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A Study on Statistical Convergence of Sequences

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Abstract – Studies on sequence spaces were additionally reached out through Sum capacity Theory. The entirety capacity hypothesis began from the endeavors made by the mathematicians as far as possible to the disparate sequences, on taking its change. O. Latrines were the main individual to consider the aggregate capacity strategies as a class of changes of complex sequences by complex interminable networks. It was trailed by the works because of I. Schur, S. Mazur, W. Orlicz, K'Knopp, G. M. Petersen, S. Banach, G. Kothe and 0. Toeplitz, are a couple to be named. The takes a shot at paranormed sequence spaces was started by H. Nakano and S. Simons. It was additionally concentrated by 1. J. Maddox, C. G. Lascarides, S. Nanda, D. Rath , G. Das , Z.U.Ahmed, B. Kuttner and numerous others.

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INTRODUCTION

A sequence space is a straight space of sequences with components in another direct space. The investigations of direct change on sequence spaces are called sum ability. The soonest thought of total capacity hypothesis were maybe contained in a letter composed by Leibnitz to C. Wolf (1713) in which the total of the oscillatory arrangement I - I + I - I+ I - as given by Leibnitz was \. hey 1880, Fresenius presented the technique for entirety capacity by number juggling mean. Later on this strategy was summed up as the (C, k) technique for total capacity.

First considered entirety capacity techniques as a class of change of complex sequences by complex unending lattices. In this manner it was concentrated by numerous remarkable mathematicians like Kojima, Steinhaus, Schur, Mazur, OrHcz, Knopp, Pali, Agnew, Brudno, Cooke, Iyer, Petersen and numerous others. With the development of useful examination, sequence spaces were concentrated with more prominent knowledge and inspiration.

Most punctual utilizations of utilitarian investigation to sum ability were made by Banach, Hahn, Mazur, Kothe and Toeplitz. It was trailed by numerous others like Lorentz, Zeller, Das, Wilanski, Russell, Sargent, Rhoades, Maddox, etc. In 1950, Robinson started the investigation of sum ability by unending frameworks of straight administrators on normed direct spaces which empowered the laborers on sum ability to broaden kick the bucket results on sum ability hypotheses and issues on double sequence spaces to this overall setting.

DEFINITIONS AND NOTATIONS

In this section we list some standard notations and concepts those will be used throughout the thesis .Throughout *N, R* and C denote the sets of **natural, real** and **complex** numbers respectively.

x = *(x*t) denote the sequence whose *k* -th term is *xk .*

Throughout *w, c,* c0, *£m* denote the spaces of **all, convergent, null** and **bounded** sequences of complex terms respectively.

The zero element of a normed linear space (n.l.s.) is denoted by *Q* . A complete n.l.s. is called as a Banach Space.

It is well known that *£* is a Banach space under the norm

$$\|x\| = \sup_k | x_k | ,$$

Called the sup-norm or uniform norm.

The spaces *c* and c0 are complete subspaces of *£m.*

l p (0 < / ? < o o) denotes the space of all complex sequences such that *xk \p* <oo, called as the space of *p* - **absolutely summable sequences.**

The space for $p > 1$ is complete under the norm defined by

$$\|x\| = \left(\sum_k | x_k |^p \right)^{\frac{1}{p}}.$$

For 0 <*p* < *l, l p* is a complete - nor med space, *p* - nor med by

$$\|x\| = \sum_{k=1}^{\infty} | x_k |^p .$$

If *X* is a linear space and *g:X~> R* is such that

***g(x)* >** 0 ;

(*i*) $g(x) \geq 0$;

(*ii*) $x = \theta \Rightarrow g(x) = 0$;

(*iii*) $g(x+y) \leq g(x) + g(y)$;

(*iv*) $g(x) = g(-x)$;

(*v*) $g(\lambda_n x_n - \lambda x) \to 0$, as $n \to \infty$, whenever $\lambda_n \to \lambda$ and $x_n \to x$, for scalars λ_n, λ and vectors x, x_n(for all $n \in N$) $\in X$,

then g is said to be a paranorm on *X* and (*X, g*) is said to be a paranormed sequence space.

A paranorm *g* which satisfies *g(x)* - 0 => x = *0* is called as a total paranorm. Let *p* - *{pk)* be a sequence of strictly positive numbers. Then the spaces *c*0, *c,*

£m, t are generalized as follows :

$$c_0(p) = \{ (x_k) \in w : |x_k|^{p_k} \to 0, \text{ as } k \to \infty \},$$

$$c(p) = \{ (x_k) \in w : |x_k - L|^{p_k} \to 0, \text{ as } k \to \infty, \text{ for some } L \},$$

$$\ell_\infty(p) = \{ (x_k) \in w : \sup_k |x_k|^{p_k} < \infty \},$$

$$\ell(p) = \{ (x_k) \in w : \sum_k | x_k |^{p_k} < \infty \}.$$

Let *H*= sup *pk* < oo. Then *c0* (/?) and c(/?) are paranormal by g(x) = sup *Xk* I where *M* -m ax (I, *H).*

The space *I m{p*) is paranormal by *g* if inf *pk*> 0 (see for instance Maddox [46]).

The space £(/?) is paranormal by

$$f(x) = \left(\sum_k |x_k|^{p_k} \right)^{\frac{1}{M}} .$$

Let *E* and *F* be two sequence spaces. Then by *(E , F)* we denote the class of all infinite matrices *A* = *(*ank) of complex entries, which transforms sequences in *E* into F defined by/fit = *(Anx),* where *Anx* - *'YJcinkxk .*

For any matrix *A* - *(*ank), the convergence field of *A* is the set

$$c_A = \{ x \in w : Ax \in c \}.$$

A matrix *A* = *(ank)* is said to be

Conservative, if *c* c *CA* ;

Regular [also written as *A* e (c, c; *P*)], if *A* is conservative and

lim *A nx* ~ lim *x k* for each *x* = *{xk)* e c ;

n-> oo *k->«>*

$$\lim_{n\to\infty} A_n x = \lim_{k\to\infty} x_k \quad \text{for each } x = (x_k) \in c ;$$

The zero operator on a n.l.s. is denoted by *6* .

The identity operator on a *n.l.s.* is denoted by *I .*

*A ~(A*nk) denotes a matrix in which *Ank* are linear operators from a Banach space *X* into a Banach space *Y. (A*llk) defines a transformation between two sequence spaces, co defined by

$$\text{defined by } Ax = (A_n x), \text{ where } \quad A_n x = \sum_{k=1}^{\infty} A_{nk} x_k$$

For two normed linear spaces *X* and *Y, B(X, Y)* denotes the set of all bounded linear mappings from *X* to *Y.*

If *(Tk*) is a sequence of bounded linear operators on *X* into *Y, U(X)* denotes the closed unit disc in *X,* the **group norm** || *(Tk*) || is defined as

$$\sup_{\substack{n \in N \\ x_k \in U(X)}} \left\| \sum_{k=1}^{n} T_k x_k \right\| .$$

Let *s0* denote the set of all sequences in *c0* with non-zero terms.

Any subsequence (x„t) of *(xk)* can be represented as a regular matrix transformation *A* - *{a*nk) times *(xk)* by letting *ak* =1 and *a* = 0 otherwise.

For any subset *A* of *N, %A* denotes the characteristic function of *A,* where *X A* >S defined by *X A (k)* = 1 , *if k e A,* 0 , otherwise. Throughout *n(n)* denotes any permutation on *N. P{n)* denotes the set of all permutations on *N.* Let x = (xn) be a sequence, then *P(x)* denotes the set of all permutation of the elements of (x„), that is *P(x)* = {(x;r(,0) : *Tin)* is a permutation on *N}.*

A sequence space *E* is said to be **symmetric** if *P(x) c _E* , for all *x e E.* A sequence space *E* is said to be **solid** if *(ylt)* e *E* whenever (x„) e *E* and | *y n* | < j *xn* | for all *n e N.*

The Kothe-Toeplitz dual, that is the *a* -dual of a set *E* of complex sequences is defined as the space

$$E^{a} = \{ y = (y_k) \in w : \sum_k |x_k y_k| < \infty \text{ for all } x = (x_k) \in E \}.$$
$$= \{ y = (y_k) \in w : (x_k y_k) \in \ell_1 \text{ for all } x = (x_k) \in E \}.$$

Throughout p denotes the class of all subsets of N. For any s e N, P s denotes the class of all cr e p , such that

The space O consists of all sequences (f> = {(f)n) such that, for all neN

$$0 \le \phi_1 \le \phi_n \le \phi_{n+1} < \infty \text{ and } n\phi_{n+1} \le (n+1)\,\phi_n .$$

We write $(\Delta\phi_n) = (\phi_n - \phi_{n-1})$.

A Bit-space (introduced by Zeller [103]) (X, ||.||) is a Banach space of complex sequences x - (xk) in which the co-ordinate maps are continuous, that is,

$$|x_k^{(n)} - x_k| \to 0, \text{ whenever } \| x^{(n)} - x \| \to 0 \text{ as } n \to \infty ,$$

Where

$$x^{(n)} = \left(x_k^{(n)}\right), \text{ for all } n \in N \text{ and } x = \left(x_k\right).$$

Throughout $\sum_{k=1}^{\infty} x_k$ denotes an infinite series and Sx denote the sequence of partial k= 1 . oO sums of $\sum_{k=1}^{\infty} x_k$ *. Throughout $\sum_{k=1}^{\infty} x_k$ * i.e. summation without limits means that the M k summation is from k = 1 to co.

STATISTICAL CONVERGENCE OF SEQUENCES

So as to expand the idea of intermingling of sequences, measurable conver-gence of sequences was presented by Fast in 1951, Buck in 1953 and Schoenberg in 1959 autonomously. It is likewise found in (see lemma in p. 181). Later on it was concentrated from sequence space perspective and connected with whole Salat and Sen [96], Kolk [39] and numerous others. The thought of measurable assembly relies upon the possibility of asymptotic thickness of subsets of the set N of characteristic numbers (one may allude to Niven, Zuckerman and Montgomery).

For any subset A of N, we say that A possesses asymptotic density (or simply density S(a) if

$$\delta(A) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_A(k) \text{ exists ,}$$

Where % a is the characteristic function of A. Clearly all finite subsets of N have zero natural density and

$$\delta(A^c) = \delta(N - A) = 1 - \delta(A)$$

We write S0 = { {xk) e w : S({k e N : xk * 0})= S (supp [xk)) = 0}

A given complex sequence x = (x,.) is said to be statistically convergent to the sum L, if for any s >0, we have $\delta(\{k \in N : |x_k - L| \ge \varepsilon\}) = 0$. =0 . We write xk — sl'" > L or stat - lim xt = L .

For two sequences (x j and (yk), we say that Xk= y k for almost all k (in short a.a.k) if 8{{k eN:xk* y k})=o. By c and c0 we denote the spaces of all statistically convergent and statistically null sequences respectively. Clearly c, ce are linear spaces. We write m = c r> £n and m0 = c0 n i m. Salat ([76], Theorem 2.1) showed that m and m0 are closed subspaces of under the sup-norm. A sequence x = (xk) is said to be statistically Cauchy (introduced by Fridy [24]), if for every s > 0, there exists = no (s) such that e N: x k ~ x „n >£ i)=0- Fridy Salat and Connor established some relations between statistical convergence and convergence of sequences. Those results are known as decomposition theorems. We procure those results below.

SEQUENCE SPACES RELATED TO THE

and n{) which are closely related to the I p spaces, where ^ e O . The space m{if>) is defined as

$$m(\phi) = \left\{ x = (x_k) \in w : \|x\|_{m(\phi)} = \sup_{s\ge 1, \sigma \in \wp_s} \tfrac{1}{\phi_s} \sum_{n\in\sigma} |x_n| < \infty \right\}.$$

Also the space n(tft) is defined as

$$n(\phi) = \left\{ x = (x_k) \in w : \|x\|_{n(\phi)} = \sup_{u \in P(x)} \left\{ \sum_{n=1}^{\infty} |u_n| \,\Delta\phi_n \right\} < \infty \right\},$$

Where

$$\Delta\phi_n = \phi_n - \phi_{n-1}, \text{ for all } n \in N.$$

Sargent showed that the spaces m() and n{f) are BK- spaces and each of the spaces m (f) , n(f) is the dual of the other in the sense of K6the and Toeplitz. The following two results are due to Sargent.

OBJECTIVES OF THE STUDY

1. Study on the space of all entire sequence of modals *(gI) and its dual space /(gI) and the subspaces of *(gI) are proved that these spaces are complete metric spaces.

2. Study on the new sequence space of interval numbers *(gI) is investigated and the topological properties of G (O) are studied.

CONCLUSION

The point of the postulation is to advance innovative work in fluffy hypothesis by considering the structure of esteemed normed direct space. The objective is to present new improvements in esteemed inward item over augmentation fields which may discover applications in future. In this setting we examined different structures, for example, Euler esteemed inward item space, t(n) esteemed internal item space, proceeded with fluffy inward item space, esteemed n-internal item space and esteemed n-utilitarian in esteemed n-inward item space.

REFERENCES

1. Brudno (1945). Summation of bounded sequences by matrices (in Russian), Recueil Math. (Mat. Sbornik), 16, pp. 191-247.

2. Esi (2011). On some new generalized di erence double sequence spaces de ned by Orlicz functions, Mathematika, 27(1), pp. 31-40.

3. Esi (1996). Some new sequence spaces de ned by a modulus function , Istanbul Univ. Fen. Fak. Mat. Derg., 55/56, 17-21, (1996/97).

4. Esi (2013). Statistical convergence of triple sequences in topological groupes, An. Univ. Craiova Ser. Mat. Inform., 40(1), pp. 29-33.

5. J. Dutta, A. Esi and B. C. Tripathy (2013). Statistically convergent triple sequence spaces de ned by Orlicz function, J. Math. Anal., 4(2), pp. 16-22.

6. Pringsheim, Zurtheorie der zweifachunendlichenzahlenfolgen, Math. Ann. 53, pp. 289-321 (1900).

7. Sahiner and B. C. Tripathy (2008). Some I - related properties of Triple sequences, Selcuk. J. Appl. Math., 9(2), pp. 9-18.

8. Sahiner, M. Gurdal and K. Duden (2007). Triple sequences and their statistical con-vergence, Selcuk. J. Appl. Math., 8(2), pp. 49-55.

9. Sahiner, M. Gurdal, S. Saltan and H. Gunawan (2007). Ideal convergence in 2-normed spaces, Taiwan. J. Math., 11(5), pp. 1477-1484.

10. Wilansky (1984). Summability through functional analysis, North-Holland mathematics studies 85, Elsevier Science publishers, Amsterdam, New York, Oxford.

11. C. Tripathy (2003). Statistically convergent double sequence, Tamkang. J. Math., 34(3), pp. 231-237.

12. C. Tripathy (1997). Matrix transformation between some classes of sequences, J. Math. Analysis and Appl., 206 (2), pp. 448-450.

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