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General Relativistic Behaviour of a Test Particle and Doppler's Effect in a Model

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Abstract – The present paper provides the behaviour of a test particle and Doppler's effect by considering suitable model.

Key Word – Model, test particle, Doppler's effect, line element geodesic.

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1. INTRODUCTION

Taking an anisotropic magneto hydrodynamic cosmological model in general relativity. Roy and Prtakash (1) have discussed the behaviour of a test particle and Doppler's effect following Tolman's technique (2). This discussion has been further extended by Singh and Yadav & Yadav & Purushottam (3, 4) for non-static cylindrically cosmological model which is spatially homogenous non-degenerate petrov type-1. In this paper we have also investigated the behaviour of a test particle and red shift (Doppler's effect) for the model given by line element.

$$(1.1) \quad ds^2 = t^{16\phi^2}(dx^2 - dt^2) + t^{4\phi}(dy^2 + dz^2)$$

which is of the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2$$

where A.B. = C are functions of t only

Clearly

$$A = t^{8\phi^2} B = C = t^{2\phi}$$

2. BEHAVIOUR OF A TEST PARTICLE IN THE MODEL

The equation of geodesic viz.

$$(2.1) \quad \frac{d^2x^i}{ds^2} + \Gamma^i_{IK}\frac{dx^k}{ds}\frac{dx^j}{ds} = 0$$

For the metric (1.1) when i = 2, 3, 4 are given by

$$(2.2) \quad \frac{d^2x}{ds^2} + 4\phi^2 t^{-1}\frac{dx}{ds}\frac{dt}{ds} = 0$$

$$(2.3) \quad \frac{d^2y}{ds^2} + 2\phi t^{-1}\frac{dy}{ds}\frac{dt}{ds} = 0$$

$$(2.4) \quad \frac{d^2y}{ds^2} + 2\phi t^{-1}\frac{dz}{ds}\frac{dt}{ds} = 0$$

$$(2.5) \quad \frac{d^2t}{ds^2} + 8\phi^2 t^{-1}\left(\frac{dx}{ds}\right)^2 - 2.\phi t^{-16\phi^2+4\phi-1}$$

If a particle is initially at rest, that is, if

$$(2.6) \quad \frac{dx}{ds} = \frac{dy}{ds} = \frac{dz}{ds} = 0$$

The from equations of geodesic we find that for all such particles the components of spatial acceleration would vanish, namely.

$$(2.7) \quad \frac{d^2x}{ds^2} = \frac{d^2y}{ds^2} = \frac{d^2z}{ds^2} = 0$$

and the particle would remain permanently at rest.

3. THE DOPPLER EFFECT IN THE MODEL

The track of a light pulse in the model is obtained by setting

$$ds^2 = 0 \text{ i.e.,}$$

$$(3.1) \quad \left(\frac{dx}{dt}\right)^2 + t^{+2-16\phi^2}\left(\frac{dy}{dt}\right)^2 + t^{+4\phi-16\phi^2}\left(\frac{dz}{dt}\right)^2 = 1$$

For the case when velocity is along z axis equation (3.1) gives

$$\left(\frac{dz}{dt}\right) = \pm t^{-2\phi + B\phi^2} = \pm t^{+2\phi(4\phi-1)} = \pm\psi(t)$$

Hence the light pulse leaving a particle at (0, 0, z) at time t_1 would arrive at a later time t_2 given by

$$(3.2) \quad \int_{t_1}^{t_2} \psi(t)dt = \int_0^z dz$$

$$(3.3) \quad \psi_2(t)\delta t_2 = \psi_1(t)\delta t_1 + \frac{dz}{dt}\delta t_1$$

$$= \psi_1(t)\delta t_1 + uz\delta t_1$$

where $\frac{dz}{dt} = U_z$ is the z-component of the velocity of the particle at the time of emission, $\psi_1(t)$ and $\psi_2(t)$ are the value of $\psi(t)$ for $t = t_1$ and $t = t_2$ respectively. From the above equation we get

$$(3.4) \quad \delta t_2 = \left\{\frac{\psi_1(t) + U_z}{\psi_2(t)}\right\}\delta t_1$$

The proper time interval δt_1^0 between the successive wave crests as measured by the local observer moving with the source is given by

$$(3.5) \quad \delta t_1^0 = \left\{t^{16\phi^2} - t^{16\phi^2}\left(\frac{dx}{dt}\right)^2 - t^{4\phi}\left(\frac{dy}{dt}\right)^2 - t^{4\phi}\left(\frac{dz}{dt}\right)^2\right\}^{1/2} dt$$

This can be written as

$$(3.6) \quad \delta t_1^0 = \left[t^{16\phi^2} - U^2\right]^{1/2} \delta t_1$$

where U is the velocity of the source at the time of emission, similarly we may write.

$$(3.7) \quad \delta t_2^0 = t^{8\phi^2}\delta t_2 = t^{8\phi^2}\left[\frac{t_1^{2\phi} + U_2}{t_2^{2\phi}}\right]$$

as the proper time interval between the reception of two successive wave crests by an observer at rest at the origin. Hence following Tolman (2) the red shift in this case is given by

$$(3.8) \quad \frac{\lambda + \delta\lambda}{\lambda} = \frac{\delta t_2^0}{\delta t_1^0} = t^{8\phi^2}\frac{(t_1^{2\phi(4\phi^{-1})} + U_z)}{t_2^{2\phi(4\phi^{-1})}(t^{16\phi^2} - U^2)^{\frac{1}{2}}}$$

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