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An Analysis upon Solving Crack Problem in Generalized Thermoelasticity: General Solution

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Abstract – The crack boundary is due to a prescribed temperature and stress distribution. By using the finite element method, the numerical solutions of the components of displacement, temperature and the stress components have been obtained. General solution for arbitrary heat flux along the crack face is obtained. For some particular cases, for example, the constant heat flux case and remote heat flux case, a closed form solution is obtained. The solution technique is effective in derivation and compact in form.

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INTRODUCTION

A unified generalized thermoelasticity formulation: Application to penny shaped crack analysis

Lately, impressive exertion has been dedicated to the investigation of breaks in solids, because of their applications in industry, in general, and in the manufacture of electronic segments, specifically, just as in geophysics and seismic tremor designing. They happen for some, reasons incorporating regular imperfections in materials, because of creation process, vulnerabilities in the stacking or condition, insufficiencies in plan and lacks in development or upkeep. Thus all structures contain breaks as assembling imperfections or as a result of administration stacking which can be either mechanical or thermal. On the off chance that the heap is oftentimes connected, the break may develop in weakness to a last crack. As the size of the split builds, the residual quality of structure stops. In the last phases of the break development, the rate expands all of a sudden prompting a disastrous structure disappointment

Investigation of such disappointment mechanics keeps up the auxiliary trustworthiness, because of splits. In 1983 the National Institute for Science and Technology and Battele Memorial Institute evaluated Mgh costs for disappointment because of crack [184]. A comparative report authorized by the European Union presumed that billions of ECU every year could be spared utilizing break mechanics technology.

The problem of penny molded breaks in flexible solids has pulled in wide consideration since Sack stretched out Griffith's theory of burst to three measurements. It is absurd to expect to audit every single pertinent production here as Fabrikant has shown that there are several papers managing this theme. Luckily intrigued perusers may allude to the survey article composed by Panasyuk et al. or on the other hand to the book composed by Kassir and Sih for progressively nitty gritty information.

In solids, thermal burdens play a significant and every now and again even a primary job in numerous fields, for example, in structure airplanes, machines, gas and stream turbines, and so on.. The aftereffect of unaccounted instigated thermal pressure might be calamitous as a rule. So a comprehension of thermally instigated worries in solids is basic for a comprehensive investigation of the assembling stages.

Presence of an opening or break in a strong causes aggravation in heat stream and the neighbourhood temperature gradient around the brokenness increments. Thermal unsettling influences of this sort can produce material disappointment through break propagation. Florence and Goodier examined stream initiated thermal worries in unbounded isotropic solids.

Hosseini-Teherani and Hosseini-Godrazi presented a boundary component formulation for the split examination using the Lord-Shulman theory of thermoelasticity. Sherief and El-Maghraby tackled a dynamical problem for a vast thermoelastic strong with an inner penny molded split which is exposed to prescribed temperature and stress circulations. They have used L-S theory of thermoelasticity and Laplace and Hankel transform method to take care of the problem. Sherief and El-Maghraby tackled a dynamical problem for a boundless thermoelastic strong with limited linear break inside the medium.

This problem has been comprehended utilizing L-S theory of thermoelasticity.

Despite these, moderately less consideration has been paid to examine the thermoelastic split problem utilizing generalized thermoelasticity theories. In this article, we take care of a dynamical problem for a penny-formed split in a limitless, homogeneous and isotropic generalized thermoelastic medium. This problem has been unraveled with regards to CCTE, Lord-Shulman and Green-Naghdi models. Laplace and Hankel transforms strategies have been utilized to take care of the problem. Reversal of twofold transform has been done numerically. Numerical reversal of Laplace transform has been finished applying the method of Bellman et al. The numerical outcomes for displacement, stress and temperature are acquired for copper material and are demonstrated graphically to look at the outcomes for CCTE, GN model II, GN model III and LS model. Variety of stress power factor against the sweep of the break and against the time have been indicated graphically for all the previously mentioned models.

FORMULATION OF THE PROBLEM

Give us a chance to consider a limitless homogeneous isotropic generalized thermoelastic medium containing a penny-formed split, which is exposed to prescribed temperature and stress disseminations. Give the body a chance to be alluded to barrel shaped co-ordinate framework (r, 6, z) and the inner split involve the district z = 0,0 < r < an as appeared in Fig.l. Since the geometry of the locale is symmetric about the split plane, the problem is diminished to a blended boundary esteem problem of thermoelasticity for the district z > 0, r > 0. Every single considered capacity will rely upon r,z and t just, i.e., the displacement vector u and temperature T can be taken in the form u = (u(r,z,t),0,w(r,z,t)) and T = T(r, 2, t). The bound together equations for classical coupled thermoelasticity (CCTE), Lord-Shulman and Green-Naghdi theories for isotropic linearly versatile strong are

$$\mu\nabla^2 u - \mu\frac{u}{r^2} + (\lambda+\mu)\frac{\partial e}{\partial r} - \gamma\frac{\partial T}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2},$$

$$\mu\nabla^2 w + (\lambda+\mu)\frac{\partial e}{\partial z} - \gamma\frac{\partial T}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2},$$

$$K\left(t_1 + t_2\frac{\partial}{\partial t}\right)\nabla^2 T + t_3 K^{\star}\nabla^2 T = \left(t_1\frac{\partial}{\partial t} + \chi\frac{\partial^2}{\partial t^2}\right)(\rho C_v T + \gamma T_0 e),$$

where t1 = l, t2 = 0, t3 = 0,x = 0 for CCTE, t1 = l, t2 = 0, t3 = 0, x = T0 for LS model, $t_1 = 0, t_2 = 1, t_3 = 1, \chi = 1$ for GN model III and $t_1 = 0, t_2 = 0, t_3 = 1, \chi = 1$ for GN

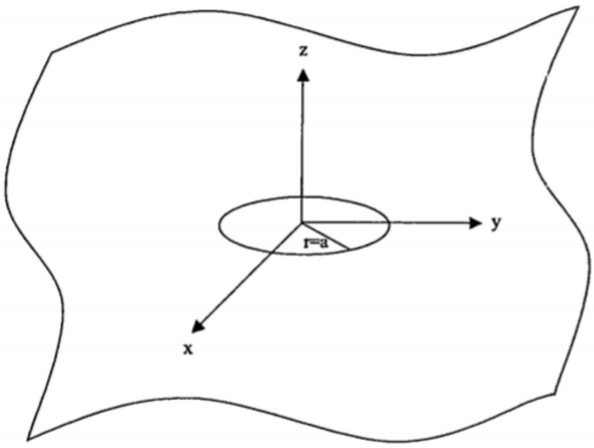

Fig.l: Co-ordinate system and geometry of the solution domain.

model II, and T is the absolute temperature and e is the cubical dilatation given by

$$e = \frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z},$$

p is the density, A and p, are Lame constants, K is thermal conductivity, K* is the material constant for GN models, 7 = (3A + 2p)at, at being the co-efficient of linear thermal expansion, To is the reference temperature assumed to be such that | |«C 1, Cv is the specific heat at constant strain, r0 is the relaxation time , V2 is the Laplacian, given in our case by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

Constitutive equations in the present case are

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma(T - T_0),$$

$$\sigma_{zz} = 2\mu\frac{\partial w}{\partial z} + \lambda e - \gamma(T - T_0),$$

$$\sigma_{rz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right).$$

Dimensionless parameters used here are

$$r' = c_1\eta r,\ z' = c_1\eta z,\ u' = c_1\eta u,\ w' = c_1\eta w,\ a' = c_1\eta a,$$

$$t' = c_1^2\eta t,\ \tau_0' = c_1^2\eta\tau_0,\ \sigma_{ij}' = \frac{\sigma_{ij}}{\mu},\ \theta = \frac{T - T_0}{T_0},$$

$$\beta^2 = \frac{\lambda+2\mu}{\mu},\ g = \frac{\gamma}{\rho C_v},\ b = \frac{\gamma T_0}{\mu},\ K^{\star\prime} = \frac{K^{\star}}{K c_1^2\eta},$$

Where

$$\eta = \frac{\rho C_v}{K},\ c_1^2 = \frac{\lambda + 2\mu}{\rho}.$$

Then, after omitting primes, equations can be rewritten as

$$\nabla^2 u - \frac{u}{r^2} + (\beta^2 - 1)\frac{\partial e}{\partial r} - b\frac{\partial \theta}{\partial r} = \beta^2 \frac{\partial^2 u}{\partial t^2},$$

$$\nabla^2 w + (\beta^2 - 1)\frac{\partial e}{\partial z} - b\frac{\partial \theta}{\partial z} = \beta^2 \frac{\partial^2 w}{\partial t^2},$$

$$(t_1 + t_2 G\frac{\partial}{\partial t})\nabla^2\theta + t_3 K^* G\nabla^2\theta = (t_1\frac{\partial}{\partial t} + \chi G\frac{\partial^2}{\partial t^2})(\theta + ge),$$

where $G = c_1^2\eta$, and $t_1 = 1, t_2 = 0, t_3 = 0, \chi = 0$ for CCTE, $t_1 = 1, t_2 = 0, t_3 = 0, \chi = \frac{\tau_0}{G}$ for LS model, $t_1 = 0, t_2 = 1, t_3 = 1, \chi = 1$ for GN model III and $t_1 = 0, t_2 = 0, t_3 = 1, \chi = 1$ for GN model II.

Constitutive equations become

$$\sigma_{rr} = 2\frac{\partial u}{\partial r} + (\beta^2 - 2)e - b\theta,$$

$$\sigma_{zz} = 2\frac{\partial w}{\partial z} + (\beta^2 - 2)e - b\theta,$$

$$\sigma_{rz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right).$$

We note that the equation retains its form.

Combining equations and using equation we obtain

$$\nabla^2 e - c\nabla^2\theta = \frac{\partial^2 e}{\partial t^2},\ \text{where } c = \frac{b}{\beta^2}.$$

The above equations are solved subject to the initial conditions

$$u = v = \theta = 0;\ \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial \theta}{\partial t} = 0 \text{ at t=0}.$$

Boundary conditions for the heat conduction problem on z=0 may be taken as

$$\theta(r,0,t) = G_1(r),\ 0 < r < a$$
$$\sigma_{zz}(r,0,t) = G_2(r),\ 0 < r < a$$
$$\frac{\partial \theta}{\partial z}(r,0,t) = 0,\ a < r < \infty$$
$$w(r,0,t) = 0,\ a < r < \infty$$
$$\sigma_{rz}(r,0,t) = 0,\ 0 < r < \infty.$$

Applying Laplace transform defined by the relation

$$\bar{f}(r,z,p) = \int_0^\infty e^{-pt} f(r,z,t)dt,\ Re(p) > 0$$

to the equations (5.1.6)-(5.1.10) we get

$$\nabla^2\bar{u} - \frac{\bar{u}}{r^2} + (\beta^2 - 1)\frac{\partial \bar{e}}{\partial r} - b\frac{\partial \bar{\theta}}{\partial r} = \beta^2 p^2 \bar{u},$$

$$\nabla^2\bar{w} + (\beta^2 - 1)\frac{\partial \bar{e}}{\partial z} - b\frac{\partial \bar{\theta}}{\partial z} = \beta^2 p^2 \bar{w},$$

$$[(t_1 + pGt_2 + GK^* t_3)\nabla^2 - p(t_1 + p\chi G)]\bar{\theta} = gp(t_1 + p\chi G)\bar{e},$$

$$(\nabla^2 - p^2)\bar{e} = c\nabla^2\bar{\theta},$$

$$\bar{\sigma}_{rr} = 2\frac{\partial \bar{u}}{\partial r} + (\beta^2 - 2)\bar{e} - b\bar{\theta},$$

$$\bar{\sigma}_{zz} = 2\frac{\partial \bar{w}}{\partial z} + (\beta^2 - 2)\bar{e} - b\bar{\theta},$$

$$\bar{\sigma}_{rz} = \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial r}\right).$$

Boundary conditions in transformed domain take the form

$$\bar{\theta}(r,0,p) = \frac{G_1(r)}{p},\ 0 < r < a$$

$$\bar{\sigma}_{zz}(r,0,p) = \frac{G_2(r)}{p},\ 0 < r < a$$
$$\frac{\partial \bar{\theta}}{\partial z}(r,0,p) = 0,\ a < r < \infty$$
$$\bar{w}(r,0,p) = 0,\ a < r < \infty$$
$$\bar{\sigma}_{rz}(r,0,p) = 0,\ 0 < r < \infty.$$

Now eliminating e between equation we get

$$[(t_1 + pGt_2 + K^* Gt_3)\nabla^4 - \{p(t_1 + p\chi G)(1 + \epsilon) + p^2(t_1 + pGt_2 + K^* Gt_3)\}\nabla^2$$

$$+ p^3(t_1 + p\chi G)]\bar{\theta} = 0,\ \epsilon = gc,$$

which can be written in the following form

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)\bar{\theta} = 0,$$

where k2 and k2 are the roots of the equation

$$[(t_1 + pGt_2 + K^* Gt_3)k^4 - \{p(t_1 + p\chi G)(1 + \epsilon) + p^2(t_1 + pGt_2 + K^* Gt_3)\}k^2 + p^3(t_1 + p\chi G)] = 0.$$

The solution of equation can be written in the form 9 = 9X + 02, where 6{ is the solution of the equation

$$(\nabla^2 - k_i^2)\bar{\theta}_i = 0, \; i = 1,2.$$

The Hankel transform with parameter a of a function f(r,z,p) is given by,

$$\bar{f}^*(\alpha,z,p) = \mathcal{H}[\bar{f}(r,z,p)] = \int_0^\infty \bar{f}(r,z,p) r J_0(\alpha r) dr,$$

where J0 is the Bessel function of the first kind of order zero. The inverse Hankel transform is given by

$$\bar{f}(r,z,p) = \int_0^\infty \bar{f}^*(\alpha,z,p)\alpha J_0(\alpha r) d\alpha.$$

Taking the Hankel transform with parameter a of both sides of equation and using the operational relation of the Hankel transform

$$\mathcal{H}\Big(\frac{\partial^2 \bar{f}(r,z,p)}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{f}(r,z,p)}{\partial r}\Big) = -\alpha^2 \bar{f}^*(\alpha,z,p)$$

we obtain

$$[\mathcal{D}^2 - (k_i^2 + \alpha^2)]\bar{\theta}_i^* = 0, \; i = 1,2, \text{where } \mathcal{D} = \frac{\partial}{\partial z}.$$

The solution of this equation, which is bounded at infinity, can be written as

$$\bar{\theta}_i^*(\alpha,z,p) = A_i(\alpha,p)(k_i^2 - p^2)e^{-q_i|z|}, \text{ where } q_i = \sqrt{\alpha^2 + k_i^2}.$$

Due to symmetry, we shall consider only the case when z > 0; we thus have

$$\bar{\theta}^*(\alpha,z,p) = \sum_{i=1}^{2} A_i(\alpha,p)(k_i^2 - p^2)e^{-q_i z} \text{ for } z > 0.$$

Taking the inverse Hankel transform of both sides of equation (5.1.23), we obtain

$$\bar{\theta}(r,z,p) = \int_0^\infty \Big[\sum_{i=1}^{2} A_i(\alpha,p)(k_i^2 - p^2)e^{-q_i z}\Big]\alpha J_0(\alpha r) d\alpha \text{ for } z > 0.$$

Similarly eliminating 6 between equations, we find that e satisfies the same differential equation as 9. The solution of this equation compatible with equation (5.1.16) is given by

$$\bar{e}^*(\alpha,z,p) = c\sum_{i=1}^{2} A_i(\alpha,p)k_i^2 e^{-q_i z} \text{ for } z > 0.$$

Using the inverse Hankel transform, we obtain

$$\bar{e}(r,z,p) = c\int_0^\infty \Big[\sum_{i=1}^{2} A_i(\alpha,p)k_i^2 e^{-q_i z}\Big]\alpha J_0(\alpha r) d\alpha \text{ for } z > 0.$$

Taking Hankel transform of equation and using equations, we obtain

$$(\mathcal{D}^2 - \alpha^2 - \beta^2 p^2)\bar{w}^* = -c\sum_{i=1}^{2}(k_i^2 - \beta^2 p^2)A_i(\alpha,p)q_i e^{-q_i z}.$$

The solution of equation (5.1.27) for z > 0, which is bounded at infinity, is given by

$$\bar{w}^*(\alpha,z,p) = B(\alpha,p)e^{-qz} - c\sum_{i=1}^{2} A_i(\alpha,p)q_i e^{-q_i z},$$

where B(a,p) is a parameter and q = y/ot2 + (32p2. Taking the inverse Hankel transform of both sides of equation (5.1.28), we obtain

$$\bar{w}(r,z,p) = \int_0^\infty \Big[B(\alpha,p)e^{-qz} - c\sum_{i=1}^{2} A_i(\alpha,p)q_i e^{-q_i z}\Big]\alpha J_0(\alpha r) d\alpha \text{ for } z > 0.$$

Taking Laplace-Hankel double transform of both sides of equation and using equations we obtain

$$\mathcal{H}\Big[\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u})\Big] = B(\alpha,p)qe^{-qz} - c\alpha^2\sum_{i=1}^{2} A_i(\alpha,p)e^{-q_i z}.$$

The solution of equation can be obtained as follows

$$\bar{u}(r,z,p) = \int_0^\infty \Big[B(\alpha,p)qe^{-qz} - c\alpha^2\sum_{i=1}^{2} A_i(\alpha,p)e^{-q_i z}\Big] J_1(\alpha r) d\alpha.$$

Substituting from equations (5.1.24), (5.1.26), (5.1.29) and (5.1.31) into equations (5.1.17a),(5.1.17b) and (5.1.17c), and using the relation

$$J_0'(z) = -J_1(z)$$

we obtain the stress components in the form

$$\bar{\sigma}_{zz} = \int_0^\infty \Big[c(q^2 + \alpha^2)\sum_{i=1}^{2} A_i(\alpha,p)e^{-q_i z} - 2B(\alpha,p)qe^{-qz}\Big]\alpha J_0(\alpha r) d\alpha,$$

$$\bar{\sigma}_{rr} = \int_0^\infty \Big[2B(\alpha,p)qe^{-qz}J_1'(\alpha r) + \sum_{i=1}^{2}\big[c(\beta^2 p^2 - 2k_i^2)J_0(\alpha r) - 2c\alpha^2 J_1'(\alpha r)\big]A_i(\alpha,p)e^{-q_i z}\Big]\alpha d\alpha,$$

$$\bar{\sigma}_{rz} = \int_0^\infty \Big[2c\alpha^2\sum_{i=1}^{2} A_i(\alpha,p)q_i e^{-q_i z} - B(\alpha,p)(q^2 + \alpha^2)e^{-qz}\Big] J_1(\alpha r) d\alpha.$$

Dual integral equation formulation

Substituting from equations into the boundary conditions, we obtain the relations

$$\int_0^\infty \left[\sum_{i=1}^{2} A_i(\alpha,p)q_i(k_i^2-p^2)\right]\alpha J_0(\alpha r)d\alpha = 0,\ a<r<\infty,$$

$$\int_0^\infty \left[B(\alpha,p) - c\sum_{i=1}^{2} A_i(\alpha,p)q_i\right]\alpha J_0(\alpha r)d\alpha = 0,\ a<r<\infty,$$

$$\int_0^\infty \left[2c\alpha^2\sum_{i=1}^{2} A_i(\alpha,p)q_i - B(\alpha,p)(q^2+\alpha^2)\right]J_1(\alpha r)d\alpha = 0,\ 0<r<\infty,$$

$$\int_0^\infty \left[\sum_{i=1}^{2} A_i(\alpha,p)(k_i^2-p^2)\right]\alpha J_0(\alpha r)d\alpha = \frac{G_1(r)}{p},\ 0<r<a,$$

$$\int_0^\infty \left[c(q^2+\alpha^2)\sum_{i=1}^{2} A_i(\alpha,p) - 2B(\alpha,p)q\right]\alpha J_0(\alpha r)d\alpha = \frac{G_2(r)}{p},\ 0<r<a.$$

Since equation is valid for all values of r, we obtain B{a,p) in the following form

$$B(\alpha,p) = \frac{2c\alpha^2[A_1(\alpha,p)q_1 + A_2(\alpha,p)q_2]}{(q^2+\alpha^2)}.$$

Substituting for B(a,p) from equation into equations, we obtain

$$\int_0^\infty \frac{1}{2\alpha^2+\beta^2p^2}\left(\sum_{i=1}^{2} A_i(\alpha,p)q_i\right)\alpha J_0(\alpha r)d\alpha = 0,\ a<r<\infty,$$

$$\int_0^\infty \frac{1}{2\alpha^2+\beta^2p^2}\left(\sum_{i=1}^{2} A_i(\alpha,p)[(2\alpha^2+\beta^2p^2)^2 - 4\alpha^2qq_i]\right)\alpha J_0(\alpha r)d\alpha = \frac{f_1(r)}{pc},\ 0<r<a$$

Equations are a set of four dual integral equations whose solution gives the unknown variables Ai(a,p) and A2(a,p). We shall now look for relations between Ai(a,p) and A2{a,p) to reduce the problem to find the solution of only two dual integral equations in one of the variables, say Ai(a,p). We assume for the case r > a that

$$A_i(\alpha,p) = \frac{1}{\alpha q_i}\int_a^\infty g_{i1}(\nu,p)\sin(\alpha\nu)d\nu,\ i=1,2.$$

Substituting from equation (5.1.44) into the equation (5.1.36) and changing the order of integration, we obtain

$$\sum_{i=1}^{2}(k_i^2-p^2)\int_a^\infty g_{i1}(\nu,p)d\nu\int_0^\infty J_0(\alpha r)\sin(\alpha\nu)d\alpha = 0.$$

Using the following integral formula of the Bessel functions

$$\int_0^\infty J_0(\alpha r)\sin(\alpha\nu)d\alpha = \frac{1}{\sqrt{\nu^2-r^2}}, r<\nu$$

$$= 0, r>\nu$$

relation reduces to

$$\sum_{i=1}^{2}(k_i-p^2)\int_r^\infty \frac{g_{i1}(u,p)}{\sqrt{u^2-r^2}}du = 0,\ a<r<\infty.$$

Multiplying both sides of equation (5.1.47) by equation with respect to r from r = v to r = oo, equation with respect to u, we obtain

$$g_{21}(\nu,p) = -\frac{(k_1^2-p^2)}{(k_2^2-p^2)}g_{11}(\nu,p),\ r>a.$$

Substituting from equation (5.1.48) into equation (5.1.44) we can write the relation between Ai(a,p) and A2(a,p), for r > o, in the form

$$A_2(\alpha,p) = -\frac{(k_1^2-p^2)q_1}{(k_2^2-p^2)q_2}A_1(\alpha,p),\ r>a.$$

Similarly, for the case r < a, we assume that

$$A_i(\alpha,p) = \frac{1}{\alpha}\int_0^a g_{i2}(\nu,p)\sin(\alpha\nu)d\nu,\ 0<r<a,\ i=1,2.$$

Substituting from equation (5.1.50) into equation (5.1.39) and using the same procedure as before, we obtain

$$g_{22}(\alpha,p) = -\frac{1}{(k_2^2-p^2)}\left(\frac{F(\nu)}{p} + (k_1^2-p^2)g_{12}(\alpha,p)\right),\ 0<r<a,$$

and then substituting this into we arrive at the relation

$$A_2(\alpha,p) = -\frac{1}{(k_2^2-p^2)}\left(\frac{H(\alpha)}{p} + (k_1^2-p^2)A_1(\alpha,p)\right),\ 0<r<a,$$

Where

$$H(\alpha) = \frac{1}{\alpha}\int_0^a F(\nu)\sin(\alpha\nu)d\nu \text{ and } F(\nu) = \frac{2}{\pi}\frac{d}{d\nu}\int_\nu^a \frac{rG_1(r)dr}{\sqrt{r^2-\nu^2}}.$$

Substituting from equations into equations we obtain two dual integral equations in the unknown parameter Ai{a,p)

$$\int_0^\infty \frac{A_1(\alpha,p)q_1}{2\alpha^2+\beta^2p^2}\alpha J_0(\alpha r)d\alpha = 0,\ r>a,$$

$$\int_0^\infty A_1(\alpha,p)K_1(\alpha,p)\alpha J_0(\alpha r)d\alpha = J(r,p),\ r<a,$$

Where

$$K_1(\alpha,p) = \frac{(2\alpha^2+\beta^2p^2)^2(k_2^2-k_1^2) - 4\alpha^2q[q_1(k_2^2-p^2) - q_2(k_1^2-p^2)]}{2\alpha^2+\beta^2p^2}$$

And

$$J(r,p)=\frac{f_1(r)(k_2^2-p^2)}{pc}+\frac{1}{p}\int_0^\infty H(\alpha)\Big[\frac{(2\alpha^2+\beta^2p^2)^2-4\alpha^2q q_2}{(2\alpha^2+\beta^2p^2)}\Big]\alpha J_0(\alpha r)d\alpha,\ r<a.$$

Solution of the dual integral equations

In order to solve the preceding system, we use the operator of fractional calculus known as the modified Hankel transform operator and defined by the relation

$$S_{a,b}[f(u)]=2^b x^{-b}\int_0^\infty u^{1-b}f(u)J_{2a+b}(xu)du.$$

Use will also be made of the two operators known as the modified Kober-Erdelyi operators and defined by the relations

$$I_{a,b}[f(u)] = \frac{2x^{-2b-2a}}{\Gamma(b)}\int_0^x (x^2-u^2)^{b-1}u^{2a+1}f(u)du,(0<b<1)$$

$$= \frac{x^{-2b-2a-1}}{\Gamma(1+b)}\frac{d}{dx}\int_0^x (x^2-u^2)^b u^{2a+1}f(u)du,(-1<b<0)$$

$$K_{a,b}[f(u)] = \frac{2x^{2a}}{\Gamma(b)}\int_x^\infty (u^2-x^2)^{b-1}u^{-2b-2a+1}f(u)du,(0<b<1)$$

$$= -\frac{x^{2a-1}}{\Gamma(1+b)}\frac{d}{dx}\int_x^\infty (u^2-x^2)^b u^{-2b-2a+1}f(u)du,(-1<b<0)$$

We now define the function ^(a,p) by the relation

$$\psi(\alpha,p)=\frac{A_1(\alpha,p)q_1}{2\alpha^2+\beta^2p^2}.$$

Using equation can be written in the form

$$\int_0^\infty \psi(\alpha,p)\alpha J_0(\alpha r)=0,\ r>a,$$

$$\int_0^\infty [K(\alpha,p)+\gamma]\psi(\alpha,p)\alpha^2 J_0(\alpha r)d\alpha=J(r,p),\ r<a,$$

Where

$$K(\alpha,p)=\frac{(2\alpha^2+\beta^2p^2)K_1(\alpha,p)}{q_1\alpha}-\gamma \text{ and } \gamma=2p^2(k_2^2-k_1^2)(\beta^2-1).$$

We note for future reference that we have chosen the constant 7 so that for large a,

$$K(\alpha,p)=O(\tfrac{1}{\alpha^2}).$$

Using the operator of the modified Hankel transform, equations can be written in the form

$$S_{0,0}\psi(\alpha,p)=0,\ r>a$$

$$S_{\frac{1}{2},-1}[K(\alpha,p)+\gamma]\psi(\alpha,p)=\frac{r}{2}J(r,p),\ r<a$$

We now write the equations (5.1.62) and (5.1.63) in the following form

$$S_{0,0}\psi(\alpha,p)=g(r,p),$$

$$S_{\frac{1}{2},-1}[K(\alpha,p)+\gamma]\psi(\alpha,p)=f(r,p),$$

Where

$$g(r,p) = g_1(r,p),0<r<a$$

$$= g_2(r,p),r>a$$

$$f(r,p) = f_1(r,p),0<r<a$$

$$= f_2(r,p),r>a$$

and in our case /2(r,p), gi(r,p) are unknown but fi(r,p), 52(r>p) are known and given by fi{r,p) = g J{r,P) and g2(r,p) = 0.

In order to reduce the dual integral equations to Eredholm integral equation of second kind, we use a method due to Cooke. Thus putting Sneddon's trial solution

$$\psi(\alpha,p)=S_{0,\frac{1}{2}}h(t,p)=\sqrt{\frac{2}{\alpha}}\int_0^\infty \sqrt{t}h(t,p)J_{\frac{1}{2}}(\alpha t)dt$$

we obtain

$$K_{0,\frac{1}{2}}h(t,p)=g(r,p)$$

and

$$\gamma I_{0,-\frac{1}{2}}h(t,p)+S_{\frac{1}{2},-1}K(\alpha,p)S_{0,\frac{1}{2}}h(t,p)=f(r,p).$$

In deriving the above equations use has been made of the following relationships established

$$S_{a+b,c}S_{a,b}=I_{a,b+c},$$

$$S_{a,b}S_{a+c,b}=K_{a,b+c}.$$

Solving the functional relation (5.1.66) and (5.1.67) we get

$$h(t,p)=K^{-1}_{0,\frac{1}{2}}g(r,p),$$

And

$$\gamma h(t,p)+I^{-1}_{0,-\frac{1}{2}}S_{\frac{1}{2},-1}K(\alpha,p)S_{0,\frac{1}{2}}h(t,p)=I^{-1}_{0,-\frac{1}{2}}f(r,p).$$

where I-1 and if-1 are the inverse operators for which the following relationship hold

$$I_{a,b}^{-1} = I_{a+b,-b},$$
$$K_{a,b}^{-1} = K_{a+b,-b}.$$

Upon utilizing these relationship, equations reduce to

$$h(t,p) = K_{\frac{1}{2},-\frac{1}{2}} g(r,p)$$

And

$$\gamma h(t,p) + S_{\frac{1}{2},-\frac{1}{2}} K(\alpha,p) S_{0,\frac{1}{2}} h(t,p) = I_{-\frac{1}{2},\frac{1}{2}} f(r,p).$$

where following relation has been used

$$I_{a+b,c} S_{a,b} = S_{a,b+c}.$$

Now writing the equation (5.1.70) in the interval [a, oo) and the equation (5.1.71) in the interval [0, a] we get respectively

$$h_2(t,p) = 0$$

and

$$\gamma h_1(t,p) + S_{\frac{1}{2},-\frac{1}{2}} K(\alpha,p) S_{0,\frac{1}{2}} h(t,p) = I_{-\frac{1}{2},\frac{1}{2}} f(r,p),$$

where hi(t,p) and h2(t,p) are the parts of h(t,p) corresponding to the intervals [0,a] and [a, oo)

After some manipulations and using equation (5.1.72), equation (5.1.73) can be written as a Fredholm integral equation of the second kind in the unknown function hi(t,p) of the form

$$h_1(u,p) + \int_0^a h_1(t,p) \bar{K}(u,t,p) dt = \bar{J}(u,p), \ 0 < u < a$$

Where

$$\bar{K}(u,t,p) = \frac{\sqrt{ut}}{\gamma} \int_0^\infty K(\alpha,p) \alpha J_{\frac{1}{2}}(\alpha u) J_{\frac{1}{2}}(\alpha t) d\alpha$$

And

$$\bar{J}(u,p) = \frac{1}{\gamma\sqrt{\pi}} \int_0^u \frac{rJ(r,p)dr}{\sqrt{u^2 - r^2}}.$$

and finally ip(a,p) can be obtained using (72), in the following form

$$\psi(\alpha,p) = \sqrt{\frac{2}{\alpha}} \int_0^a \sqrt{t} h_1(t,p) J_{\frac{1}{2}}(\alpha t) dt.$$

We note that with our choice of 7, we made the improper integral defining K(u,t,p) convergent. Solving the integral equation (5.1.74), we can obtain the function hi(t,p) . Prom equation (5.1.77) we obtain Tj){a,p). Prom equation (5.1.49), (5.1.52) and (5.1.59) we obtain unknown parameters Ai(or,p) and A2(a:,p). Knowing Ai(a,p), A2(a,p) we can obtain temperature distribution, displacement components and stresses in Laplace transform domain from equations (5.1.24), (5.1.29), (5.1.31) and (5.1.33)-(5.1.35) respectively.

The methods for solving the integral equation numerically [52] and for inverting the Laplace transform numerically [21] are discussed in Appendix.

NUMERICAL RESULTS AND DISCUSSIONS

For numerical purpose we shall take

$$G_1(r) = \theta_0 (\text{constant}), \ 0 < r < a,$$

$$G_2(r) = -1, \ 0 < r < a,$$

then we get from

$$F(\nu) = \frac{-2\nu\theta_0}{\pi\sqrt{a^2 - \nu^2}},$$

$$H(\alpha) = \frac{-a\theta_0 J_1(\alpha a)}{\alpha}.$$

Changing the order of integration, the function J(u,p) in equation (5.1.76) can be written in the form

$$\bar{J}(u,p) = \frac{1}{\gamma p\sqrt{\pi}} \Big[-\frac{(k_2^2 - p^2)u}{c} + \int_0^\infty \alpha H(\alpha) \Big[\frac{(2\alpha^2 + \beta^2 p^2)^2 - 4\alpha^2 q q_2}{(2\alpha^2 + \beta^2 p^2)} \Big] d\alpha \int_0^u \frac{rJ_0(\alpha r)dr}{\sqrt{u^2 - r^2}} \Big],$$

and upon using the relation

$$\int_0^1 \frac{xJ_0(xy)}{\sqrt{1-x^2}} dx = \frac{\sin(y)}{y}, \ y > 0$$

we can write J(u,p) in its simplest form as

$$\bar{J}(u,p) = -\Big[\frac{(k_2^2 - p^2)u}{\gamma pc\sqrt{\pi}} + \frac{a\theta_0}{\gamma p\sqrt{\pi}} \int_0^\infty \Big[\frac{(2\alpha^2 + \beta^2 p^2)^2 - 4\alpha^2 q q_2}{(2\alpha^2 + \beta^2 p^2)} \Big] \frac{J_1(\alpha a)\sin\alpha u}{\alpha} d\alpha \Big].$$

For numerical computation copper material has been chosen whose material constants have been taken as follows

$$\lambda = 7.76 \times 10^{10} Nm^{-2}, \mu = 3.86 \times 10^{10} Nm^{-2}, \rho = 8954 Kg\, m^{-3},$$
$$K = 386 Wm^{-1}deg^{-1}, C_v = 383.1 JKg^{-1}deg^{-1},$$
$$T_0 = 293K, \alpha_t = 1.78 \times 10^{-5} deg^{-1},$$
$$\epsilon = 0.0168, \theta_0 = 1, a = 1, K^* = 1.$$

Figs.2-5 are drawn to give a comparison of results for temperature, radial and axial displacements and axial stress against radial distance r and Figs. 6-7 are drawn to give a comparison of stress intensity factor against crack radius and time respectively for CCTE, GN model II, GN model III and Lord-Shulman model. Fig. 2 depicts variation of temperature distribution 6 against radial distance r for z = 0.2, £ = 0.2623592. It is

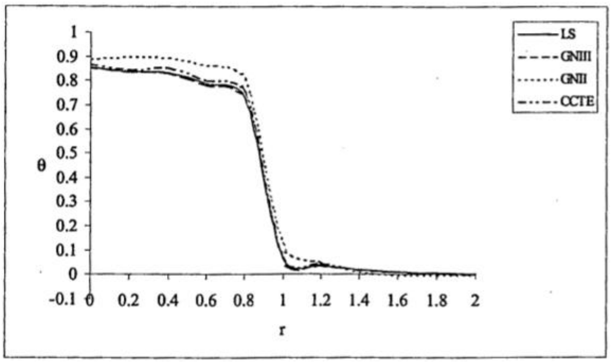

Fig.2: Variation of temperature 0 for z=0.2 and t=0.2623592.

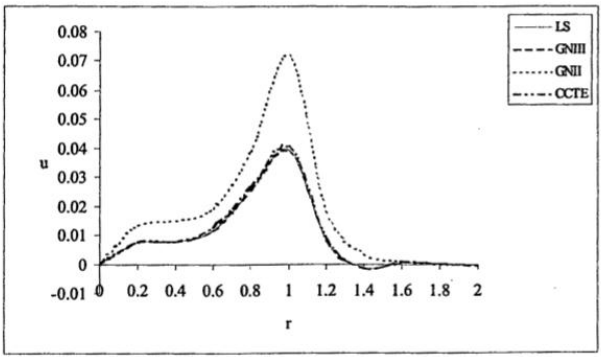

Fig.3: Variation of radial displacement u for z=0.2 and t=0.2623592.

clear from the graph that 6 has maximum value at the centre of the crack, it begin to fall just near the crack edge, where it experiences sharp decreases. It is also observed that for GN model II it has larger value than CCTE, GN model III and LS model because of the presence of the energy dissipative term (see equation (5.1.8)). Besides this the four theories begin to coincide when the radial distance r is beyond the crack radius a and then reduce to the reference temperature of the solid.

Fig.3. & Fig.4. are plotted for radial displacement u and axial displacement w versus radial distance r for the same set of parameters as mentioned above. Fig.3 shows that the radial displacement u increases to reach its maximum magnitude just after the crack circumference and beyond it u falls again to try to attain zero for all the theories as r increases. The Table 1 shows that there is slight difference in magnitude for two models (LS model and GN model III) which is not clear in the figure because of the scaling used here.

Table I: Radia J distribution of Temperature and Radial displacement for LS model and GN model III

	Temperature(θ)		Radial displacement(u)	
r	LS model	GN model III	LS model	GN model III
0.0	0.855848000	0.855012300	0.000000000	0.000000000
0.2	0.838495800	0.837349200	0.007535812	0.007546790
0.4	0.834185500	0.832181500	0.007727411	0.007692841
0.6	0.786120500	0.783120200	0.011460240	0.011402890
0.8	0.728654600	0.726838400	0.025502810	0.025403980
1.0	0.064196950	0.057513000	0.039127930	0.038902070
1.2	0.038188580	0.035973860	0.008115759	0.008006049
1.4	0.020476070	0.020361330	-0.001416521	-0.001428750
1.6	0.010606650	0.011152960	0.000733573	0.000729547
1.8	0.004880605	0.005630486	0.000194748	0.000209995
2.0	0.002167820	0.002833664	-0.000581768	-0.000576406

From Fig.4 we see that the axial displacement w has its maximum value at centre of the crack and begins to fall near the crack edge and becomes zero for the four theories. It is observed that the magnitude of w is larger in case of GN model II than GN model III, which is again larger than LS model and CCTE. The displacement components u and w show different behaviour, because of the elasticity of the solid that tends to resist vertical displacement. Fig.5 represents axial stress azz for the same set of parameters.

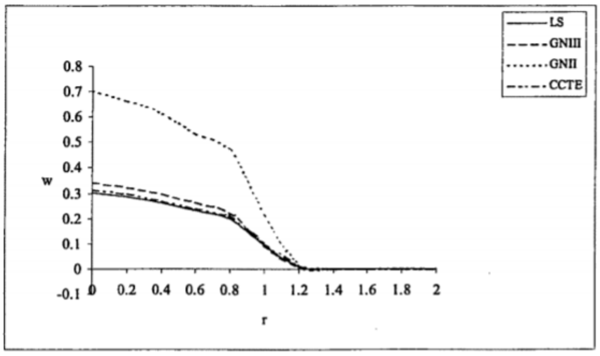

Fig. 4: Variation of axial displacement w for z=0.2 and t=0.2623592.

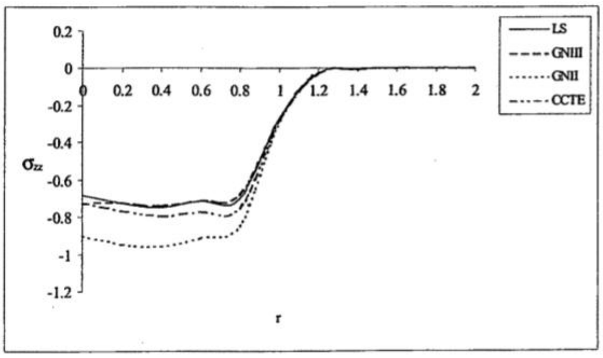

Fig.5: Variation of axial stress σ_{zz} for z=0.2 and t=0.2623592.

Here also the magnitude of stress is larger in case of GN model II than CCTE, GN model III and LS model. From the figure it is observed that beyond the crack the axial stress tends to vanish for all theories which is physically plausible. The stress intensity factor K has been calculated numerically for t=0.2623592 from the relation

$$\bar{K}(p) = \lim_{r \to a^+} \sqrt{2(r-a)}\bar{\sigma}_{zz}(r, 0, p).$$

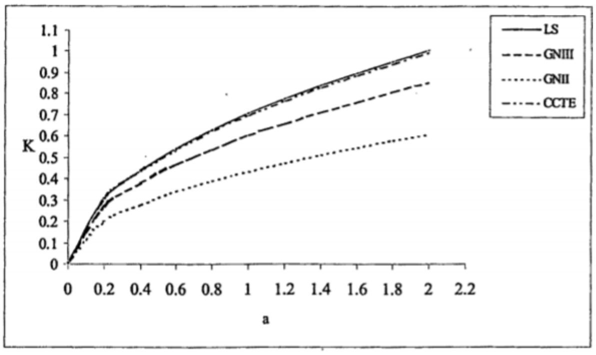

Fig. 6: Variation of stress intensity factor against crack radius a for t=0.2623592.

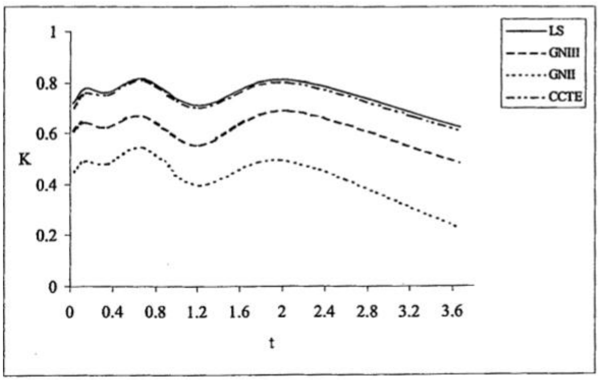

Fig.7: Variation of stress intensity factor against time t for a=1.2

Fig.6 and Fig.7 show the variation of stress intensity factor against the crack radius and time respectively for all four theories. In Figs.2-6 (for t=0.2623592) the result for L-S theory shows the similar behavior with those of Sherief and El-Maghraby [224] (for t=0.25) where they have considered only LS model.

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