Journal of Advances and Scholarly Researches in Allied Education
Vol. 13, Issue No. 2, July-2017, ISSN 2230-7540

Generalized Fractional Integral Operators Introduced by Saigohaving the Gauss Hypergeometric Function as Kernel

Vijaya Bai*[1] Dr. Ashwini Kumar[2]

[1]Research Scholar

[2]Associate Professor, Faculty of Mathematics, OPJS University, Churu, Rajasthan

Abstract – Because of the general nature of the multivariable H-function and Saigo operators, a large number of new and referred to results can be inferred as specific instances of the fundamental results. In this way, first we acquire the image of the r-product of various H-functions in the Saigo operators. This image is likewise very broad in nature and is of intrigue and significance independent from anyone else. Next, we get the image of r-product of Bessel function of the principal kind in Saigo operators.

----------------------------------x----------------------------------

INTRODUCTION

The Multivariable H-Function

The multivariable H-function effectively given in chapter 1 is characterized and spoken to in the following manner [139, pp.251-252, Eqs.(C.1)-(C.3)]:

$$H[z_1,\ldots z_r] = H^{0,N:M_1,N_1;\ldots;M_r,N_r}_{P,Q:P_1,Q_1;\ldots;P_r,Q_r}$$

$$\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} \left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,P} : \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P_r} \\ \left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,Q} : \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q_r} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r} \psi(\xi_1,\ldots,\xi_r)\prod_{i=1}^{r}\left\{\phi_i(\xi_i)\,z_i^{\xi_i}\right\} d\xi_1\ldots d\xi_r \qquad (2.1.1)$$

where $i=\sqrt{(-1)}$ and

$$\psi(\xi_1,\ldots,\xi_r) = \frac{\prod_{j=1}^{N}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)}{\prod_{j=N+1}^{P}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=1}^{Q}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)} \qquad (2.1.2)$$

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\xi_i\right)\prod_{j=1}^{N_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i\right)}{\prod_{j=M_i+1}^{Q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}\xi_i\right)\prod_{j=N_i+1}^{P_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)}$$

$$\forall\, i\in(1,\ldots,r) \qquad (2.1.3)$$

The nature of shapes $L_1,\ldots,L_r$, different arrangements of convergence conditions of the integral given by (2.1.1), the extraordinary instances of this function and its different properties have been given in chapter 1. We should accept all through the present work that the suitable conditions of presence of this function specified in chapter 1 are additionally fulfilled.

FRACTIONAL INTEGRAL OPERATORS

The generalized fractional integral operators presented by Saigo [108] will be characterized and spoken to in the following way:

$$\left(I_{0+}^{\alpha,\beta,\eta} f\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\,{}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x}\right) f(t)\,dt,\ (x>0) \qquad (2.1.7)$$

and

$$\left(I_{-}^{\alpha,\beta,\eta} f\right)(x) = \frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta}\,{}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t}\right) f(t)\,dt,\ (x>0) \qquad (2.1.8)$$

where $\alpha,\beta,\eta\in C,\ Re(\alpha)>0$

At the point when $\beta=-\alpha$, the above operators (2.1.7) and (2.1.8) diminish to the following established Riemann-Liouville fractional integral operators [111, p.94, Eqs.(5.1),(5.3)].

$$\left(I_{0+}^{\alpha,-\alpha,\eta} f\right)(x)=\left(I_{0+}^{\alpha} f\right)(x)=\frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)\,dt, \quad (x>0)$$

(2.1.9)

$$\left(I_{-}^{\alpha,-\alpha,\eta} f\right)(x)=\left(I_{-}^{\alpha} f\right)(x)=\frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)\,dt, \quad (x>0)$$

(2.1.10)

Once more, if $\beta=0$, equations (2.1.7) and (2.1.8) reduce to the following Erdélyi – Kober fractional integral operators [111, p.322, Eqs.(18.5),(18.6)].

$$\left(I_{0+}^{\alpha,0,\eta} f\right)(x)=\left(I_{\eta,\alpha}^{+} f\right)(x)=\frac{x^{-\alpha-\eta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} t^{\eta} f(t)\,dt, \quad (x>0) \qquad (2.1.11)$$

$$\left(I_{-}^{\alpha,0,\eta} f\right)(x)=\left(K_{\eta,\alpha}^{-} f\right)(x)=\frac{x^{\eta}}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\eta} f(t)\,dt, \ (x>0) \qquad (2.1.12)$$

REQUIRED RESULTS

The following known formulae [67, p.871, Eq.(15) to (18)]; p.872, Eq.(21) to (24)] will be required to build up our fundamental discoveries.

Formula 1:

$$(I_{0+}^{\alpha,\beta,\eta}\, t^{\sigma-1})(x)=\frac{\Gamma(\sigma)\Gamma(\sigma+\eta-\beta)}{\Gamma(\sigma-\beta)\Gamma(\sigma+\alpha+\eta)}x^{\sigma-\beta-1}, \quad (x>0) \qquad (2.1.13)$$

where $\alpha,\beta,\eta\in C$, $Re(\alpha)>0$, $Re(\sigma)>\max\{0,Re(\beta-\eta)\}$

In particular, if $\beta=-\alpha$ and $\beta=0$ in equation (2.1.13), we have

$$(I_{0+}^{\alpha}\, t^{\sigma-1})(x)=\frac{\Gamma(\sigma)}{\Gamma(\sigma+\alpha)}x^{\sigma+\alpha-1}, \quad Re(\alpha)>0, Re(\sigma)>0$$

(2.1.14)

$$(I_{\eta,\alpha}^{+}\, t^{\sigma-1})(x)=\frac{\Gamma(\sigma+\eta)}{\Gamma(\sigma+\alpha+\eta)}x^{\sigma-1}, \ Re(\alpha)>0, Re(\sigma)>-Re(\eta)$$

(2.1.15)

Formula 2:

$$(I_{-}^{\alpha,\beta,\eta}\, t^{\sigma-1})(x)=\frac{\Gamma(\beta-\sigma+1)\Gamma(\eta-\sigma+1)}{\Gamma(1-\sigma)\Gamma(\alpha+\beta+\eta-\sigma+1)}x^{\sigma-\beta-1} \qquad (2.1.16)$$

where $\alpha,\beta,\eta\in C$, $Re(\alpha)>0$ and $Re(\sigma)<1+\min[Re(\beta),Re(\eta)]$

In particular, if $\beta=-\alpha$ and $\beta=0$ in equation (2.1.16), we have

$$(I_{-}^{\alpha}\, t^{\sigma-1})(x)=\frac{\Gamma(1-\alpha-\sigma)}{\Gamma(1-\sigma)}x^{\sigma+\alpha-1}, \quad 0<Re(\alpha)<1-Re(\sigma) \qquad (2.1.17)$$

$$(K_{\eta,\alpha}^{-}\, t^{\sigma-1})(x)=\frac{\Gamma(\eta-\sigma+1)}{\Gamma(\alpha+\eta-\sigma+1)}x^{\sigma-1}, \quad Re(\sigma)<1+Re(\eta) \qquad (2.1.18)$$

MAIN RESULTS

Result 1

$$\left\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\sigma-1} H_{P,Q:P_1,Q_1;\ldots;P_r,Q_r}^{0,N:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1 t^{\rho_1} \\ \vdots \\ z_r t^{\rho_r}\end{matrix}\right]\right)\right\}(x)$$

$$= x^{\sigma-\beta-1} H_{P+2,Q+2:P_1,Q_1;\ldots;P_r,Q_r}^{0,N+2:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r}\end{matrix}\middle| \begin{matrix} A^*:C^* \\ B^*:D^*\end{matrix}\right]$$

(2.2.1)

where

$$A^*=(1-\sigma;\rho_1,\ldots,\rho_r),(1-\sigma+\beta-\eta;\rho_1,\ldots,\rho_r),\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,P}$$

$$B^*=(1-\sigma+\beta;\rho_1,\ldots,\rho_r),(1-\sigma-\alpha-\eta;\rho_1,\ldots,\rho_r),\left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,Q}$$

$$C^*=\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,P_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,P_r}$$

$$D^*=\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,Q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,Q_r}$$

The adequate circumstances of validity of (2.2.1) are

(i) $\alpha,\beta,\eta,\sigma,z_i\in C$ and $\rho_i>0\ \forall i\in\{1,2,\ldots,r\}$

(ii) $|\arg z_i|<\frac{1}{2}\Omega_i\pi$ and $\Omega_i>0$

where

$$\Omega_i\equiv-\sum_{j=n+1}^{P}\alpha_j^{(i)}-\sum_{j=1}^{Q}\beta_j^{(i)}+\sum_{j=1}^{N_i}\gamma_j^{(i)}-\sum_{j=N_i+1}^{P_i}\gamma_j^{(i)}+\sum_{j=1}^{M_i}\delta_j^{(i)}-\sum_{j=M_i+1}^{Q_i}\delta_j^{(i)}>0$$

$\forall\ i\in\{1,\ldots,r\}$

(iii) $Re(\alpha)>0$ and

$$Re(\sigma)+\sum_{i=1}^{r}\rho_i\min_{1\le j\le M_i} Re\left(d_j^{(i)}/\delta_j^{(i)}\right)>\max[0,Re(\beta-\eta)]$$

Proof: To demonstrate result 1, we first express the multivariable H-function happening in left – hand side of (2.2.1) as far as different Mellin-Barnes shape integral with the assistance of equation (2.1.1) and trade the request of coordination (which is allowable under the conditions expressed), it takes the following structure (say I_1) after a little rearrangements:

$$I_1 = \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\psi(\xi_1,\ldots,\xi_r)\prod_{i=1}^{r}\left\{\phi_i(\xi_i)z_i^{\xi_i}\right\}d\xi_{1,\ldots,}d\xi_r$$

$$\times\left(I_{0+}^{\alpha,\beta,\eta}t^{\sigma+\rho_1\xi_1+\ldots.\rho_r\xi_r-1}\right)(x) \quad (2.2.2)$$

Presently on applying equation (2.1.13) to assess the I administrator happening in the above result lastly reinterpreting the different Mellin-Barnes form integral in this way got as far as the H-function of N+2 factors, we land at the correct hand side of (2.2.1) after a little improvement.

On the off chance that we put $\beta=-\alpha$ in Result 1, we touch base at the following new and fascinating formula concerning Riemann – Liouville fractional integral operators.

Result 2

$$\left\{I_{0+}^{\alpha}\left(t^{\sigma-1}H_{P,Q:P_1,Q_1;\ldots;P_r,Q_r}^{0,N:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1t^{\rho_1} \\ \vdots \\ z_rt^{\rho_r}\end{matrix}\right]\right)\right\}(x)$$

$$= x^{\sigma+\alpha-1}H_{P+1,Q+1:P_1,Q_1;\ldots;P_r,Q_r}^{0,N+1:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1x^{\rho_1} \\ \vdots \\ z_rx^{\rho_r}\end{matrix}\left|\begin{matrix} A_1^*:C^* \\ B_1^*:D^*\end{matrix}\right.\right]$$

(2.2.3)

where

$$A_1^* = (1-\sigma;\rho_1,\ldots,\rho_r),\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,P}$$

$$B_1^* = (1-\sigma-\alpha;\rho_1,\ldots,\rho_r),\left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,Q}$$

C^* and D^*, are same as given in (2.2.1) and the conditions of presence of the above result take after effortlessly with the assistance of Result 1.

Once more, in the event that we put $\beta=0$ in Result 1, we get the following result which is additionally accepted to be new and relates to Erdélyi – Kober fractional integral operators.

Result 3

$$\left\{I_{\eta,\alpha}^{+}\left(t^{\sigma-1}H_{P,Q:P_1,Q_1;\ldots;P_r,Q_r}^{0,N:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1t^{\rho_1} \\ \vdots \\ z_rt^{\rho_r}\end{matrix}\right]\right)\right\}(x)$$

$$= x^{\sigma-1}H_{P+1,Q+1:P_1,Q_1;\ldots;P_r,Q_r}^{0,N+1:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1x^{\rho_1} \\ \vdots \\ z_rx^{\rho_r}\end{matrix}\left|\begin{matrix} A_2^*:C^* \\ B_2^*:D^*\end{matrix}\right.\right]$$

(2.2.4)

where

$$A_2^* = (1-\sigma-\eta;\rho_1,\ldots,\rho_r),\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,P}$$

$$B_2^* = (1-\sigma-\alpha-\eta;\rho_1,\ldots,\rho_r),\left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,Q}$$

C^* and D^*, are same as given in (2.2.1) and the conditions of legitimacy of the above result can be effortlessly acquired from the presence conditions given with Result 1

Result 4

$$\left\{I_{-}^{\alpha,\beta,\eta}\left(t^{\sigma-1}H_{P,Q:P_1,Q_1;\ldots;P_r,Q_r}^{0,N:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1t^{-\rho_1} \\ \vdots \\ z_rt^{-\rho_r}\end{matrix}\right]\right)\right\}(x)$$

$$= x^{\sigma-\beta-1}H_{P+2,Q+2:P_1,Q_1;\ldots;P_r,Q_r}^{0,N+2:M_1,N_1;\ldots;M_r,N_r}\left[\begin{matrix} z_1x^{-\rho_1} \\ \vdots \\ z_rx^{-\rho_r}\end{matrix}\left|\begin{matrix} A^{**}:C^* \\ B^{**}:D^*\end{matrix}\right.\right]$$

(2.2.5)

where

$$A^{**} = (\sigma-\beta;\rho_1,\ldots,\rho_r),(\sigma-\eta;\rho_1,\ldots,\rho_r),\left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,P}$$

$$B^{**} = (\sigma;\rho_1,\ldots,\rho_r),(\sigma-\alpha-\beta-\eta;\rho_1,\ldots,\rho_r),\left(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,Q}$$

C^* and D^*, are same as characterized in (2.2.1) and gave that the following conditions are fulfilled

(i) $\mathrm{Re}(\alpha)>0$ and

$$\mathrm{Re}(\sigma)-\sum_{i=1}^{r}\rho_i \min_{1\le j\le M_i} \mathrm{Re}\left(d_j^{(i)}\Big/\delta_j^{(i)}\right)<1+\min[\mathrm{Re}(\beta),\mathrm{Re}(\eta)]$$

(ii) and the conditions (I) and (ii) given with Result 1 are additionally fulfilled.

The verification of Result 4 can be produced on the lines like those given with confirmation of Result 1 with the assistance of equation (2.1.16).

If we place $\beta=-\alpha$ and $\beta=0$ in Result 4, we might effectively touch base at the relating results concerning Riemann – Liouville and Erdélyi – Kober fractional integral operators individually.

SPECIAL CASES OF RESULT 1

(I) If we take N=P=Q=0 in equation (2.2.1), we effortlessly acquire the following result including r-product of various H-functions [139, p.10, Eq.(2.1.1)] in the Saigo operators

$$\left\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\sigma-1}\prod_{i=1}^{r}H_{P_i,Q_i}^{M_i,N_i}\left[z_i t^{\rho_i}\right]\right)\right\}(x)$$

$$=x^{\sigma-\beta-1}H^{0,2:M_1,N_1;\dots;M_r,N_r}_{2,2:P_1,Q_1;\dots;P_r,Q_r}\left[\begin{array}{c|c} z_1x^{\rho_1} & A_3^*:C^* \\ \vdots & \\ z_rx^{\rho_r} & B_3^*:D^*\end{array}\right] \quad (2.3.1)$$

where

$$A_3^*=(1-\sigma;\rho_1,..,\rho_r),(1-\sigma+\beta-\eta;\rho_1,...,\rho_r)$$

$$B_3^*=(1-\sigma+\beta;\rho_1,...,\rho_r),(1-\sigma-\alpha-\eta;\rho_1,..,\rho_r)$$

C^* and D^*, are same as specified in (2.2.1) and the conditions of legitimacy of the above result can be effortlessly determined with the assistance of Result 1.

The above image is likewise very broad in nature and is of intrigue and significance without anyone else's input. Thus,if we lessen r-product of various H-functions happening in the left hand side of (2.3.1) to the r-product of Bessel functions of the principal kind [139, p.18, Eq.(2.6.5)], we touch base at the following result after a little disentanglement

$$\left(I_{0+}^{\alpha,\beta,\eta}\left[t^{\sigma-1}\prod_{i=1}^{r}J_{\upsilon_i}\left(a_i t^{\rho_i}\right)\right]\right)(x)$$

$$=x^{\sigma-\beta-1}\left(\prod_{i=1}^{r}\left(\frac{a_i x^{\rho_i}}{2}\right)^{\upsilon_i}\right)H^{0,2:1,0;\dots;1,0}_{2,2:0,2;\dots;0,2}\left[\begin{array}{c|c} \frac{a_1^2x^{2\rho_1}}{4} & A_4^*:- \\ \vdots & \\ \frac{a_r^2x^{2\rho_r}}{4} & B_4^*:D_4^*\end{array}\right] \quad (2.3.2)$$

where

$$A_4^*=\left(1-\sigma-\sum_{i=1}^{r}\rho_i\upsilon_i;2\rho_1,...,2\rho_r\right),\left(1-\sigma+\beta-\eta-\sum_{i=1}^{r}\rho_i\upsilon_i;2\rho_1,...,2\rho_r\right)$$

$$B_4^*=\left(1-\sigma+\beta-\sum_{i=1}^{r}\rho_i\upsilon_i;2\rho_1,...,2\rho_r\right),\left(1-\sigma-\alpha-\eta-\sum_{i=1}^{r}\rho_i\upsilon_i;2\rho_1,...,2\rho_r\right)$$

$$D_4^*=(0,1),(-\upsilon_1,1);...;(0,1),(-\upsilon_r,1)$$

The conditions of legitimacy of the above result take after specifically from those given with (2.2.1).

Presently, diminishing the H-function of a few factors happening in the correct hand side of (2.3.2) to generalized Lauricella function [135], we touch base at a current vital result got by Kilbas and Sebastian [69, p.164, Eq.(2.4)].

(ii)On lessening the multivariable H-function to the product of two Fox H-function and after that diminish one H-function to the exponential function by taking $\rho_1=1$ in the result given by (2.3.1), we get the following result on rearrangements.

$$\left\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\sigma-1}e^{-z_1t}H^{M_2,N_2}_{P_2,Q_2}\left[z_2t^{\rho_2}\left|\begin{array}{c}(c_j,\gamma_j)_{1,P_2}\\(d_j,\delta_j)_{1,Q_2}\end{array}\right.\right]\right)\right\}(x)$$

$$=x^{\sigma-\beta-1}H^{0,2:1,0;M_2,N_2}_{2,2:0,1;P_2,Q_2}\left[\begin{array}{c|c} z_1x & (1-\sigma;1,\rho_2),(1-\sigma+\beta-\eta;1,\rho_2):-\ ;(c_j,\gamma_j)_{1,P_2} \\ z_2x^{\rho_2} & (1-\sigma+\beta;1,\rho_2),(1-\sigma-\alpha-\eta;1,\rho_2):(0,1);(d_j,\delta_j)_{1,Q_2}\end{array}\right] \quad (2.3.3)$$

The conditions of validity of the above result can be effortlessly gotten from those of (2.2.1).

Advance on letting $z_1\to 0$ in the above equation, it takes the following structure:

$$\left\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\sigma-1}H^{M_2,N_2}_{P_2,Q_2}\left[z_2t^{\rho_2}\left|\begin{array}{c}(c_j,\gamma_j)_{1,P_2}\\(d_j,\delta_j)_{1,Q_2}\end{array}\right.\right]\right)\right\}(x)$$

$$= x^{\sigma-\beta-1} H^{M_2,N_2+2}_{P_2+2,Q_2+2}\left[z_2 x^{\rho_2}\left|\begin{matrix}(c_j,\gamma_j)_{1,N_2},(1-\sigma,\rho_2),\\ (d_j,\delta_j)_{1,Q_2},(1-\sigma+\beta,\rho_2),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-\sigma+\beta-\eta,\rho_2),(c_j,\gamma_j)_{N_2+1,P_2}\\ (1-\sigma-\alpha-\eta,\rho_2)\end{matrix}\right] \quad (2.3.4)$$

The conditions of legitimacy of the above result take after effortlessly from the conditions given with Result 1.

In the event that we put $\beta=-\alpha$ and make appropriate alterations in the parameters, we touch base at a known result recorded in the book by Kilbas and Saigo [66, p.52, Eq.(2.7.9)].

Further, in the event that we put $\beta=0$ in the equation (2.3.4), we might effectively touch base at the relating result concerning Erdélyi – Kober fractional integral operators separately.

(iii) In (2.3.4), in the event that we diminish the H-function to the generalized Wright hypergeometricfunction [139, p.19, Eq.(2.6.11)], we acquire the following result on disentanglement

$$\left\{I^{\alpha,\beta,\eta}_{0+}\left(t^{\sigma-1}\ {}_P\Psi_Q\left[z_2 t^{\rho_2}\left|\begin{matrix}(c_j,\gamma_j)_{1,P}\\ (d_j,\delta_j)_{1,Q}\end{matrix}\right.\right]\right)\right\}(x)$$

$$= x^{\sigma-\beta-1}\ {}_{P+2}\Psi_{Q+2}\left[z_2 x^{\rho_2}\left|\begin{matrix}(c_j,\gamma_j)_{1,P}, & (\sigma,\rho_2), & (\sigma+\eta-\beta,\rho_2)\\ (d_j,\delta_j)_{1,Q}, & (\sigma-\beta,\rho_2), & (\sigma+\alpha+\eta,\rho_2)\end{matrix}\right.\right]$$

(2.3.5)

The conditions of legitimacy of (2.3.5) can be effectively gotten from those of (2.2.1).

If we put $\beta=-\alpha$ in the above result, we get a known critical result given by Kilbas [65, p.117, Eq.(11)].

Again if we put $\beta=0$ in the equation (2.3.5), it reduces to

$$\left\{I^{+}_{\eta,\alpha}\left(t^{\sigma-1}\ {}_P\Psi_Q\left[z_2 t^{\rho_2}\left|\begin{matrix}(c_j,\gamma_j)_{1,P}\\ (d_j,\delta_j)_{1,Q}\end{matrix}\right.\right]\right)\right\}(x)$$

$$= x^{\sigma-1}\ {}_{P+1}\Psi_{Q+1}\left[z_2 x^{\rho_2}\left|\begin{matrix}(c_j,\gamma_j)_{1,P}, & (\sigma+\eta,\rho_2)\\ (d_j,\delta_j)_{1,Q}, & (\sigma+\alpha+\eta,\rho_2)\end{matrix}\right.\right] \quad (2.3.6)$$

The conditions of legitimacy of the above result can be effectively determined with the assistance of Result 1.

(iv) Further on taking $z_2=-1, \rho_2=1$ in the equation (2.3.4) and reducing the H-function happening in that to generalized Mittag – Leffler function [70, p.67, Eq.(1.12.65)], we effectively get the following intriguing result which is likewise accepted to be new.

$$\left\{I^{\alpha,\beta,\eta}_{0+}\left(t^{\sigma-1} E^{\upsilon}_{u,v}(t)\right)\right\}(x)$$

$$= \frac{x^{\sigma-\beta-1}}{\Gamma(\upsilon)} H^{1,3}_{3,4}\left[-x\left|\begin{matrix}(1-\upsilon,1), & (1-\sigma,1), & (1-\sigma+\beta-\eta,1), & \\ (0,1), & (1-\sigma+\beta,1), & (1-\sigma-\alpha-\eta,1), & (1-v,u)\end{matrix}\right.\right]$$

(2.3.7)

The conditions of validity of the above result follow directly from those given with (2.2.1).

If we put $\beta=-\alpha$ in the above result (2.3.7), we get a known result due to Saxena et al. [120, p.168, Eq.(2.1)].

If we put $\beta=0$ in the result given by (2.3.7), we arrive at the following result after a little simplification

$$\left\{I^{+}_{\eta,\alpha}\left(t^{\sigma-1} E^{\upsilon}_{u,v}(t)\right)\right\}(x)$$

$$= \frac{x^{\sigma-1}}{\Gamma(\upsilon)} H^{1,2}_{2,3}\left[-x\left|\begin{matrix}(1-\upsilon,1), & (1-\sigma-\eta,1) & \\ (0,1), & (1-\sigma-\alpha-\eta,1), & (1-v,u)\end{matrix}\right.\right] (2.3.8)$$

The conditions of legitimacy of (2.3.8) can be effectively gotten from those of (2.2.1).

(v) If we diminish the H – function to the Whittaker function [139, p.18, Eq.(2.6.7)] in the equation (2.3.4) and take $z_2=\rho_2=1$, we get the following fascinating result which is additionally accepted to be new.

$$\left\{I^{\alpha,\beta,\eta}_{0+}\left(t^{\sigma-1} e^{-t/2} W_{a,b}(t)\right)\right\}(x)$$

$$= x^{\sigma-\beta-1} H^{2,2}_{3,4}\left[x\left|\begin{matrix}(1-\sigma,1), & (1-\sigma+\beta-\eta,1), & (1-a,1)\\ \left(\frac{1}{2}\pm b,1\right), & (1-\sigma+\beta,1), & (1-\sigma-\alpha-\eta,1)\end{matrix}\right.\right] (2.3.9)$$

The conditions of validity of (2.3.9) can be easily obtained from those of (2.2.1).

If we take $z_2 = \frac{1}{4}, \rho_2 = 2$ and reduce the H-function to the Bessel function of first kind in the equation (2.3.4), we get known results established by Kilbas and Sebastian [67, p.873, Eq.(25) to (29)].

Further, on the off chance that we decrease the multivariable H-function happening in the left hand side of (2.2.1) to the product of two natural H-functions of one variable, we land at the current image got by Gupta et al.[41, p.206, Eq.(21)].

A number of a few unc

REFERENCES

Agrawal R. (2017). *A Study of Unified Special Functions and Generalized Fractional Integrals with Applications*, Ph.D. Thesis, Approved by MNIT, Jaipur.

Agrawal, R.P. (1965). An extension of Meijer's G-function, *Proc. Nat. Inst. Sci. India,Sect. A***31**, pp. 536-546.

Appell P. (1880). Sur les sérieshypergéométriques de deux variables, et surdesequations, differentielleslinearies aux deriveespartielles, *C.R. Acad. Sci. Paris* **90**, pp. 296-298.

Appell P. and Kampedé Fériet J. (1926). *Functions Hypergéométriques et Hypersphériques: Polynômesd'Hermite*, Gauthier-Villars, Paris.

Barnes E.W. (1901). The theory of the double gamma function, *Philos. Trans. Roy. Soc. (A)* **196**, pp. 265-387.

Bateman H. (1996). Two systems of polynomials for the solution of Laplace's integral equation, *Duke Math. J.***2**, pp. 569-577.

Bedient P.E. (1959). *Polynomials Related to Appell Functions of Two variables***,** Ph.D. Thesis, University of Michigan.

Corresponding Author

Vijaya Bai*

Research Scholar

E-Mail –