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A Study of Multiplicative Diagram of Linear Transformations and Linear Algebra

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Abstract – An important concept in mathematics is the linear transformation because linear models can approximately match many real-world phenomena. In contrast to a linear function, a linear transformation works on both vectors and numbers. A linear mathematical transformation to turn a geometric figure (or matrix or vector) in another format, using a formula of a certain format. The original components have to be formatted in a linear combination. In a linear algebra, a transition between two vector spaces is a rule that assigns a vector to a vector within a given space. Linear transformations are transformations that fulfill a specific additional and scaffolding property. The present lecture discusses the fundamental notation of transformations, what 'pic' and what is meant by 'range,' and what distinguishes a linear transformation from other transformations.

Key Words – Linear Algebra, Linear Transformations, Linear Models, Linear Function

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INTRODUCTION

The interrelationships between the theory of linear transformations which preserve the invariant matrix and various branches of mathematics are examined. These methods and motives are given the expectations for studying these changes which are derived from general algebra.

A *linear transformation*, T:U→VT:U→V, is a function that carries elements of the vector space UU (called the *domain*) to the vector space VV (called the *codomain*), and which has two additional properties

1. $T(\mathbf{u}_1 + \mathbf{u}_2) = T(\mathbf{u}_1) + T(\mathbf{u}_2)$ for all $\mathbf{u}_1, \mathbf{u}_2 \in U$
2. $T(\alpha \mathbf{u}) = \alpha T(\mathbf{u})$ for all $\mathbf{u} \in U$ and all $\alpha \in \mathbb{C}$

The two defining conditions "feel linear" anyway in a linear transformation concept. On the other hand, these terms may be understood as exactly the significance of linearity. — linear transformation property comes from both of these determining properties, because the addition of the vector and the scalar multiplication are the characteristic of each vector space. Although these conditions are very different, don't confuse them. They remember how we measure subspaces.

The basic features of both distinguishing features of a linear transformation are shown in two diagrams. In every case, start from the upper left corner and follow the arrows in the lower right corner around the rectangle, follow two routes and carry out the operations specified on the bracelets. For a linear transformation, these two expressions are the same.

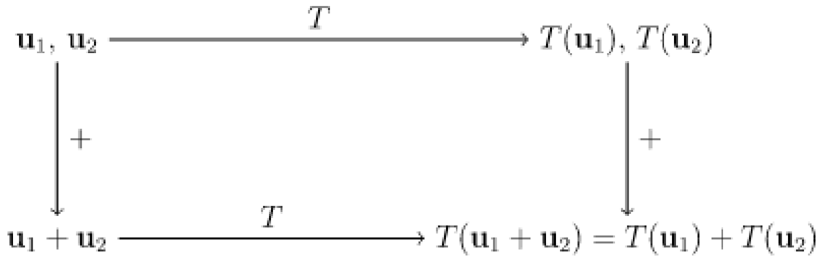

Diagram DLTA Definition of Linear Transformation, Additive

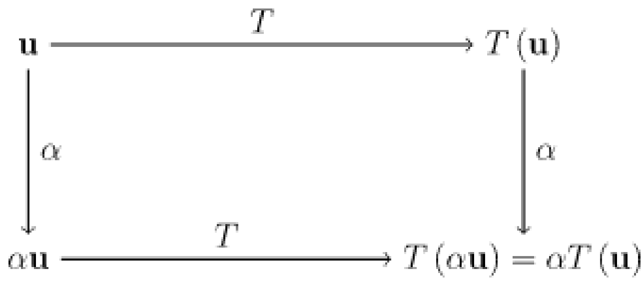

Diagram DLTM Definition of Linear Transformation, Multiplicative

A couple of words about notation. TT is the *name* of the linear transformation, and should be used when we want to discuss the function as a whole. T(u)T(u) is how we talk about the output of the function, it is a vector in the vector space VV. When we write T(x+y)=T(x)+T(y)T(x+y)=T(x)+T(y), the plus sign on the left is the operation of vector addition in the vector space UU, since xx and yy are elements of UU. The plus sign on the right is the operation of vector addition in the vector space VV, since T(x)T(x) and T(y)T(y) are elements of the vector space VV. These two instances of vector addition might be wildly different.

Linear Transformation and Linear combination

It is the relationship between linear and linear transformations that is at the core of many important linear algebra theorems. This is the basis of the next theorem. The evidence is not conclusive, the outcome is not surprising, but it is sometimes stated. This theorem says that we can "throw" linear transformations "down" on "linear" combinations, or "pull" linear transformations "up" onto linear combinations. We've already passed it by for a time in the evidence of theorem MLTCV. We will be able to push and pull them.

T:U→VT:U→V *is a linear transformation,* u1,u2,u3,…,utu1,u2,u3,…,ut *are vectors from* UU *and* a1,a2,a3,…,ata1,a2,a3,…,at *are scalars from* CC. *Then*

$$T(a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3 + \cdots + a_t\mathbf{u}_t) = a_1T(\mathbf{u}_1) + a_2T(\mathbf{u}_2) + a_3T(\mathbf{u}_3) + \cdots + a_tT(\mathbf{u}_t)$$

Linear Algebra/Linear Transformations

A linear transformation is an important concept in mathematics because many real world phenomena can be approximated by linear models.

Say we have the vector $\begin{pmatrix}1\\0\end{pmatrix}$ in $\mathbb{R}^2$ in , and we rotate it through 90 degrees, to obtain the vector $\begin{pmatrix}0\\1\end{pmatrix}$

Another example instead of rotating a vector, we stretch it, so a vector $\mathbf{v}$ becomes , $2\mathbf{v}$; for example.

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ becomes } \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

Or, if we look at the *projection* of one vector onto the *x* axis - extracting its *x* component - , e.g. from

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ we get } \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

These examples are all an example of a *mapping* between two vectors, and are all linear transformations. If the rule transforming the matrix is called *T*., we often write Tv for the mapping of the vector $\mathbf{v}$ by the rule *T*.*T* is often called the transformation.

Linear Operators

If you have a field K, and let x be a field element. Let O be a feature that takes values from K where O(x) is a J field variable. Define O to be a linear form if and only if:

O(x+y)=O(x)+O(y)

O(λx)=λO(x)

Linear Forms

Assume that one has a space vector V, and that x is part of this vector space. Let F be a V values function where F(x) is a K field unit. Define F to be a linear form if and only if:

F(x+y)=F(x)+F(y)

F(λx)=λF(x)

Linear Transformation

Let us consider this time functions from one vector space to another vector space instead of a field. *T* can be a value-taking function from one space vector *V* where *L*(*V*) is an element of another space vector. Define *L* to be a linear transformation when it:

preserves scalar multiplication: T(λx) = λTx

preserves addition: T(x+y) = Tx + Ty

Remember, not everyone's linear transformations. Many simple transformations which are also non-linear in the real world. Your research is harder and will not be carried out here. The transformation, for example *S* (whose input and output are both vectors in $\mathbf{R}^2$) defined by

$$S\mathbf{x} = S\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xy \\ \cos(y) \end{pmatrix}$$

We can learn about nonlinear transformations by studying easier, linear ones.

We often describe a transformation T in the following way

$$T : V \to W$$

This means that T, whatever transformation it may be, maps vectors in the vector space V to a vector in the vector space W.

The actual transformation could be written, for instance, as

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ x - y \end{pmatrix}$$

Examples and proofs

Some examples of such linear transformations are given here. Let's simultaneously explore how we can show that a transition we find is or cannot be linear..

Projection

Let us take the projection of vectors in $\mathbf{R}^2$ to vectors on the *x*-axis. Let's call this transformation T.

We know that T maps vectors from $\mathbf{R}^2$ to $\mathbf{R}^2$, so we can say

$T : \mathbb{R}^2 \to \mathbb{R}^2$

and we can then write the transformation itself as

$$T\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ 0 \end{pmatrix}$$

Clearly this is linear. (Can you see why, without looking below?)

Let's go through a proof that the conditions in the definitions are established.

Scalar multiplication is preserved

We wish to show that for all vectors v and all scalars λ, T(λv)=λT(v).

Let,

$$\mathbf{v} = \begin{pmatrix} v_0 \\ v_1 \end{pmatrix}.$$

Then

$$\lambda\mathbf{v} = \begin{pmatrix} \lambda v_0 \\ \lambda v_1 \end{pmatrix}$$

Now

$$T(\lambda\mathbf{v}) = T\begin{pmatrix} \lambda v_0 \\ \lambda v_1 \end{pmatrix} =$$
$$\begin{pmatrix} \lambda v_0 \\ 0 \end{pmatrix}$$

If we work out λT(**v**) and find it is the same vector, we have proved our result.

$$\lambda T\mathbf{v} = \lambda\begin{pmatrix} v_0 \\ 0 \end{pmatrix} =$$
$$\begin{pmatrix} \lambda v_0 \\ 0 \end{pmatrix}$$

This is the same vector as above, so under the transformation T, *scalar multiplication is preserved*.

Addition is preserved

We wish to show for all vectors x and y, T(x+y)=Tx+Ty.

Let

$$\mathbf{x} = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}.$$

and

$$\mathbf{y} = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}.$$

Now

$$T(\mathbf{x}+\mathbf{y}) = T\left(\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} + \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}\right) =$$
$$T\begin{pmatrix} x_0 + y_0 \\ x_1 + y_1 \end{pmatrix} =$$
$$\begin{pmatrix} x_0 + y_0 \\ 0 \end{pmatrix}$$

Now if we can show T**x**+T**y** is this vector above, we have proved this result. Proceed, then,

$$T\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} + T\begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ 0 \end{pmatrix} + \begin{pmatrix} y_0 \\ 0 \end{pmatrix} =$$
$$\begin{pmatrix} x_0 + y_0 \\ 0 \end{pmatrix}$$

So we have that the transformation T preserves addition.

Zero vector is preserved

Clearly we have

$$T\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

Disproof of linearity

In order to refute linearity-in other words to show that a transformation is not linear, only a counter-example is required.

If we can find only one case that does not sustain addition, scale-based multiplication or zero vectors in the transformation, we can infer that the transformation is not linear.

For example, consider the transformation

$$T\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x^3\\y^2\end{pmatrix}$$

We suspect it is not linear. To prove it is not linear, take the vector

$$\mathbf{v}=\begin{pmatrix}2\\2\end{pmatrix}$$

then

$$T(2\mathbf{v})=\begin{pmatrix}64\\16\end{pmatrix}$$

but

$$2T(\mathbf{v})=\begin{pmatrix}16\\8\end{pmatrix}$$

so we can immediately say T is not linear because it doesn't preserve scalar multiplication.

Problem set

Given the above, determine whether the following transformations are in fact linear or not. Write down each transformation in the form T:V -> W, and identify V and W. (Answers follow to even-numbered questions):

1. $T\begin{pmatrix}v_0\\v_1\end{pmatrix}=\begin{pmatrix}v_0^2+v_1\\v_1\end{pmatrix}$
2. $T\begin{pmatrix}v_0\\v_1\end{pmatrix}=\begin{pmatrix}1\\v_0\end{pmatrix}$
3. $T\begin{pmatrix}v_0\\v_1\end{pmatrix}=\mathbf{0}$
4. $T\begin{pmatrix}v_0\\v_1\\v_2\end{pmatrix}=\begin{pmatrix}v_0-v_2\\v_1\end{pmatrix}$

CONCLUSION

A linear transformation is a function which respects the underlying (linear) structure of each vector space. A linear transformation is often known as a linear operator or map. The transformation range may be identical with that of the domain and, when it occurs, a finalomorphism or automatic Orphism is known to occur if inverted. To support the two vector spaces, the same field must be used. Linear transformations are advantageous because they maintain a space vector structure. Under some circumstances, a linear transformation image is automatically kept in several qualitative estimations of a vector space which are the domain of a linear transformation. For example, the kernel and image are the subspaces of the linear transformation spectrum immediately indicated (not just subsets). Most linear functions can be interpreted as linear transitions in the correct setting. Linear and most geometric change in base shifts include rotations, reflections and contractions/dilations. In some rather than linear functions, linear algebra techniques may even more effectively use approximation through lineary functions and reinterpretation as a linear function in unusual vector spaces. There are many relationships between mathematical fields in an analysis of linear changes.

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