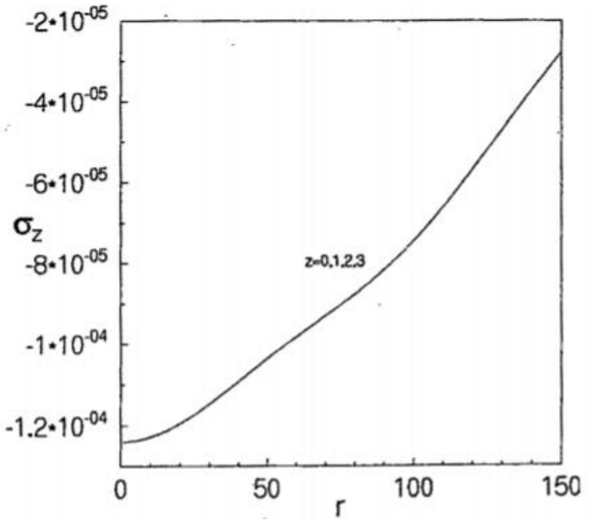

Figure 6: Normal stress vs r (identical loads)

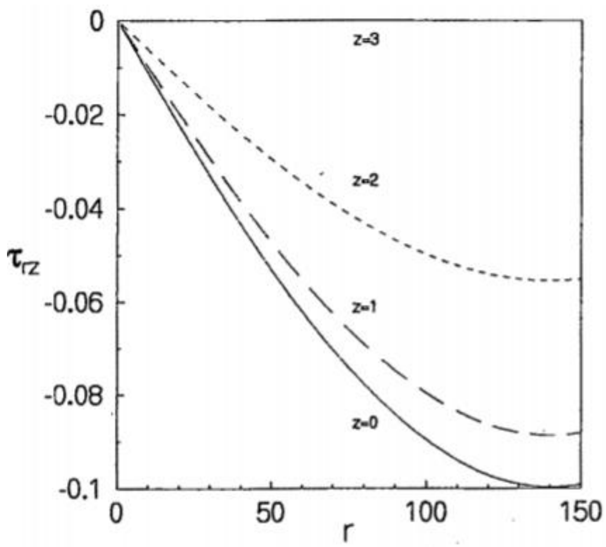

Figure 7: Shearing stress vs r (different loads)

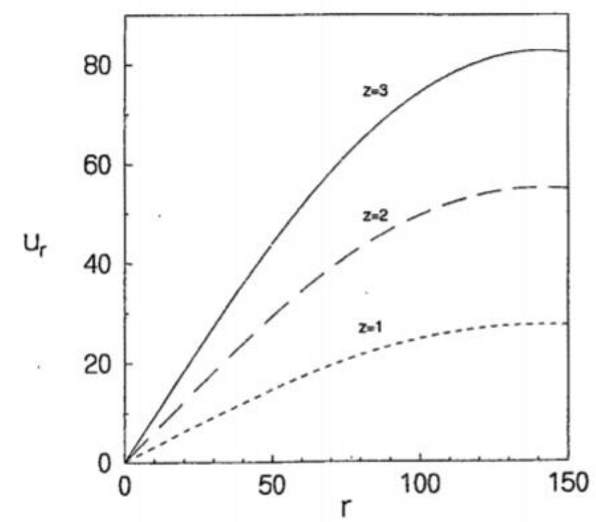

Figure 8: Radial displacement vs r (different loads)

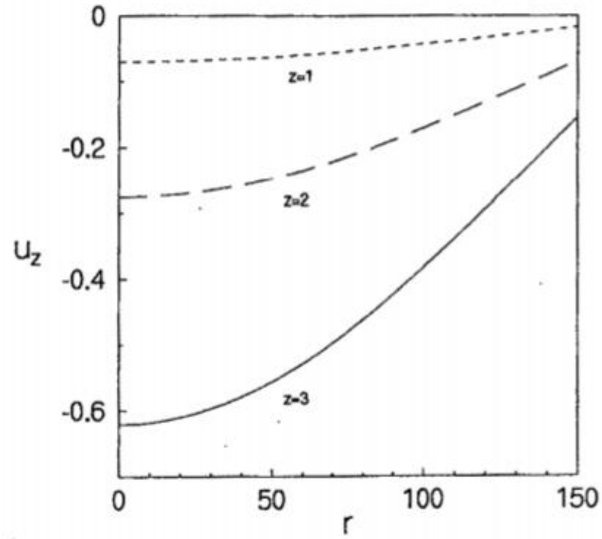

Figure 9: Normal displacement vs r (different loads)

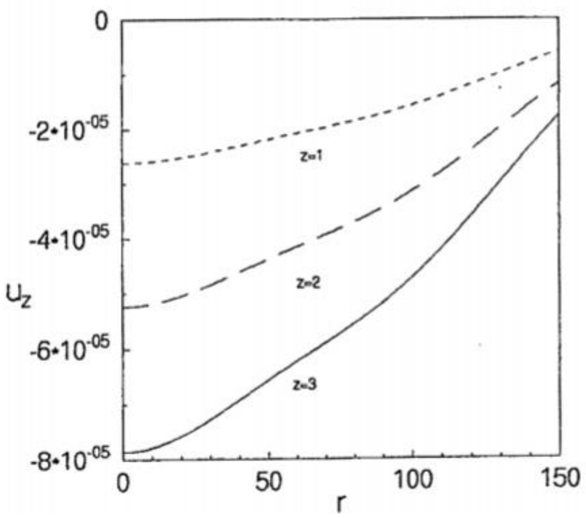

Figure 10: Normal displacement vs r (identical loads)

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