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An Overview of Boolean Algebras and Pre A* - Algebras

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Abstract – Boolean was a mathematician and scholar who created methods for communicating sensible procedures utilizing arithmetical images, in this way making a part of science known as representative rationale, or Boolean variable based math. It wasn't until years after the fact that Boolean polynomial math was connected to processing by John Vincent Atanasoff. He was endeavoring to manufacture a machine dependent on a similar innovation utilized by Pascal and Babbage, and needed to utilize this machine to settle direct arithmetical conditions. In the wake of battling with rehashed disappointments, Atanasoff was so baffled he chosen to take a drive.He was living in Ames, Iowa, at the time, however gotten himself 200 miles away in Illinois before he all of a sudden acknowledged how far he had driven. Atanasoff had not planned to drive that far, yet since he was in Illinois where he could legitimately purchase a beverage in a bar, he sat down, requested a whiskey, and acknowledged he had driven a significant separation to get a beverage (Atanasoff consoled the creator that it was not the beverage that driven him to the accompanying disclosures—truth be told, he left the beverage immaculate on the table.) Exercising his material science and arithmetic foundations and concentrating on the disappointments of his past processing machine, he made four basic achievements important in the machine's new structure

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INTRODUCTION

Although each solid Boolean algebra is a Boolean algebra, few out of every odd Boolean algebra need be concrete. Give n a chance to be a without square positive whole number, one not separable by the square of a number, for example 30 yet not 12. The operations of greatest normal divisor, least basic numerous, and division into n (that is, $\neg x = n/x$), can be appeared to satisfy all the Boolean laws when their arguments range over the positive divisors of n. Subsequently those divisors structure a Boolean algebra. These divisors are not subsets of a set, making the divisors of n a Boolean algebra that isn't concrete according to our definitions.

In any case, on the off chance that we represent each divisor of n by the arrangement of its prime factors, we find that this nonconcrete Boolean algebra is isomorphic to the solid Boolean algebra comprising of all arrangements of prime factors of n, with association comparing to least regular various, crossing point to greatest normal divisor, and supplement to division into n. So this example while not technically concrete is at least "morally" concrete via this representation, called an isomorphism. This example is an instance of the accompanying idea.

BOOLEAN ALGEBRAS

A Boolean algebra is called representable when it is isomorphic to a solid Boolean algebra. The conspicuous next inquiry is answered decidedly as pursues. Each Boolean algebra is representable. That is, up to isomorphism, abstract and cement Boolean algebras are the same thing. This very nontrivial result relies upon the Boolean prime ideal hypothesis, a decision guideline somewhat weaker than the axiom of decision, and is treated in more detail in the article Stone's representation hypothesis for Boolean algebras. This strong relationship recommends a more fragile result reinforcing the perception in the past subsection to the going with simple consequence of representability. The laws fulfilled by every Boolean polynomial math match with those fulfilled by the prototypical Boolean variable based math.

It is flimsier as in it doesn't of itself suggest representability. Boolean algebras are extraordinary here, for instance a connection variable based math is a Boolean polynomial math with extra structure however it isn't the situation that each connection polynomial math is representable in the sense fitting to connection algebras.

The above meaning of a unique Boolean polynomial math as a set and activities fulfilling "the" Boolean laws brings up the issue, what are those laws? A stupid answer is "all Boolean laws," which can be characterized as all conditions that hold for the Boolean polynomial math of 0 and 1. Since there are infinitely numerous such laws this is definitely not a horribly satisfactory answer by and by, prompting the following inquiry: does it get the job done to require just finitely numerous laws to hold?

On account of Boolean algebras the appropriate response is yes. Specifically the finitely numerous conditions we have recorded above do the trick. We state that Boolean variable based math is

FINITELY AXIOMATIZABLE OR FINITELY BASED

Could this once-over be made shorter yet? Again the appropriate response is yes. In any case, a portion of the above laws are induced by a portion of the others. A satisfactory subset of the above laws contains the sets of associativity, commutativity, and absorption laws, distributivity of ∧ over ∨ (or the other distributivity law—one takes care of business), and the two enhancement laws. Truth be told this is the conventional axiomatization of Boolean polynomial math as an enhanced distributive grid.

By presenting extra laws not recorded above it ends up conceivable to shorten the rundown yet further. In 1933, Edward Huntington demonstrated that if the essential tasks are taken to be x∨yand ¬x, with $x \wedge y$ considered a derived operation (e.g. via De Morgan's law in the form $x \wedge y = \neg(\neg x \vee \neg y)$), then the equation $\neg(\neg x \vee \neg y) \vee \neg(\neg x \vee y) = x$ along with the two equations expressing associativity and commutativity of ∨ completely axiomatized Boolean algebra. When the only basic operation is the binary NAND operation $\neg(x \wedge y)$, Stephen Wolfram has proposed in his book *A New Kind of Science* the single axiom $((xy)z)(x((xz)x)) = z$ as a one-equation axiomatization of Boolean algebra, where for convenience here xy denotes the NAND rather than the AND of x and y.

Concrete Boolean algebras

A solid Boolean algebra or field of sets is any nonempty set of subsets of a given set X shut under the set operations of association, crossing point, and supplement relative to X.[4]

(As an aside, historically X itself was required to be nonempty as well to avoid the degenerate or one-component Boolean algebra, which is the one special case to the standard that all Boolean algebras satisfy the same equations since the degenerate algebra satisfies each equation. Anyway this prohibition clashes with the preferred simply equational meaning of "Boolean algebra," there being no real way to discount the one-component algebra utilizing just equations—0 ≠ 1 does not check, being a negated equation. Consequently current authors allow the degenerate Boolean algebra and let X be vacant.)

Example 1

The power set 2X of X, comprising of all subsets of X. Here X may be any set: unfilled, limited, boundless, or even uncountable.

Example 2

The unfilled set and X. This two-component algebra demonstrates that a solid Boolean algebra can be limited notwithstanding when it comprises of subsets of an unending set. It very well may be seen that each field of subsets of X must contain the unfilled set and X. Thus no smaller example is conceivable, other than the degenerate algebra obtained by taking X to be vacant in order to make the unfilled set and X correspond.

Example 3

The arrangement of limited and cofinite sets of whole numbers, where a cofinite set is one discarding just limitedly many numbers. This is clearly shut under supplement, and is shut under association because the association of a cofinite set with any set is cofinite, while the association of two limited sets is limited. Convergence behaves like association with "limited" and "cofinite" interchanged.

Example 4

For a less trivial example of the point made by Example 2, consider a Venn diagram framed by n shut bends partitioning the diagram into 2n districts, and let X be the

The Binary System

In the decimal framework when a number is perused from directly to left every digit is increased by a dynamically higher intensity of 10. These are regularly alluded to as ones, tens, hundreds, thousands, etc. In the twofold framework a similar idea applies with the exception of that the number being raised are forces of two, the digits accordingly speaks to ones, twos, fours, eights, etc. In this tallying framework the main numbers that are utilized are ones.

Example of the comparison of Binary and Decimal Systems.

Decimal system Binary System

10^3 10^2 10^1 10^0 2^4 2^3 2^2 2^1 2^0

The number-crunching in the twofold framework utilizes indistinguishable tasks from the decimal framework however might be viewed as more straightforward. The expansion includes gathering things in gatherings of twos with conveying to the following higher power.

Example (a) 10 + 10

10

+ 10

100

carrying to the next higher power.

NUMBER CONVERSION

Binary to Decimal conversion: In a binary number each position corresponds to a power of two.

Example (a) 110 means $2^2 + 2^1 + 0$

$=4 + 2 + 0$

$= 6$

(b) 101 means $2^2 + 0 + 2^0(1)$

$=4 + 0 + 1$

$=5$

(c) 11001 means $2^4 + 2^3 + 0^2 + 0^1 + 2^0$

$=16 + 8 + 0 + 0 + 1$

$= 25$

Multiplication is also a straight forward procedure, since each digit is either 0 Or 1; therefore each potential product is either zero or one.

Example 10 x 01

10	10
x01	x 10
10	00
	10 partial products
	100

How to design circuits:

The initial phase in the plan of a circuit is to build up a reality table that demonstrates the yield for every single imaginable information.

Truth Tables

(a) A	A	(b) A	B	A.B	(c) A	B	A + B
0	1	0	0	0	0	0	0
1	0	0	1	0	0	1	1
not		1	0	0	1	0	1
		1	1	1	1	1	1
			and			or	

With appropriate info electronic advanced circuits (rationale circuits) build up legitimate control ways. By going twofold flags through different blend of rationale circuits, any ideal data for processing can be worked on; each flag speaks to a paired conveying one "piece" of data.

Application to Digital Computer Circuits

Computer performs among other things, all kinds of arithmetic operations. The most basic operation is the addition of two binary digits, which consists of 0 + 0 = 0, 0 + 1 = 1, 1 + 1 = 10 and 1 + 0 = 1. The first three operations had sum that is a single digit, but 1 + 1 has a sum of two digits. The higher bit of this sum is called a carry. In adding two multiple digits numbers a carry is to be added to the next higher digit. The circuit that performs the addition of two bits is called a half adder. The circuit that performs the addition of three bits is called a full adder. A half adder needs two binary inputs (A and B) and two binary out puts (S = sum; C= carry).

Truth Table for Half adder.

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

The fourth row shows 1 + 1 = 10, here 1 is the carry to the next higher power of two.

Boolean Algebra applied to Electrical problems.

This thought can be connected to voltages that are available in a physical circuit. There are high voltages and low voltages. High voltage implies that current is streaming, low voltage connotes that there is no current. These circumstances must be given some numerical hugeness.

let high voltage = 1 or let high = 0

low voltage = 0 low = 1

from these we can write truth tables to show the desired operations (or, and).

Input Output

A	B	F
low	low	low
low	high	high
high	low	high
high	high	high

or operation

Input Output input output

A	B	F	A	B F
0	0	0	1	1 1
0	1	1	1	0 0
1	0	1	0	1 0
1	1	1	0	0 0

In conclusion if the dualities of high and low, on and off are given values from the elements of the set of boolean algebra many physical electrical problems can be solved.

BOOLEAN ALGEBRA:

Definition: A Boolean algebra is an algebra (B, Ú, Ù, (-)~, 0, 1) with two binary operations, one unary operation (called complementation), and two nullary operations which satisfies :

is a distributive lattice;

$(B, \vee, \wedge)$

$x \wedge 0 = 0, x \vee 1 = 1$ for all $x \in B$; (3) $x \wedge x' = 0$ $x \vee x' = 1$ for all $x \in B$.

We can easily prove that

$= x, (x \vee y)' = x' \wedge y', (x \wedge y)' = x' \vee y'$ for all $x, y \in$ B.

Definition: Give X a chance to be a set. The Boolean algebra of subsets of X, P(X), has as its universe P(X) and as operations $\vee, \wedge, ', \phi,$, X. The Boolean Algebra **2** = {0, 1} is given by $(2, \vee, \wedge, ', 0, 1),$ Where $(2, \vee, \wedge)$ is a 2 component lattice with 0 < 1 and where 0¢ = 1, 1¢ = 0.Alternative frameworks of postulates for Boolean Algebras were seriously examined amid the decades 1900 - 1940. E.V. Huntington composed an influential early paper [3] regarding this matter. No attempt will be made here to overview the broad literature on such postulate frameworks. We present here Huntington's postulates in 1.1.3

Huntington's Theorem

Let B have one binary operationÚand oneunary operation (-)¢ and define (i)$g \vee p = (g' \wedge p')$ for all a, $b \in B$. . Suppose for all a, b, c Î B, (ii)$a \vee b = b \vee a$; ; (iii)$a \vee (b \vee c) = (a \vee b) \vee c(iv)(g \vee p) \wedge (g \vee p')$ = a. Then B is a Boolean algebra.

Theorem

Let B have one binary operationÙand one unary operation (-)¢and define (i) $a \vee b = (a' \wedge b')'$ for all $a, b \in$ B. Suppose for a, b, c Î B,

(ii) $a \vee b = b \vee a,$ (iii) $(a \vee b) \vee c = a \vee (b \vee c)$ (iv) $(a \wedge b) \vee (a \wedge b')$ = a. Then B is a Boolean algebra.

Pre A* - Algebra:

Definition**:** An algebra (A,Ú,Ù, (-)˜) satisfying:

(a) $(x^{\sim})^{\sim} = x, \forall x \in A$

(b) $x \wedge x = x, \forall x \in A$

(c) $x \wedge y = y \wedge x, \forall x,y \in A$

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}, \forall x,y \in A$

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \forall x,y,z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \forall x,y,z \in A$

(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \forall x,y \in A$

is called a Pre A* - algebra.

Note

If (mn) is an axiom in Pre A* - algebra, then (mn)˜is its dual.

Examples

3 = {0,1,2} with $\vee, \wedge, (-)^{\sim}$ defined below is a Pre A* - algebra

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^{\sim}$
0	1
1	0

CONCLUSION

So as to characterize a Boolean algebra, we need the additional idea of complementation. Definition: Complemented Lattice. Let L;, contains a least component, 0, and a greatest component, 1. L; is called a supplemented lattice if and if for each component a œ L, there exists a component an in L with the end goal that an a = 0 and an a = 1. Such a component an is called a supplement of the component a.

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