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Some Application of Fixed Point Theorem: A Case Study of Fuzzy Metric Space

Sandeep*

Assistant Professor in Mathematics, CRSU, Jind, Haryana

Abstract – Main aim of this paper is to prove some fixed point theorems in Fuzzy Metric spaces through rational inequality. Our results extends and generalizes the results of many other authors existing in the literature. In this paper, we manage iterative methods for approximation of fixed points and their applications. We initially talk about fixed point theorems for a non-expansive mapping or a group of non-expansive mappings. we study those fuzzy metrics M on X, in the George and Veeramani's sense, such that $A_{t>0}$ M(x, Y, t) > 0. The continuous extension M^0 of M to X^2 ×[0,+∞[is called extended fuzzy metric. We prove that M^0 generates a metrizable topology on X, which can be described in a similar way to a classical metric. M^0 can be used for simplifying or improving questions concerning M; in particular, we expose the interest of this kind of fuzzy metrics to obtain generalizations of fixed point theorems given in fuzzy metric spaces.

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INTRODUCTION

The foundation of fuzzy mathematics is laid by Lofti A Zadeh with the introduction of fuzzy sets in 1965. This foundation represents a vagueness in everyday life. Subsequently several authors have applied various form general topology of fuzzy sets and developed the concept of fuzzy space. In 1975, Kramosil and Michalek (1975) introduced concept of fuzzy metric spaces. In 1988, Mariusz Grabiec (1988) extended fixed point theorem of banach and eldestien to fuzzy metric spaces in the sense of Kramosil and Michalek. In 1994, George et al. (1994) modified the notion of fuzzy metric spaces with the help of continuous *t*-norms. A number of fixed point theorem have been obtained by various authors in fuzzy metric space by using the concept of compatible map, implicit relation, weakly compatible map, *R* weakly compatible map. Also Vishal Gupta et al. (2008) proved some fixed points theorems, On expansion type maps and common coincidence points of *R*-Weakly commuting fuzzy maps, in Fuzzy Metric Space.

The beginning stage of fixed point theory lies in the method of dynamic approximations used for exhibiting nearness of arrangements of differential equations introduced independently by Joseph Liouville in 1837 and Charles Emile Picard in 1890. Regardless, formally it was started in the beginning of twentieth century as a basic bit of examination. The pondering of this conventional theory is the leading work of the impressive Polish mathematician Stefan Banach disseminated in 1922 which gives a significant method to find the fixed points of a guide.

In any case, on chronicled point of view, the genuine set up result in fixed point theory is a direct result of L. E. J. Brouwer given in 1912. The watched Banach withdrawal standard (BCP) states that "a compression mapping on a whole metric space has an exceptional fixed point". Banach used contracting mapto get this key outcome. The Brouwer fixed point theory is of phenomenal centrality in the numerical treatment of equations. It accurately communicates that "a nonstop guide on a close unit ball in Rn has a fixed point". A basic extension of this is the Schauder's fixed point speculation of 1930 communicating "a persistent guide on an angled preservationist subspace of a Banach space has a fixed point". These adulated comes about have been used, summed up and connected in various courses by a couple of mathematicians, analysts, budgetary specialists for single esteemed and multivalued mappings under different contractive conditions in various spaces. Kannan showed a fixed point theory for the maps not by any means nonstop. This was another crucial progression in fixed point theory. In this way unique outcomes identifying with fixed points, essential fixed points, chance occasion points, et cetera have been explored for maps satisfying unmistakable contractive conditions in different settings.

In this way fixed point theory has been broadly inspected, summed up and enhanced in different methodologies, for instance, metric, topological and orchestrate theoretic. This progress in fixed point theory widened the uses of different fixed point brings about various domains, for instance, the nearness theory of differential and basic equations, dynamic programming, fractal and anarchy theory, discrete elements, masses elements, differential contemplations, system examination, break number-crunching, advancement and diversion theory, variational disparities and control theory, flexibility and pliancy theory and different requests of numerical sciences.

Fixed point theory has reliably been stimulating in itself and its applications in new domains. Starting at now, it has found new and hot regions of activity. The beginning of fixed point theory in software engineering enhances its propriety in different areas. On account of the presence of convenient and speedy computational devices, another horizon has been given to fixed point theory. The fixed point equations are handled by methods for some iterative methodology. In context of their strong applications, it is of wonderful excitement to know whether these iterative methodology are numerically consistent or not. The examination of quality of iterative procedures acknowledges a praised put in significant arithmetic in view of wild direct of capacities in discrete elements, fractal designs and diverse other numerical figurings where PC writing computer programs is incorporated. This kind of issue for certified esteemed capacities was first discussed by M. Urabe (1956) in 1956. The main outcome on the dauntlessness of iterative methodologies on metric spaces is a direct result of Alexander M. Ostrowski. This outcome was extended to multivalued managers by Singh and Chadha (1995). Czerwik et al (2002) extended this to the setting of summed up metric spaces.

FIXED POINT THEOREMS WITHIN COMPLETE METRIC SPACES

The STUDY in fixed point theory has generally created in three principle bearings: generalization of conditions which guarantee presence, and, if conceivable, uniqueness, of fixed points; examination of the character of the sequence of iterates $\{T^n x\}_{n=0}^{\infty}$, where $T:X \to X$, X a total metric space, is the map under thought; investigation of the topological properties of the arrangement of fixed points, at whatever point T has more than one fixed point. This note treats just a few parts of the first and second inquiry, along a line followed by numerous different creators.

More precisely we consider maps $T:X \to X$, which satisfy conditions of the type

$d(Tx, Ty) \leqslant \varphi(d(x, y)) + \psi(d(Tx, x)) + \chi(d(Ty, y))$ for each $x, y \in X$, what's more, for these mappings we demonstrate, under appropriate speculations, presence and uniqueness of fixed points.

Through all the examination, X indicates a total metric space and $T, T:X \to X$, an asymptotically regular mapping; i.e., a function satisfying $\lim_n d(T^n x, T^{n+1} x) = 0$ for each $x \in X$.

Furthermore, we suppose that there exist three functions φ, ψ, χ, from $[0, +\infty[$ into $[0, +\infty[$, which satisfy the assumptions:

(I_1) $\varphi(r) < r \quad \text{if } r > 0,$

(I_2) there exists $\lim_{r \to \bar{r}+} \varphi(r) \leqslant \varphi(\bar{r}) \quad \text{for each } \bar{r} \in [0, +\infty[$

(I_3) $\psi(0) = \chi(0) = 0.$

Moreover, we suppose that *T,* satisfy the inequality

(I_4) $d(Tx, Ty) \leqslant \varphi(d(x, y)) + \psi(d(Tx, x)) + \chi(d(Ty, y))$
for each $x, y \in X$.

Lemma. Under the above assumptions on X and T and if, in addition, ψ and χ are continuous at $r = 0$, then, for each $x \in X$, there exists $z \in X$ such that $\{T^n x\}_{n=0}^{\infty}$ converges to z.

Proof. Suppose that there exists $x \in X$ such that the sequence of iterates is not a Cauchy sequence. Then, following there exist $\varepsilon > 0, \{m(j)\}_{j=0}^{\infty}, \{n(j)\}_{j=0}^{\infty}$ which satisfy the conditions

$$m(j) > n(j) \quad \text{for each } j \in N \qquad (3)$$

$$\lim_j n(j) = +\infty \qquad (4)$$

$$d(T^{m(j)}x, T^{n(j)}x) \geqslant \varepsilon \qquad (5)$$

$$d(T^{m(j)-1}x, T^{n(j)}x) < \varepsilon. \qquad (6)$$

Then, we have

$$\varepsilon \leqslant d(T^{m(j)}x, T^{n(j)}x) \leqslant d(T^{m(j)}x, T^{m(j)-1}x) + d(T^{m(j)-1}x, T^{n(j)}x) < \varepsilon + d(T^{m(j)}x, T^{m(j)-1}x)$$

which implies

$$\lim_j d(T^{m(j)}x, T^{n(j)}x) = \varepsilon. \qquad (7)$$

On the other hand

$$d(T^{m(j)}x, T^{n(j)}x) \leqslant d(T^{m(j)}x, T^{m(j)+1}x) + d(T^{n(j)}x, T^{n(j)+1}x) + d(T^{m(j)+1}x, T^{n(j)+1}x) \leqslant d(T^{m(j)}x, T^{m(j)+1}x) + d(T^{n(j)}x, T^{n(j)+1}x) + \varphi(d(T^{m(j)}x, T^{n(j)}x)) + \psi(d(T^{m(j)+1}x, T^{m(j)}x)) + \chi(d(T^{n(j)+1}x, T^{n(j)}x))$$

that is

$$d(T^{m(j)}x, T^{n(j)}x) - \varphi(d(T^{m(j)}x, T^{n(j)}x)) \leqslant d(T^{m(j)}x, T^{m(j)+1}x) + d(T^{n(j)}x, T^{n(j)+1}x) + \psi(d(T^{m(j)+1}x, T^{m(j)}x)) + \chi(d(T^{n(j)+1}x, T^{n(j)}x))$$

and, letting $j \to +\infty$

$$\varepsilon - \lim_j \varphi(d(T^{m(j)}x, T^{n(j)}x)) \leqslant 0.$$

FIXED POINT THEOREM ON FUZZY METRIC SPACES

FUZZY metric space is a generalization of metric space. The study on uncertainty and on randomness began to explore with the concept of fuzziness in mathematics. Fuzzy set is used in fuzzy metric space, which is initiated by Lofti. A. Zadeh. After that Kramosil and Michalek[4] introduced the concept of fuzzy metric space. A very important notion of fuzzy metric space with continuous tnorm is laid by Georage and Veeramani. Grabiec extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Compatible mapping is generalized from commutatively mappings by Jungck. After that Jungck and Rhodes [3] initiated the notion of weak compatible and proved that compatible maps are weakly compatible but converse is not true. A common E.A property is the generalization of the concept of non compatibility is introduced under strict contractive conditions by Aamri and El. Moutawakil.

In present time, Fuzzy set theory and Fuzzy logic is not only active field of research in mathematics but also in other field of engineering, medicine, communication, physics, biology etc. are field in which the applicability of fuzzy theory was accepted. Since, Many authors regarding the theory of fuzzy sets and its applications have developed a lot of literature.

In this paper, we prove a new fixed point theorem on fuzzy metric space by using above results. We also give an example which satisfies our main result in the paper.

APPLICATIONS

In this section, we gives some applications. Let us define $\Psi : [0,\infty) \to [0,\infty)$,

As $\Psi(t) = \int_0^t \varphi(t)dt \ \forall\, t > 0$, be a non-decreasing and continuous function. Moreover for each $\varepsilon > 0, \ \varphi(\varepsilon) > 0$. Also implies that $\varphi(t) = 0$ iff t = 0.

Theorem 1. Let (X, M, *) be a complete fuzzy metric space and $f : X \to X$ be a mapping satisfying

$$M(x,y,t) = 1$$

$$\int_0^{M(fx,fy,kt)} \varphi(t)dt \geq \int_0^{\lambda(x,y,t)} \varphi(t)dt$$

where

$$\lambda(x,y,t) = \min\left\{\frac{M(y,fy,t)[1+M(x,fx,t)]}{[1+M(x,y,t)]}, M(x,y,t)\right\}$$

for all $x, y \in X, \varphi \in \Psi$ and $k \in (0,1)$. Then f has a unique fixed point.

Proof By taking $\varphi(t) = 1$, we obtain the result.

Theorem 2. Let (X, M, *) be a complete fuzzy metric space and $f : X \to X$ be a mapping satisfying

$$\lambda(x,y,t) = \min\left\{\frac{M(y,fy,t)[1+M(x,fx,t)]}{[1+M(x,y,t)]}, M(x,y,t)\right\}$$

$$M(x,y,t) = 1$$

$$\int_0^{M(fx,fy,kt)} \varphi(t)dt \geq \phi\left\{\int_0^{\lambda(x,y,t)} \varphi(t)dt\right\}$$

Where for all $x, y \in X \ \varphi \in \Psi, k \in (0,1)$ ami $\phi \in \Phi$. Then f has a unique fixed point.

Proof. Since $\phi(a) > a$ for each $0 < a < 1$, therefore result follows immediately from Theorem 1.

FIXED POINTS BY GIVEN ITERATIVE SCHEMES WITH APPLICATIONS

Over the latest couple of decades examinations of fixed points by some iterative designs have pulled in various mathematicians. With the present quick changes in fixed point theory, there has been a restored energy for iterative designs. The properties of emphasess between the kind of arrangements and kind of managers have not been completely thought about and are presently under talk. The theory of heads has included a central place in present day examine using iterative designs because of its assurance of titanic utility in fixed point theory and its applications. There are different papers that have considered fixed points by some iterative designs. It is to some degree charming to observe that the kind of chairmen accept a significant part in examinations of fixed points.

The Mann iterative arrangement was composed in 1953, and it is used to get joining to a fixed point for a few classes of mappings. Considering fixed point emphasis methods with botches begins from convenient numerical computations. This subject of research expect a basic part in the robustness issue of fixed point emphasess. In 1995, Liu (1995) began an examination of fixed point emphasess with botches. A couple of makers have exhibited some fixed point theorems for Mann-type cycles with botches using a couple of classes of mappings.

Suppose that *H* is a real Hilbert space and *A* is a nonlinear mapping of *H* into itself. The map *A* is said to be accretive if $\forall x,y \in D(A)$, we have that

$$\langle Ax - Ay, x - y\rangle \geq 0, \quad (1)$$

and it is said to be strongly accretive if $A-kI$ is accretive, where $k \in (0,1)$ is a constant and I denotes the identity operator on *H.*

The map *A* is said to be ϕ-strongly accretive if $\forall x,y \in E$, exists a strictly increasing function $\phi:[0,\infty)\to[0,\infty)$ with $\phi(0)=0$ such that $\langle Ax-Ay,x-y\rangle \geq \phi(\|x-y\|)\|x-y\|,$ and it is called uniformly accretive if there exists a strictly increasing function $\psi:[0,\infty)\to[0,\infty)$ with $\psi(0)=0$ such that

$$\langle Ax - Ay, x - y\rangle \geq \psi(\|x-y\|).$$

Let $N(A)=\{x^* \in H : Ax^* = 0\}$ denote the null space (set of zero) of *A.* If $N(A)\neq\phi$ and (1) holds for all $x \in D(A)$ and $y \in N(A),$ then *A* is said to be quasi-accretive. The notions of strongly, ϕ-strongly, uniformly quasi-accretive are similarly defined. *A* is said to be ϕ-accretive if $\forall r > 0$ the operator $(I + rA)$ is surjective. Closely related to the class of accretive maps is the class of pseudo-contractive maps.

A map $T: H \to H$ is said to be pseudo-contractive if $\forall x,y \in D(T)$ we have that

$$\langle (I-T)x - (I-T)y, x - y\rangle \geq 0, \quad (2)$$

Observe that *T* is pseudo-contractive if and only if $A=(I-T)$ is accretive.

A mapping $T: H\to H$ is called Lipschitzian (or L-Lipschitzian) if there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x-y\|, \quad \forall x,y \in H.$$

In the sequel we use $L > 1.$

EXTENDED FUZZY METRICS

We begin this section introducing the announced concept of extended fuzzy metric space.

Definition 1. The term (X,M^0, *) is called an extended fuzzy metric space if X is a (non-empty) set, * is a continuous t-norm and M^0 is a fuzzy set onsatisfying the foil owing conditions, for each

$x,y,z \in X$ and $t,s \geq 0$ $X^2 \times [0,+\infty[$

(EFM1) $M^0(x,y,t) > 0;$

(EFM2) $M^0(x,y,t) = 1$ if and only if x = y;

(EFM3) $M^0(x,y,t) = M^0(y,x,t);$

(EFM4) $M^0(x,y,t) * M^0(y,z,s) \leq M^0(x,z,t+s);$

(EFM5) $M^0_{x,y} : [0,+\infty[\to]0,1]$ is continuous, where $M^0_{x,y}(t) = M^0(x,y,t).$

It is also said that $(M^0,*)$, or simply M^0, is an extended fuzzy metric on X. If * is a continuous f-norm satisfying $\star \leq *,$ then $(M^0,\star)$ is also an extended fuzzy metric on X.

Remark 1. Recently, by Mehmood, F. et al. (2017), it was introduced the concept of extended fuzzy b-metric space, with the aim of generalizing the notion of fuzzy b-metric space. Both notions generalize the concept of fuzzy metric by means of relaxing the triangle inequality. Nevertheless, the goal of introducing Definition 1 is to "extend" in the concept of fuzzy metric, given by George and Veeramani, the domain of definition of the t parameter to Thus, extended fuzzy b-metrics are not related to the new concept introduced above. $[0,+\infty[.$

The following theorem shows the relationship between fuzzy metrics and extended fuzzy metrics that one can observe in the last example.

Theorem 1. Let M be a fuzzy set on $X^2 \times]0,+\infty[$ and denote by M^0 its extension to $X^2 \times [0,+\infty[$ given by $M^0(x,y,t) = M(x,y,t)$ for all $x,y \in X, t > 0,$ and $M^0(x,y,0) = \bigwedge_{t>0} M(x,y,t).$ Then, $(X,M^0,*)$ is an extended fuzzy metric space if and only if $(X,M,*)$ is a fuzzy metric space satisfying for each $x,y \in X$ the condition $\bigwedge_{t>0} M(x,y,t) > 0.$

Proof. Suppose that $(M^0,*)$ is an extended fuzzy metric on X. Then, clearly, $(M,*)$ is a fuzzy metric on X. Now, we will see that $\bigwedge_{t>0} M(x,y,t) > 0$ for all $x,y \in X.$

Take $x,y \in X.$ Since $M_{x,y}$ is not decreasing on $]0,+\infty[$ and $M^0_{x,y}$ is continuous at t=0, then

$$\bigwedge_{t>0} M(x,y,t) = \lim_{t\to 0} M(x,y,t) = \lim_{t\to 0} M^0_{x,y}(t) = M^0_{x,y}(0) = M^0(x,y,0) > 0.$$

Conversely, let (X, M, *) be a fuzzy metric space satisfying $\bigwedge_{t>0} M(x,y,t) > 0$ for each $x,y \in X.$

Attending to the hypothesis and by construction of M°, we have that (EFM1) and (EFM3) are fulfilled. We will show the rest of the axioms.

(EFM2) Suppose $M^0(x,y,t)=1$ for some $t > 0.$ Then, $M(x,y,t)=1$ and so x = y. If $M^0(x,y,0)=1$, then

$\wedge_{t>0} M(x,y,t) = 1$ and thus $M(x,y,t) = 1$ for all $t > 0$, and so x = y. Obviously, if x = y, then $M^0(x,y,t) = 1$ for all $t \geq 0$.

(EFM4) Let $x,y,z \in X$. We will distinguish three possibilities on $t,s \geq 0$.

1. If t, s > 0, then (EFM4) is fulfilled since M is a fuzzy metric.

2. Suppose t > 0 and s = 0 (the case t = 0 and s > 0 is analogous). Then, for $\varepsilon \in]0,t[$, we have that

$$M^0(x,z,t+0) = M^0(x,z,t) = M(x,z,t) \geq M(x,y,t-\varepsilon) * M(y,z,\varepsilon).$$

Then, taking limits as ε tends to 0 in the last inequality, we obtain

$$M^0(x,z,t+0) \geq \lim_{\varepsilon \to 0} (M(x,y,t-\varepsilon) * M(y,z,\varepsilon)) =$$

$$= \left(\lim_{\varepsilon \to 0} M(x,y,t-\varepsilon)\right) * \left(\lim_{\varepsilon \to 0} M(y,z,\varepsilon)\right) =$$

$$= M(x,y,t) * \left(\bigwedge_{\varepsilon>0} M(y,z,\varepsilon)\right) = M^0(x,y,t) * M^0(y,z,0).$$

3. Suppose t = s = 0. Then, we have that

$$M^0(x,z,0+0) = M^0(x,z,0) = \bigwedge_{t>0} M(x,z,t) = \lim_{t \to 0} M(x,z,t) \geq$$

$$\geq \lim_{t \to 0} (M(x,y,t/2) * M(y,z,t/2)) = \left(\lim_{t \to 0} M(x,y,t/2)\right) * \left(\lim_{t \to 0} M(y,z,t/2)\right) =$$

$$= \left(\bigwedge_{t>0} M(x,y,t)\right) * \left(\bigwedge_{t>0} M(y,z,t)\right) = M^0(x,y,0) * M^0(y,z,0).$$

(EFM5) Since $M_{x,y}$ is continuous on $]0,+\infty[$, and $]0,+\infty[$ is open in $[0,+\infty[$, with the usual topology of $\mathbb{R}$ restricted to $[0,+\infty[$, then is continuous at each point of $]0,+\infty[$ for each $x,y \in X$.

For t = 0, we have that $M^0_{x,y}$

$$\lim_{t \to 0} M^0(x,y,t) = \lim_{t \to 0} M(x,y,t) = \bigwedge_{t>0} M(x,y,t) = M^0(x,y,0),$$

and so $M^0_{x,y}$ is continuous at t = 0.

Hence, (X, M°, *) is an extended fuzzy metric space.

An immediate consequence of the preceding result is that, given an extended fuzzy metric space (X,M^0, *), then $M^0_{x,y} : [0,+\infty[\to]0,1]$ is a non-decreasing continuous function satisfying $M^0_{x,y}(0) = \wedge_{t>0} M^0(x,y,t)$, for all $x,y \in X$. Furthermore, we can deduce the following result proved by Gregori et al. (2014).

Proposition 1. Let (X, M, *) be a fuzzy metric space. Define

$$N_M(x,y) = \bigwedge_{t>0} M(x,y,t).$$

Then, (N_m, *) is a stationary fuzzy metric on X if and only if $\wedge_{t>0} M(x,y,t) > 0$ for all $x,y \in X$.

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Corresponding Author

Sandeep*

Assistant Professor in Mathematics, CRSU, Jind, Haryana