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A Study of Algebraic Singularities of the Differential Forms of Degree (N-1) Without Zero on One Variety of N-Dimensions

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***Abstract** – The present work comprises of the study of algebraic singularities of the differential of degree (n-1) without zero on one assortment of n measurements (A particular point for one such from □ is where the differential d□ is invalid). Martinet J.71 Studies the main idea of it in the year 1979 and we introduced idea of singularities with applications. Differential forms are a rich source of invariants in algebraic Singularities. This approach was very successful for smooth varieties, but the singular case is less well understood. We explain how the use of the h-topology (introduced by Suslin and Voevodsky in order to study motives) gives a very good object also in the singular case, at least in characteristic zero. We also explain problems and solutions in positive characteristic. Differential forms originally show up when integrating or differentiating on manifolds. The object has very many important uses. The one we are concentrating on is as a source of invariants used in order to classify varieties. This approach was very successful for smooth varieties, but the singular case is less well-understood.*

***Keywords:** Algebraic Differential Forms, Cohomological Invariants, H-Topology, Singular Varieties.*

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INTRODUCTION

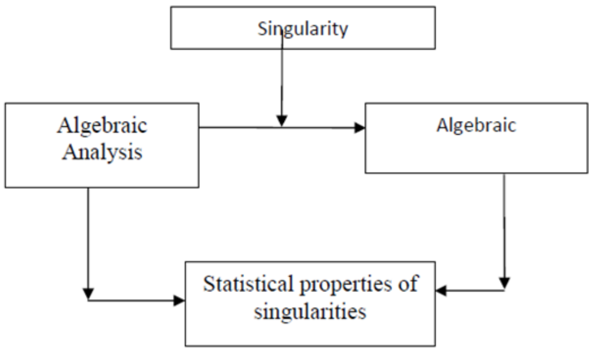
One (n-1) frame □ without zero one assortment M of measurements n characterizes a folio (□) of measurement one of M and □ itself means the one transverse volume from at □. One speculation of the issue examined. W will comprises of the study of the singularities of structures (M, □, □) where □ is a folio of co-measurement p of the assortment M and □ a p-shape (totally decomposable and integrable) speaking to a transverse volume at □. We examines and get some productive results in the soundness and the models for the nonexclusive singularities.

SINGULARITIES:

Algebraic structure deals with singularity theory. Singularity theory is an impartent Zeta Function, Hyper Functions, Empirical process and Statistics.

i) By using this particular bridge we can think of the behaviour of any learning Machine based on the resolution of singularities.

ii) The main domain of singularities is as shown below



Description of singularities: We assumed that for all fix on the variety M a folio of dimension one, transversably orientable.

We introduced the fiber $J^{n+1}(\Lambda^{n-1}E)$ on M of (n+1) jets of sections of fiber $\Lambda^{n-1}E$ and construct on $J^{n+1}(\Lambda^{n-1}E)$ a set of sub-verities which shall be the singularities of transverse volume on $\mathfrak{I}$.

We consider a system of local coordinates $(X_1,\ldots\ldots\ldots,X_n)$ under M such that at $X_2,\ldots\ldots\ldots\ldots X_n$ be the first local integrals of the system E (or of folio $\mathfrak{I}$); under such a system, called the adopted system of local coordinates on $\Im$, all $\omega \in J^{n+1}(\Lambda^{n-1}E)$ will be written in a unique manner as:

$$\omega = f dX_2 \Lambda \ldots\ldots \Lambda dX_n$$

Where $f \in J^{n+1}(\Re^n)$, are the (n+1) jet fibers of the function of n variables.

By the choice of the local coordinates $(X_1,\ldots\ldots X_n)$ one has an isomorphism of $J^{n+1}(\Lambda^{n-1}E)/U$ under $J^{n+1}(\Re^n)/U$ where U is the domain of the system of the coordinates considered.

All $f \in J^{n+1}(\Re^n)$ of source $a = (a_1,\ldots\ldots,a_n) \in U$ identify with a polynomial of degree n+1, in the variables $X_I = X_i - a_i$, $i = 1,\ldots\ldots,n$

we write

$$f = \sum_{|\alpha| \le n+1} A_\alpha . X^\alpha$$

where according to the usage, $\alpha = (\alpha_1,\ldots\ldots,\alpha_n)$ is a multi index such that for all i, $\alpha_i > 0$. One has posed $|\alpha| = \alpha_1 + \alpha_2 + \ldots\ldots + \alpha_n$

The functions (a_i, A_α) contribute a system of coordinates on $J^{n+1}(\Re^n)/U$, and therefore on $J^{n+1}(\Lambda^{n-1}E)/U$ through the isomorphism of the choice of coordinates $(\alpha_1,\ldots\ldots\alpha_n)$,

We define $\Sigma^i = E^{1,\ldots,1}_{\;\;i}$, $i = 1,\ldots\ldots,n$, under $J^{n+1}(\Lambda^{n-1}E)/U$ as the sub-variety of the equations

$$A_{1,0,\ldots\ldots,0} = A_{2,0,\ldots\ldots,0} = \ldots\ldots = A_{i,0,\ldots\ldots,0} = 0$$

$$A_{0,0,\ldots\ldots,0} \neq 0$$

Trivially it can be verified that this definition does not depend on local system of coordinates $(X_1,\ldots\ldots X_n)$ adopted on the considered $\Im$. Thus, we define on $J^{n+1}(\Lambda^{n-1}E)$ a sequence of sub-varieties $\overset{1}{\Sigma} 1 \supset \overset{1_2}{\Sigma} \supset \ldots\ldots \supset \overset{1_n}{\Sigma}$

such that the codimension $\overset{1_i}{\Sigma} = i$

As per the construction the sub-varieties $\overset{1_i}{\Sigma}$ are invariant (globally) on M, leaving the folio $\Im$ and invariant.

We take

$$\Sigma^{1_i,0} = \Sigma^{1_i} - \Sigma^{1_i+1}, i \le n$$

If ω is a section of $\Lambda^{n-1}E$ we design by J^{n+1} ω the section of $J^{n+1}(\Lambda^{n-1}E)$ defined by J_x^{n+1} $\omega = (n+1)^{th}$ jet of ω on X.

We say that a volume ω presents the singularity $\Sigma^{1_i,0}$ of X or rather X is a singular point of ω of type $\Sigma^{1_i,0}$ and write $X \in \Sigma^{1_i,0}(\omega)$ if $J_x^{n+1}\omega \in \Sigma^{1_i,0}$

Analytical description

If one writes $\omega = f dX_2, \Lambda \ldots \Lambda dX_n$ under a local system of coordinates adapted with $\Im$, then ω presents at the origin the singularity $\Sigma^{1_i,0}$, $i \le n$, if and only if.

$$\frac{\partial f}{\partial x}(0) = \ldots\ldots = \frac{\partial^i f}{\partial x^i}(0) = 0, \; \frac{\partial^{i+1} f}{\partial x_1^{i+1}}(0) \neq 0$$

This singularities that $\frac{\partial f}{\partial x_1}$ is, at the origin, a regular function of order I with respect to the variable X_1. The verifications are trivial if one refers to the definition further, we suppose that $J_0^{n+1}\omega \in \Sigma^{1_i,0}$, besides the section of J^{n+1} ω, being transverse, at the origin at $\Sigma^{1_i,0}$. This signifies the differential forms d $\left(\frac{\partial f}{\partial x_1}\right)$......d$\left(\frac{\partial^i f}{\partial x_i}\right)$ are linearly independent at the origin.

Taking into consideration the conditions (1) it amounts to the differential forms

$$d\left(\frac{\partial f}{\partial x_1}\right) \ldots\ldots d\left(\frac{\partial^{i-1} f}{\partial x_1^{i-1}}\right)$$ being linearly independent at the origin.

GEOMETRICAL DESCRIPTION

We recall the previous notations:

The folio $\Im$ is constituted by the parallels to the axis OX_1 we have

$$d\omega = \frac{\partial f}{\partial x_1} dx_1 \Lambda \ldots \Lambda dx_n$$

The set $\Sigma^1(\omega)$ is thus the set of points where dω is zero.

Now suppose that $J^{n+1}\omega$ transverses in O at $\Sigma^{1,0}$ so that $\frac{\partial^2 f}{\partial x_1^2}(0) \neq 0$; $\Sigma^1(\omega)$ is thus a hypersurface and the condition $\frac{\partial^2 f}{\partial x_1^2} \neq 0$; geometrically signifies that at the origin $\Sigma^1(\omega)$ is transverse at $\Im$.

Now suppose that $J^{n+1}\omega$ transverses in O at $\Sigma^{1,0}$ so that $\frac{\partial^2 f}{\partial x_1^2}(0) \neq 0$; $\Sigma^1(\omega)$ is thus a hypersurface and the condition $\frac{\partial^2 f}{\partial x_1^2} \neq 0$; geometrically signifies that at the origin $\Sigma^1(\omega)$ is transverse at $\Im$.

Now suppose that $J_0^{n+1}\omega \in \Sigma^{1_2,0}$ such that $\frac{\partial f}{\partial x_1} = \frac{\partial^2 f}{\partial x_1^2}(0) = 0$ and that $\frac{\partial^3 f}{\partial x_1^3}(0) \neq 0$, and further that $J^{n+1}\omega$ is transverse at $\Sigma^{1_2,0}$ in O such that d$\left(\frac{\partial f}{\partial x_1}\right)$ be not zero at 0; thus $\Sigma^1(\omega)$ is a hypersurface and it is tangent to $\Im$ at the origin. It is a sub-variety of $\Sigma^1(\omega)$ of codimension one which is transverse to $\Im$ at the origin.

It is natural to interpret as follows: Let Π be the projection parallel to the folio of $\Im$ on one transversal at $\Im$, for example, the sub-space $\Re^{n-1}$ of coordinates $(X_2,\ldots\ldots X_n)$; thus $J^{n+1}\omega$ is transverse in O at $\Sigma^{1_2,0}$ if and only if the restriction Π : $\Sigma^1(\omega)$ → $\Re^{n-1}$ presents one fold on O.

If $J_0^{n+1}\omega \in \Sigma^{1_3,0}$ and if $J^{n+1}\omega$ is transverse in O at $\Sigma^{1_3,0}$, it signifies geometrically that the restriction Π : $\Sigma^1(\omega)$ → $\Re^{n-1}$ presents contraction in O i.e. a singularity $\Sigma^{1_2,0}$ is the sense of [2].

In general, if $J_0^{n+1}\omega \in \Sigma^{1_i,0}$ and if $J^{n+1}\omega$ is transverse in O at $\Sigma^{1_i,0}$ it will signifies geometrically that the restriction Π signifies a singularity $\Sigma^{1_i-1,0}$ in the sense of O.

In the end we give examples of each type of singularities (which are transversely present)

Let $\omega_1 = (1 + x_1^2) dX_2 \Lambda ..... \Lambda dX_n$

$\omega_2 = (1 + x_1 x_n + x_1^3) dX_2 \Lambda ..... \Lambda dX_n$

:

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$\omega_i = (1 + x_1 x_n + x_1^2 x_{n-1} + ... + x_1^{i-1} x_{n-i+2} + x_i^{i+1}) dX_2 \Lambda ..... \Lambda dX_n$

:

:

$\omega_n = (1 + x_1 x_n + x_1^2 x_{n-1} + .. + x_1^{2-1} x_2 + x_1^n) dX_2 \Lambda ..... \Lambda dX_n$

It is easily verified that ω_i presents transversally a singularity of type $\Sigma^{1_i,0}$ in O

TRANSVERSALITY

The general idea of transversally has for outcome in following recommendation:-

Let $\Im$ be a folio of dimension one, transversely oreintable of one variety M of dimension n, paracompact generically under the set of volumes ω, transverse at $\Im$, and for the C^{n+1} topology, $J^{n+1}\omega$ is transverse at all the singularities $\Sigma^{1_i,0}$ i = 1,….,n.

In particular, the set $\Sigma^{1_i,0}$ is in this case if non empty, a sub-variety of co dimension i of M.

Truth be told this recommendation deals with the arrangements of (n-1) shapes, without zero on one assortment of M of measurement n, standard minimized, contaminate each (n-1) frame, without zero on M, in a transverse volume at a folio of measurement one of M transversely situate capable.

GENERALITIES

We conseder two triplets $(M, \Im, \omega)$ and $(M', \Im', \omega')$

Their germs at points $X \in M$ and $X' \in M'$ are isomorphic if these exists a local deffeomorphism ϕ of a neighbourhood U of X in the neighbourhood U' of X' such that

$\phi(X) = X'$

$\phi(\Im/U) = \Im'/U'$

$\phi^*(\omega'/U') = \omega/U$

We propose to make a classification of the germs of the triplets (M, □, □) at close isomorphism.

Let us consider one triple $(M, \Im, \omega)$. Its germ in $X \in M$ is isomorphic (by the choice of the system of local coordinates in the neighbourhood of X_1 adopted at folio $\Im$) to the triple formed in the neighbourhood V of $0 \in \Re^n$, of the restriction V of folio $\Im_0$ formed by the parallels to the axis OX_1, and of a germ of differential form :

$f(X_1,\ldots\ldots,X_n) dX_n \Lambda ..... \Lambda dX_n$ with $f(0) \neq 0$.

We fix once and for all on $\Re^n$ the folio $\Im_0$.

The classification of germs of the triplets $(M, \Im, \omega)$ (at close isomorophism) thus leads to the germs on $0 \in \Re^n$: $\omega = f(X_1,\ldots\ldots,X_n) dX_n \Lambda ..... \Lambda dX_n$ $(f(0) \neq 0)$ through the group of local diffeomorphisms of $\Re^n$ of the source, leaving $\Im_0$ invariant, i.e. the forms;

$$(X_1,\ldots X_n) \rightarrow \phi(X_1,\ldots X_n) = \left| \begin{array}{l} \phi_1(X_1,\ldots\ldots,X_n) \\ \phi_2(X_2,\ldots\ldots,X_n) \\ : \\ : \\ \phi_n(X_2,\ldots\ldots,X_n) \end{array} \right.$$

Notations and definitions

By $\sum_n$ (respectively Σ_{n-1}) we design the ring of germs in $0 \in \Re^n$ (respectively on $0 \in \Re^n$) of the functions variables $(X_1,\ldots X_n)$ (respectively $(X_2,\ldots\ldots,X_n)$ and by $\bar{\sum_n}$ (respectively $\bar{\sum_{n-1}}$) the ring of the germs in $0 \in \Re^n \times \Re$ (respectively on $\Re^{n-1} \times \Re^{n-1}$) of functions of the variables $(X_1, X_2,\ldots,X_n,t)$ (respectively $X_2,\ldots,X_n,t$). The set of $\omega = f(X_1,\ldots,X_n)\, dX_2 \Lambda ..... \Lambda dX_n$ which we consider as such that

$f \in \Sigma_n, f(0) \neq 0$

GENERALITIES AND GENERIC SINGULARITIES

Geometrically the classification of the generic singularities rests on the behaviour of the hyper surface $\Sigma^1(\omega)$ (set of) the equation $\omega = 0$; with the couple $(\Im, \Sigma^1(\omega))$ is associated in equation way as the germ of application of $\Re^{n-1}$ under $\Re^{n-1}$ (representing the application of $\Sigma^1(\omega)$ on one local transversal at $\Im$). Thus the corresponding applications on generic singularities are the germs of the type $\Sigma^{1\ldots1,0}$ of which the stability is known. But the structure envisaged use in much richer.

We demonstrated the security of singularities of (n-1) shapes, without zero, comprises of a reversal of a differential administrator of request one and of one homomorphism of modules over a ring of capacities. It is consequently important to utilize the obtained theorems of arrangement and the resolution of an arrangement of incomplete differential conditions.

It is there that the basic distinction with instance of the similitudes of differential likenesses of their applications lies.

The principle results are as per the following:

The singularities of request sub-par or equivalent to 'n' are steady and we give them the neighborhood models.

The singularities of request "n+1" are steady.

We characterize the singularities on the space of fly structures and we compose the nonexclusive singularities utilizing the transverseability.

We characterize the dependability of germs of the types of degree (n-1) and we demonstrate that the singularities of request mediocre or meet "n" are steady. At that point we reason the neighborhood models for the singularities. At last we demonstrate that the singularities of request (n+1) are instable.

All objects considered shall be C□

Suppose M in a variety of n dimension, ℑa folio of M of dimension (n- p), which in transversably orientable. This signifies that ℑis defined by a system of pfaff E on M of rank p (that is to say a sub-fiber E of cotangent fiber T*M), completely interable such that the fiber on the right □p E (p-ieme exterior power of E) be trivial.

We call transverse volume folio ℑ all sections without zero of fiber □p E . One transverse volume is thus a p-form on M, without zero, completely decomposable, integrable and defining the folio ℑ.

Let us suppose □ is a transverse volume on ℑ which is fixed. At a point *x* □ M where the differential d□ of □ is not zero, one can always choose a system of local coordinates $(x_1,\ldots,x_n)$ under which □ can be written as

$\omega = (1+X_1)\, dX_{n-p+1} \wedge \ldots\ldots\ldots \wedge dX_n$ [1] where $X_{n-p+1},\ldots\ldots\ldots X_n$ are the local first integrals of ℑ.

If d ω is identically zero (one writes d ω= 0).

This signifies that the volume ω is invariant by the actions of the field vectors tangent to the folio □ (i.e. whatever the field of vectors X, tangent to □ the Lie derivative of □ with respect to X, written □ (X). □ is zero).

If d□ □ 0 it in natural to call all points X □ M singular points where d□ is zero.

In the general case it is difficult to make a study of the singularities of the structures (M, □, □). A particular case where the dimension of □ is one will be treated.

CONCLUSION:

This is note is an extended version of my plenary talk at the AMS-EMS-SPM International Meeting 2015 in Porto. Most of it is aimed at a very general audience. The last sections are more technical and written in a language that assumes a good knowledge of algebraic geometry. We hope that it will be of use for people in the field. Differential forms originally show up when integrating or differentiating on manifolds. However, the concept also makes perfect sense on algebraic varieties because the derivative of a polynomial is a polynomial. The present examination on the real utilizations of the algebraic Singularities matrix in quantum science and we take a portion of the inorganic frameworks to which the said lattice has been connected. Such frameworks are of particular enthusiasm as the every now and again contain no portable □ - electrons. As per Mather J.72 strength of C□ mappings expresses that "Tensor Fields can be neither made retrofitted nor anticipated in practical frame by maps are not diffeomorphisms. The instance of singularities is for the most part valuable in organic fields i.e. changes in Regional Myocardial this is amid the Cardiac cycle suggestions for transmural blood stream and cardiovascular structure. We explain how the use of the h-topology gives a very good object also in the singular case, at least in characteristic zero. The approach unifies other ad-hoc notions and implies many proofs. We also explain the necessary modifications in positive characteristic and the new problems that show up.

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