Journal of Advances and Scholarly Researches in Allied Education
Vol. XV, Issue No. 1, April-2018, ISSN 2230-7540

Effect of Varying Temperature on Waves in Anisotropic Thermoelastic Plate under Liquid Loading

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Abstract – This paper aims to investigate the circulation of Lamb waves in a transversely isotropic, homogeneous thermoplastic solid plate in the context of the non-classical theories of thermo-elasticity. The surfaces of the solid plate are added with the finite thickness of non-viscous fluid layers on both sides with varying temperatures. Helmholtz decomposition technique is used to find out the solutions. After developing the mathematical model, formal solution, and boundary conditions, the frequency equations are derived for a homogeneous, transversely isotropic solid plate for both symmetric and asymmetric modes. Some regions of frequency equations are deduced depending on the characteristic roots. The numerical simulation is prepared for a transversely isotropic zinc material plate cladded with water and represented graphically. The theoretical and numerical computations are found to be in close agreement.

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1. INTRODUCTION

An infinite speed of heat transference is predicted by the conventional theory of heat conduction which denies the physical proofs. To improve this ambiguity generalized theories are established. For the fixed speed in thermal signals, the generalized theory is formulated by including a flux-rate term in Fourier's law of heat conduction by Lord and Shulman [1]. If the considered body has a center of symmetry then Green and Lindsay [2] incorporated a temperature rate term amongst the fundamental relations to formulate a temperature-rate-dependent thermoelasticity that does not overrule the conventional Fourier's law of heat conduction and also this theory forecasts a limited speed of propagation of heat. By using these generalized theories, propagation of energy can be regarded as a wave-type instead of a diffusion-type. Dhaliwal and Sherief [3] elaborated on the findings of Lord and Shulman by deriving the equations for an anisotropic medium of generalized thermoelasticity and to these equations' uniqueness theorem is proved. Chandrasekharaiah [3] developed a thermal wave disturbance that he mentioned as 'second sound'. Ackerman et.al. [5] and Ackerman and Overtone [6] proved experimentally that the heat waves traveling with limited and high velocity for solid helium also exist, while the conforming frequency window i .e.the frequency range of thermal variations in which heatwaves are noticed is restricted for most of the solids.

Firstly Lamb [7] studied the propagation of the waves in a uniform elastic plate. Since then disturbance traveling in an elastic solid material with free borders is referred to as 'Lamb wave'. The applications of Lamb waves are used in many fields like the checking of defects in thin-walled constituents and multi-sensors etc. The Lamb waves are constructed on the belief that when the solid plate is accompanied by the fluid it varies the amplitude and propagation velocity of the Lamb waves in the borders due to viscous and inertial properties of the fluid in the plate. The implications of the-viscous fluid on the Lamb waves propagation in a solid material of fixed-width inserted between homogeneous liquid half-space on its both sides investigated by Schoch [8] and found that some amount of energy in the plate is attached with the fluid in the form of radiation, while the most of energy remains in the solid. Chadwick and Seet [9] developed the time-harmonic plane wave propagation in a consistent anisotropic plate. Watkins et al. [10] used an acoustic impedance method to calculate the reduction in the magnitude of Lamb waves due to the existence of the non-viscous fluid. The growing attention in Lamb waves was moderately originated by its implementation in the multi-sensors [11]. Wu and Zhu [12] evaluated the impact of non-viscous fluid layers on both sides of the material plate for the Lamb wave propagation.

Sharma and Pathania [13] investigated Lamb wave propagation in isotropic generalized thermoelastic homogeneous solid material fringe with layers or half-spaces of fluid on the surfaces of the plate. They showed that the shear horizontal component of waves decouples from the primary stream of wave motion. Sharma and Pathania [14] also derived the expressions for frequency equations, short-wavelength waves, and long-wavelength waves in a transversely isotropic homogeneous thermoelastic plate exposed to inviscid fluid loading. Propagation of wave in a liquid saturated porous solid with a micro-polar elastic skeleton at the boundary surface has been evaluated by [15, 16]. Sharma and Kumar [17] studied the effect of micropolarity on Lamb waves in thermally conducting elastic solid subjected to non-viscous fluid layers (or half-spaces) with varied temperatures and discussed the characteristic length and coupling factors impact on phase velocity. Pathania et. al. [18] investigated the Lamb waves in transversely isotropic thermoelastic plates immersed in viscous fluid layers in non-classical theories of thermo-elasticity. Pathania et. al. [19] analyzed the characteristics of the circular waves in a homogeneous anisotropic thermo-elastic material surrounded by conducting viscous fluid loading layers (or half-spaces) on the boundary surface of the plate. The effect of the loosely bonded interface on wave propagation between two half-space has been obtained by Barak and Kaliraman [20]. Recently, Pathania et. al. [21] investigated the impact of varying temperatures on the Lamb waves propagation in a homogeneous thermoelastic isotropic plate in the presence of ideal fluid in the frame of reference of the classical theory of thermoelasticity.

In this paper, an effort has been done to discuss the propagation of plane waves in an anisotropic homogeneous thermo-elastic material having width 2d under the effect of non-viscous liquid with varying temperatures on borders of the plate, in the framework of coupled and generalized models of thermoelasticity. The liquid is supposed to be homogeneous and inviscid. The frequency equations are determined by using appropriate boundary conditions for Lamb waves. Also for Lamb waves, the frequency equation is solved analytically for the classical and non-classical models of thermo-elasticity and further deduced for uncoupled thermoelasticity. The equations for various regions have been deduced from the secular equation depending upon the type of characteristic roots.

2. FORMULATION OF THE PROBLEM

Consider a homogeneous anisotropic solid plate of width '2d' consisted of a thermoelastic solid material. The solid plate is cladded with an ideal fluid of width h on both borders i.e. on top and the bottom. The solid material is supposed at an undisturbed state and unvarying temperature T_0 initially.

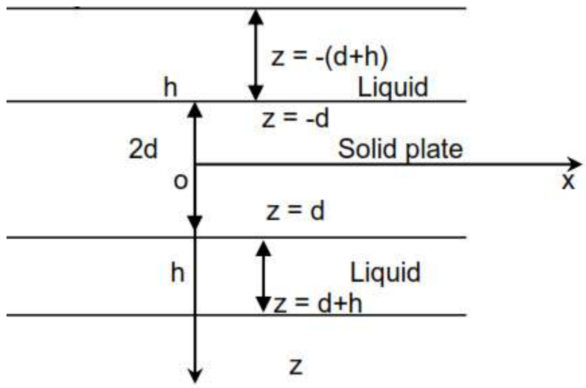

Figure 1: Geometry of the Problem

In the cartesian coordinate system $o-xyz$, the origin "o" is chosen in the center of the plate and the z-axis is taken in a vertically downward direction along the width of the plate. The wave is traveled along the x-axis direction and the field parameters remain explicitly independent of y-coordinate but depend implicitly on y-coordinate such that the shear component of displacement is non-vanishing**.** The complete configuration of the problem is illustrated in Figure-1.

The basic governing equations in non-dimensional form for homogenous transversely isotropic solid plate and liquid layers in the non-classical model of thermoelasticity, in the non-attendance of energy sources and body forces are given by [14] and [21] respectively as

$$\frac{\partial^2 u}{\partial x^2}+c_2\frac{\partial^2 u}{\partial z^2}+c_3\frac{\partial^2 w}{\partial xz}-\frac{\partial}{\partial x}\left(T+t_1\delta_{2k}\dot{T}\right)=\frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$c_3\frac{\partial^2 u}{\partial xz}+c_2\frac{\partial^2 w}{\partial x^2}+c_1\frac{\partial^2 w}{\partial z^2}-\bar{\beta}\frac{\partial}{\partial z}\left(T+t_1\delta_{2k}\dot{T}\right)=\frac{\partial^2 w}{\partial t^2} \quad (2)$$

$$\frac{\partial^2 T}{\partial x^2}+\bar{K}\frac{\partial^2 T}{\partial z^2}-\left(\frac{\partial T}{\partial t}+t_0\frac{\partial^2 T}{\partial t^2}\right)=\varepsilon\left(\frac{\partial \dot{u}}{\partial x}+\bar{\beta}\frac{\partial \dot{w}}{\partial z}+t_0\delta_{1k}\left(\frac{\partial \ddot{u}}{\partial x}+\bar{\beta}\frac{\partial \ddot{w}}{\partial z}\right)\right) \quad (3)$$

$$\nu_L\frac{\partial}{\partial t}\nabla^2\vec{u}_{L,j}-\frac{\bar{\beta}_L\rho}{\rho_L}\nabla T_{L,j}=\frac{\partial^2\vec{u}_{L,j}}{\partial t^2} \quad (4)$$

$$T_{L,j}=-\frac{\varepsilon_L\rho_L\delta_L^2}{\bar{\beta}_L\rho}\nabla\vec{u}_{L,j}\ ;j=1,2 \quad (5)$$

Where

$$\beta_1=\left(c_{11}+c_{12}\right)\alpha_1+c_{13}\alpha_3,\ \beta_3=2c_{13}\alpha_1+c_{33}\alpha_3,$$

$$\bar{\beta}=\frac{\beta_3}{\beta_1},\bar{\beta}_L=\frac{\beta_L^*}{\beta_1},\ \delta_L^2=\frac{c_L^2}{v_1^2},\ c_L^2=\frac{\lambda_L}{\rho_L},$$

$$\bar{K}=\frac{K_3}{K_1},\varepsilon=\frac{\beta_1^2 T_0}{\rho C_e c_{11}},\varepsilon_L=\frac{\beta_L^{*2} T_0^*}{\rho_L C_v^* \lambda_L}$$

$$\beta_L^*=3\lambda_L\alpha^*,$$

For Lord-Shulman (LS) theory the value of suffix is $k=1$ and for Green-Lindsay(GL) theory the value of suffix is k = 2 in Kronecker's delta δ_{ik}. v_1 is the velocity of the longitudinal wave in the thermoelastic half-space, t_0 and t_1 are the thermal relaxation times, ε is the thermomechanical coupling constant in the solid plate. For the top $(j=2)$ and the bottom liquid layers $(j=1)$. λ_L is the bulk modulus, c_L is the velocity of sound, ε_L is the thermomechanical coupling constant, ρ_L is the density of the liquid, C_v^* is the specific heat at a constant volume, α^* is the coefficient of volume thermal expansion. Here dot over a symbol represents the time derivative.

In the liquid layers, we take

$$u_{Lj}=G_{Lj,x}+H_{Lj,z}\quad,\quad w_{Lj}=G_{Lj,z}-H_{Lj,x}\quad, j=1,2$$

where H_{Lj} is the vector velocity potential and G_{Lj} is the scalar velocity potential element in the positive direction of z-axis. Since the shear motion is not supported by the inviscid liquid, therefore the shear motion of liquid vanishes, and thus we have

$$u_{Lj}=G_{Lj,x},\quad w_{Lj}=G_{Lj,z},\ j=1,2 \qquad (6)$$

Here the comma in the subscript represents spatial derivatives.

Plugging (6) in equations (4) and (5), we get equations in terms of the potential functions G_{Lj} as

$$\nabla^2 G_{Lj}-\frac{1}{\left(\delta_L^2(1+\varepsilon_L)\right)}\ddot{G}_{Lj}=0 \quad (7)$$

$$T_{Lj}=-\frac{\varepsilon_L\rho_L\delta_L^2}{\bar{\beta}_L\rho}\nabla^2 G_{Lj},\qquad (j=1,2)\,(8)$$

3. BOUNDARY CONDITIONS

The stress traction, displacement, and heat flux at the solid-fluid interfaces $z=\pm d$ may be written as:

Mechanical Conditions

(i) For the solid plate, the dimensionless normal element of the stress tensor must be equal to the pressure of the fluid. This implies that

$$\sigma_{zz}=\left(\sigma_{zz}\right)_L \qquad (9.1)$$

(ii) The dimensionless shear element of the stress tensor of the solid plate must be zero, implying that

$$\sigma_{xz}=\left(\sigma_{xz}\right)_L \qquad (9.2)$$

(iii) The dimensionless normal displacement element of the plate must be equal to the fluid. This yields to

$$w=w_L \qquad (9.3)$$

Thermal Conditions

(iv) The boundary condition for conduction-convection case is agreed by

$$T_{,z}+H(T-T_L)=0 \qquad (9.4)$$

Where *H* is Biot's heat transfer coeffcient.

The boundary condition (9.4) refers to the thermally insulated case if $H\to 0$ and corresponds to the isothermal case if $H\to\infty$, in the nonappearance of liquid $\rho_L=0$ i.e. for thermoelastic half-space.

4. SOLUTION OF THE PROBLEM

Analyzing the disturbances into the normal modes, the solutions of the considered problem are of the form

$$\{u,v,w,T\}=\{1,V,W,S\}U\,e^{i\xi(x\sin\theta+mz-ct)} \qquad (10.1)$$

$$\{G_{L1},G_{L2}\}=\{\Phi_{L1},\Phi_{L2}\}Ue^{i\xi(x+mz-ct)} \qquad (10.2)$$

where $c=\omega/\xi$ ω is the frequency, ξ is the wavenumber, and c is the dimensionless phase velocity. The inclination angle of wave normal with the z-axis (axis of symmetry) is denoted by θ and m is an unidentified parameter. The displacements v,w, temperature *T,* and potentials G_{L1},G_{L2} concerning the displacement u have the amplitude ratios *V,W,S* and Φ_{L1},Φ_{L2} respectively.

Plugging the solutions (10.1) in equations (1)-(3), *u, w, t,* can be obtained as

$$(u,w,T)=\sum_{q=1}^{6}(1,W_q,S_q)U_q\exp\left[i\xi(xs+m_q z-ct)\right],\ -d<z<d \qquad (11)$$

Where

$$m_q, q=1,2,3,4,5,6.$$

$$\sum m_1^2 = \frac{Ps^2 - Jc^2}{c_1c_2} + \frac{s^2 - c^2\tau_0}{\bar{K}} + \frac{i \in \omega c^2 \bar{\beta}^2 \tau_0' \tau_1}{c_1 \bar{K}}$$

$$\sum m_1^2 m_2^2 = \frac{(c_2s^2 - c^2)(s^2 - c^2)}{c_1c_2} + \frac{(s^2 - c^2\tau_0)(Ps^2 - Jc^2)}{c_1c_2\bar{K}} + \frac{i \in \omega c^2 \tau_0' \tau_1}{c_1c_2\bar{K}}\left[(\bar{\beta}^2 - 2c_3\bar{\beta} + c_1)s^2 - c^2\bar{\beta}^2\right]$$

$$\sum m_1^2 m_2^2 m_3^2 = \frac{c_2s^2 - c^2}{c_1c_2\bar{K}}\left[(s^2 - c^2\tau_0)(s^2 - c^2) + i \in s^2 \omega c^2 \tau_0' \tau_1\right] \tag{12}$$

The amplitude ratios W_q and S_q are given by

$$W_q = \begin{cases} \dfrac{m_q a_q}{s}, & q = 1,2,3,4 \\ -\dfrac{\left[c_2 m_q^2 + s^2 - c^2 + sc\tau_1 S_q\right]}{c_3 m_q s}, & q = 5,6 \end{cases},$$

$$S_q = \begin{cases} -\dfrac{\left[(c_2 + c_3 a_q)m_q^2 + s^2 - c^2\right]}{sc\tau_1}, & q = 1,2,3,4 \\ \dfrac{\left[c_1c_2 m_q^4 + (Ps^2 - Jc^2)m_q^2 + (c_2s^2 - c^2)(s^2 - c^2)\right]a_q}{sc\bar{\beta}\tau_1\left(c_2 m_q^2 + (1 - c_3/\bar{\beta})s^2 - c^2\right)}, & q = 5,6 \end{cases} \tag{13}$$

where

$$s = \sin\theta, \tau_0 = t_0 + i\omega^{-1}, \tau_0' = t_0\delta_{1k} + i\omega^{-1}, \tau_1 = t_1\delta_{2k} + i\omega^{-1}, P = c_1 + c_2^2 - c_3^2, J = c_1 + c_2,$$

$$a_q = \bar{\beta}\frac{c_2 m_q^2 + \left(1 - c_3/\bar{\beta}\right)s^2 - c^2}{\left(c_1 - c_3\bar{\beta}\right)m_q^2 + c_2s^2 - c^2}, \quad q = 1,2,3,4,5,6 \tag{14}$$

Upon using solutions (10.2) in equations (7)-(8) for the bottom liquid layer $d < z < d + h$ and top liquid layer $-(d+h) < z < -d$, we have

$$\Phi_{L1} = A_7 \sin \xi m_7\left[z - (d + h)\right]e^{i\xi(x - ct)} \tag{15}$$

$$\Phi_{L2} = A_8 \sin \xi m_8\left[z + d + h\right]e^{i\xi(x - ct)} \tag{16}$$

$$T_{L1} = S_L\xi^2 A_7 \sin \xi m_7\left[z - (d + h)\right]e^{i\xi(x - ct)} \tag{17}$$

$$T_{L2} = S_L\xi^2 A_8 \sin \xi m_8\left[z + d + h\right]e^{i\xi(x - ct)} \tag{18}$$

where

$$S_L = \frac{\varepsilon_L \rho_L \delta_L^2}{\beta_L \rho[\delta_L^2(1 + \varepsilon_L)]}c^2, \; m_7, m_8 = \pm\sqrt{\frac{c^2}{\delta_L^2(1 + \varepsilon_L)} - 1} \tag{19}$$

Upon using the above formal solution (11), the stresses, displacements, and temperature change are obtained as

$$(\sigma_{zz}, \sigma_{xz}, w, T_{,z}) = \sum_{q=1}^{8} i\xi\left(D_{1q}, D_{2q}, D_{3q}, D_{4q}\right)U_q \exp\left[i\xi(xs + m_q z - ct)\right]$$

Here

$$D_{1q} = \begin{cases} (c_3 - c_2)s + c_1 m_q W_q + c\bar{\beta}\tau_1 S_q, & q = 1,2,\ldots,6 \\ -\omega^2\rho_L/i\xi\rho, & q = 7,8 \end{cases},$$

$$D_{2q} = \begin{cases} c_2\left(m_q + sW_q\right), & q = 1,2,\ldots,6 \\ 0, & q = 7,8 \end{cases}$$

$$D_{3q} = \begin{cases} W_q/i\xi, & q = 1,2,\ldots,6 \\ im_q, & q = 7,8 \end{cases},$$

$$D_{4q} = \begin{cases} \left(m_q + H/i\xi\right)S_q, & q = 1,2,\ldots,6 \\ -i\xi HS_L, & q = 7,8 \end{cases} \tag{20}$$

5. DERIVATION OF THE SECULAR EQUATIONS

In this topic, the dispersal relation for Lamb waves propagation in the transversely isotropic plate under inviscid liquid loading is derived. An arrangement of eight linear equations in eight unknown amplitudes $U_q, q = 1,2,\ldots 8$ is obtained by applying thermal and mechanical boundary conditions at external plate surfaces $z = \pm d$. We get the determinant of the coefficient of $U_q, q = 1,2,\ldots 8$ equal to zero when this system of equations has a non-zero, which gives the frequency equation for the propagation of Lamb waves in the thermoelastic plate. After straight forward algebraicsimplifications and reductions along with conditions $\gamma^* = \xi h$ is non zero and not odd multiple of $\pi/2$, the distinctive equation for the thermoelastic plate yields to the frequency equations

$$\left[\frac{T_1}{T_5}\right]^{\pm1} - \frac{D_{13}G_5}{D_{11}G_1}\left[\frac{T_3}{T_5}\right]^{\pm1} - \frac{D_{17}T_7}{D_{41}D_{11}G_1[T_5]^{\pm1}}\left[D_{11}G_1 - D_{13}G_3 + D_{15}G_5\right] - \frac{2iD_{45}''T_7}{D_{41}D_{11}G_1'[T_5]^{\pm1}}\left[D_{41}G_1' - D_{43}G_3' + D_{45}G_5'\right] = -\frac{D_{15}G_3}{D_{11}G_1} \tag{21}$$

Here $\gamma = \xi d, \gamma^* = \xi h, T_k = \tan\gamma m_k, k = 1,3,5$ and $T_7 = \tan\gamma^* m_7$. The skew-symmetric mode corresponds to superscripted + sign and symmetric mode corresponds to superscripted – sign and

$$G_1 = D_{23}\left(D_{45}' + iD_{45}''T_5^{\pm1}\right) - D_{25}\left(D_{43}' + iD_{43}''T_3^{\pm1}\right),$$

$$G_3 = D_{21}\left(D_{45}' + iD_{45}''T_5^{\pm1}\right) - D_{25}\left(D_{41}' + iD_{41}''T_1^{\pm1}\right),$$

$$G_5 = D_{21}\left(D_{43}' + iD_{43}''T_3^{\pm1}\right) - D_{23}\left(D_{41}' + iD_{41}''T_1^{\pm1}\right),$$

$$G_1' = \left(D_{23}\left(D_{15}T_5\right) - D_{25}\left(D_{13}T_3\right)\right),$$

$$G_3' = \left(D_{21}\left(D_{15}T_5\right) - D_{25}\left(D_{11}T_1\right)\right),$$

$$G_5' = \left(D_{21}\left(D_{13}T_3\right) - D_{23}\left(D_{11}T_1\right)\right) \tag{22}$$

Where

$$D'_{4q} = m_q S_q, \quad D''_{4q} = \frac{HS_q}{i\xi} - i\xi HS_L$$

For an inviscid fluid at a uniform temperature and homogeneous isotropic thermoelastic plate equation (21) get reduced to form which is the same as obtained and discussed by [13,14].

6. RAYLEIGH–LAMB MODES

The complex transcendental frequency equation (21) gives evidence about the attenuation coefficient, phase velocity, and wavenumber of the Lamb waves. To solve these secular equations, we yield

$$c^{-1} = V_P^{-1} + i\omega^{-1} Q \qquad (23)$$

Implies $\xi = R + iQ$, and $V_p = \frac{\omega}{R}$, where R and Q are real quantities. Thus these complex quantities i.e. the phase velocity of the waves and wavenumber are attenuated in space. The exponential $e^{i(\xi(x-ct))}$ in the solutions of time-harmonic plane waves (10.1) and (10.2) respectively become

$$i\{Rx\sin\theta - \omega t\} - Qx\sin\theta \qquad (24)$$

$$i\{Rx - \omega t\} - Qx \quad (24.2)$$

Here Q is the attenuation coefficient and V_p is the phase velocity of plane waves respectively. The various modes of propagating wave for the attenuation coefficient Q and phase velocity V_p can be obtained by substituting relation (23) in equation (21).

7. HOMOGENEOUS ISOTROPIC MATERIAL

For homogeneous isotropic thermoelastic solid plate, we have

$c_1 = 1$, $c_2 = \delta^2$, $c_3 = 1-\delta^2$, $\bar{\beta} = 1$, $\bar{K} = 1$ and $m_q = \alpha_q$

Therefore the secular equation (21) becomes

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{(\alpha_1^2+1-c^2)}{(\alpha_3^2+1-c^2)}\frac{(\xi\alpha_1 - HT_1^{\pm 1})}{(\xi\alpha_3 - HT_3^{\pm 1})}\left[\frac{T_3}{T_5}\right]^{\pm 1} + \frac{i\omega^2\rho_L\alpha_L}{\rho\xi^2\alpha_5\delta^2}\frac{(\alpha_5^2+1)}{(\alpha_5^2-1)^2}\frac{T_L}{[T_5]^{\pm 1}}\left[1 - \frac{\alpha_3(\alpha_1^2+1-c^2)(\xi\alpha_3 - HT_1^{\pm 1})}{\alpha_1(\alpha_3^2+1-c^2)(\xi\alpha_3 - HT_3^{\pm 1})}\right]$$

$$-\frac{2H\xi S_L T_1\left(2\alpha_5^{-1} + \frac{\alpha_5^2-1}{\alpha_5}\right)(\alpha_3 T_3 - \alpha_1 T_3)}{\alpha_5\frac{\alpha_5^2-1}{\alpha_5}(\alpha_3^2+1-c^2)(\xi\alpha_3 - HT_3^{\pm 1})} = -\frac{4\alpha_1\alpha_5}{(\alpha_5^2-1)^2}\left[1 - \frac{\alpha_3(\alpha_1^2+1-c^2)(\xi\alpha_1 - HT_1^{\pm 1})}{\alpha_1(\alpha_3^2+1-c^2)(\xi\alpha_3 - HT_3^{\pm 1})}\right] \qquad (25)$$

For an inviscid fluid at a uniform temperature and homogeneous isotropic thermoelastic plate equation (25) get reduced to form which is the same as obtained and discussed by [13,14].

8. DISCUSSION OF THE SECULAR EQUATION

Based on the values of characteristic roots m_1, m_3, m_5 to be imaginary, zero, or real, the dispersion equation (21) is converted as given below.

8.1 Region I

If the frequency equation has characteristic roots in the form $m_k^2 = -m_k'^2,\ k = 1,3,5$; then $m_k = im_k'$ which is purely imaginary. Therefore the partial wave superposition confirms that they have the "exponential decay" property. Therefore the secular equation (21) can be changed to hyperbolic tangent functions $m_k',\ k = 1,3,5$ by replacing the circular tangent functions $m_k',\ k = 1,3,5$

$$\left[\frac{\tanh\gamma m_1'}{\tan\gamma m_5}\right]^{\pm 1} - \frac{D_{13}}{D_{11}}\frac{\bar{G}_3'}{\bar{G}_1'}\left[\frac{\tanh\gamma m_3'}{\tan\gamma m_5}\right]^{\pm 1} \pm \frac{iD_{17}T_7}{D_{57}D_{11}\bar{G}_1[\tan\gamma m_5]^{\pm 1}}\left[D_{31}\bar{G}_1 - D_{33}\bar{G}_3' + D_{35}\bar{G}_5'\right]$$

$$\pm \frac{D_{47}^{*}T_7}{D_{57}D_{11}\bar{G}_1[\tan\gamma m_5]^{\pm 1}}\left[D_{31}\bar{G}_1^{*} - D_{33}\bar{G}_3^{*} + D_{35}\bar{G}_5^{*}\right] = \pm\frac{iD_{35}}{D_{11}}\frac{\bar{G}_5'}{\bar{G}_1'} \qquad (26)$$

where

$$\overline{G_1} = D_{23}\left(D'_{45} - D''_{45}[\tanh\gamma m_5']^{\pm 1}\right) - D_{25}\left(D'_{43} - D''_{43}[\tanh\gamma m_3']^{\pm 1}\right),$$

$$\overline{G_3} = D_{21}\left(D'_{45} - D''_{45}[\tanh\gamma m_5']^{\pm 1}\right) - D_{25}\left(D'_{41} - D''_{41}[i\tanh\gamma m_5']^{\pm 1}\right)$$

$$\overline{G_5} = D_{21}\left(D'_{43} - D''_{43}[\tanh\gamma m_3']^{\pm 1}\right) - D_{23}\left(D'_{41} - D''_{41}[\tanh\gamma m_1']^{\pm 1}\right),$$

$$\overline{G_1^{*}} = \left(D_{23}\left(D_{15}[i\tanh\gamma m_5']^{\pm 1}\right) - D_{25}\left(D_{13}[i\tanh\gamma m_3']^{\pm 1}\right)\right)$$

$$\overline{G_3^{*}} = \left(D_{21}\left(D_{15}[i\tanh\gamma m_5']^{\pm 1}\right) - D_{25}\left(D_{11}[i\tanh\gamma m_1']^{\pm 1}\right)\right)$$

$$\overline{G_5^{*}} = \left(D_{21}\left(D_{13}[i\tanh\gamma m_3']^{\pm 1}\right) - D_{23}\left(D_{11}[i\tanh\gamma m_1']^{\pm 1}\right)\right)$$

Herecorresponding expressions $D_{1q}, D_{2q}, D_{3q}, D_{4q}, W_q, S_q, q = 1,2 \ldots 8$ can be found by substituting m_k by $im_k',\ k = 1,3,5$.

8.2 Region II

If two roots of the equation are in the form $m_k^2 = -m_k'^2,\ k = 1,3$, then the secular equation (21) can be altered with hyperbolic tangent functions in $m_k',\ k = 1,3$ by substituting circular tangent functions $m_k,\ k = 1,3$

$$\left[\frac{\tanh\gamma m_1'}{\tan\gamma m_5}\right]^{\pm 1} - \frac{D_{13}}{D_{11}}\frac{\bar{G}_3'}{\bar{G}_1'}\left[\frac{\tanh\gamma m_3'}{\tan\gamma m_5}\right]^{\pm 1} \pm \frac{iD_{17}T_7}{D_{57}D_{11}\bar{G}_1'[\tan\gamma m_5]^{\pm 1}}\left[D_{31}\bar{G}_1' - D_{33}\bar{G}_3' + D_{35}\bar{G}_5'\right]$$

$$\pm 2\frac{D_{47}^{*}T_7}{D_{57}D_{11}\bar{G}_1'[\tan\gamma m_5]^{\pm 1}}\left[D_{31}\bar{G}_1^{*} - D_{33}\bar{G}_3^{*} + D_{35}\bar{G}_5^{*}\right] = \pm\frac{iD_{35}}{D_{11}}\frac{\bar{G}_5}{\bar{G}_1'} \quad (27)$$

where

$$\overline{G_1} = D_{23}\left(D'_{45} + iD^*_{45}[\tan \gamma m_5]^{\pm1}\right) - D_{25}\left(D'_{43} - D^*_{43}[\tanh \gamma m'_3]^{\pm1}\right),$$

$$\overline{G_5} = D_{21}\left(D'_{45} + iD^*_{45}[\tan \gamma m_5]^{\pm1}\right) - D_{25}\left(D'_{41} - D^*_{41}[\tanh \gamma m'_1]^{\pm1}\right),$$

$$\overline{G_9} = D_{21}\left(D'_{43} - D^*_{43}[\tanh \gamma m'_3]^{\pm1}\right) - D_{23}\left(D'_{41} - D^*_{41}[\tanh \gamma m'_1]^{\pm1}\right)$$

$$\overline{G_1^*} = \left(D_{23}(D_{15}[\tan \gamma m'_5]) - D_{25}\left(D_{13}[i \tanh \gamma m'_3]^{\pm1}\right)\right)$$

$$\overline{G_5^*} = \left(D_{21}(D_{15}[\tan \gamma m'_5]) - D_{25}\left(D_{11}[i \tanh \gamma m'_1]^{\pm1}\right)\right)$$

$$\overline{G_9^*} = \left(D_{21}\left(D_{13}[i \tanh \gamma m'_3]^{\pm1}\right) - D_{23}\left(D_{11}[i \tanh \gamma m'_1]^{\pm1}\right)\right)$$

Herecorresponding expressions $D_{1q}, D_{2q}, D_{3q}, D_{4q}, W_q, S_q, q = 1,2,\ldots,8$ can be found by substituting m_k with $im'_k,\ k = 1,3$.

8.3 Region III

when the roots of the characteristic equation $m_k^2,\ k = 1,3,5$ are complex numbers then the dispersion equation remains the same as given by equation (21).

9. NUMERICAL RESULTS AND DISCUSSION

We now present some numerical solutions to the theoretical results obtained in the earlier sections. The following values of material constants are taken for solid zinc plate and liquid water.

$c_1 = 0.3851, c_2 = 0.2365, c_3 = 0.5485, \bar{\beta} = 0.8817, \bar{K} = 1, \varepsilon = 0.0221, T_0 = 296^0 K,$

$C_e = 3.9 \times 10^2\ JKgm^{-1}\ \deg$

$\rho = 7.14 \times 10^3\ Kgm^{-3},$

$\omega^* = 5.01 \times 10^{11} s^{-1}, c_L = 1.5 \times 10^3\ m/s,$

$t_1 = 0.5 \times 10^{-13} s, t_0 = 0.75 \times 10^{-13} s, d = 1, h = 0.25, H = 0.1, \rho_L = 1000\ Kg\,m^{-3}$

Table 1: The values of specific heat of water at diverse temperatures

$T_0(^\circ C)$	0	10	20	30	40	50	60	70	80	90	100
$C_p^*(J/Kg^\circ C)$	1.008	1.002	.9995	.9987	.99869	.9919	1.001	1.0013	1.003	1.00	1.0076

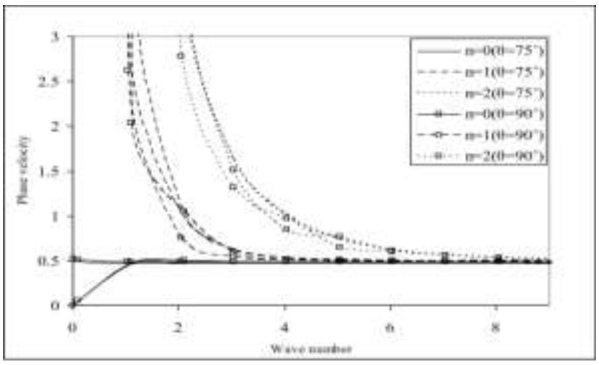

Figure 2: Variation of phase velocity with wave number for symmetric and asymmetric modes

Figure 2 presents the profiles of phase velocity (V_p) with wavenumber ® with liquid loaded temperature for the angle of inclination 0=75^0, 90^0 for symmetric and asymmetric modes. At the higher wavenumbers, the phase velocities of several optical symmetric and asymmetric styles achieve very high values and then significantly decrease to become nearer to the shear wave velocity asymptotically but the lowest (acoustic) asymmetric mode (A_0) approaches the Rayleigh wave speed at higher wavenumbers. The trembling energy is mostly spread by the surfaces (interfaces) of the solid material in the situations when a plate of finite thickness seems to be half-space. The free sides possess Rayleigh-type waves having complex phase velocity and wavenumber. The acoustic symmetric mode (S_0) turns in to a non-scattered phase velocity concerning the wavenumber. For higher modes, the value of phase velocity is found to progress at an amount nearly n-times the phase velocity magnitude of the initial mode (n=1). The lowest asymmetric mode(A_0) is detected most sensitive and affected.

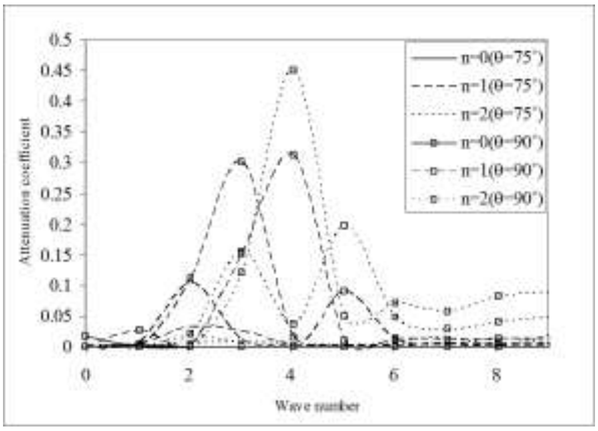

Figure 3: Variation of the attenuation coefficient with wavenumber for symmetric and asymmetric modes

Figure 3 indicates the variations of attenuation coefficients concerning the liquid loaded temperature of wave propagation for symmetrical and asymmetrical manners for the angle of inclination 0=75^0, 90^0 and it is observed that for n=0 and n=1, the attenuation coefficient has an insignificant variation concerning wavenumber along the direction of wave propagation in the framework of generalized theories of thermo-elasticity. For the other optical modes, the magnitude of the attenuation coefficient shoots up to achieve maximum value and then decreases monotonically to zero with the increasing wavenumber.

Figure 4 depicts the variation of phase velocity with liquid temperature for symmetrical and asymmetrical modes for the angle of inclination 45^O. It is evident from the figure that for both symmetrical and asymmetrical modes, the phase velocity of the liquid has constant values with changing liquid loading temperature. For higher modes, the value of phase velocity is found to progress at an amount nearly n-times the values of phase velocity of the initial mode (n=1). The behavior and trend of asymmetric and symmetric modes be similar to each other concerning the liquid loading temperature except for the difference in their values or magnitudes. For symmetric modes, the value of the phase velocity is found to be high to that of the asymmetric one at different liquid temperatures.

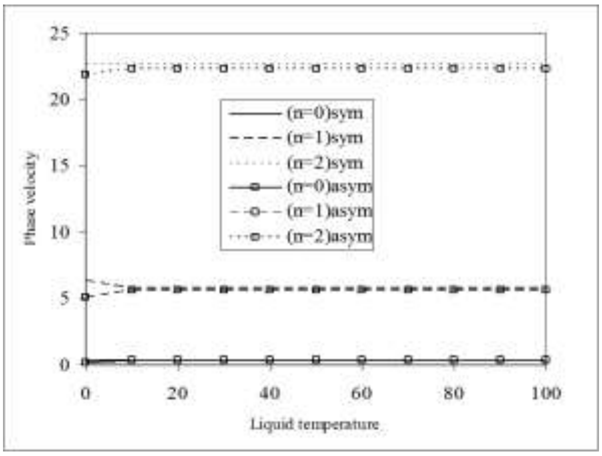

Figure 4: Variation of phase velocity with liquid temperature for symmetric and asymmetric modes

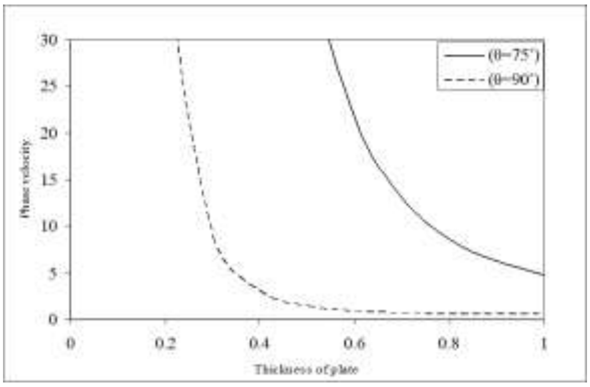

Figure 5: Variation of phase velocity with the thickness of plate for asymmetric mode

In Figure 5, the phase velocity of the asymmetric mode of wave propagation achieves quite high values at a small value of the plate thickness that significantly decreases with the increasing values of the thickness of the plate and remains constant. This happens because the coupling effect of various interacting fields increases as the thickness of the plate increases, resulting in a decrease in the values phase velocity. Also, it is found that with the increasing values of the plate thickness the Rayleigh wave velocity is attained as the heat transmission primarily occurs in the vicinity of the free sides of the plate.

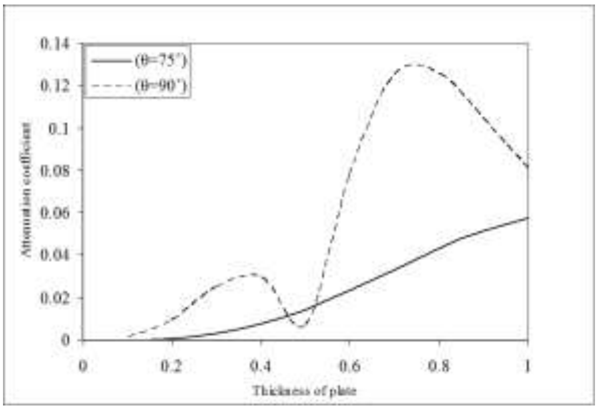

Figure 6: Variation of attenuation coefficient with the thickness of plate for asymmetric mode

Figure 6 shows the variation of the attenuation coefficient with the thickness of the plate for asymmetric mode for the angle of inclination $0=75^0$, 90^0. It is evident from the figure that the values of the attenuation coefficient increases moderately to achieve a maximum value in case of $0=90^0$ and then decreases with the increase in the thickness of the plate while for $0=75^0$, the profile of attenuation coefficient increases linearly with plate thickness. For $0=90^0$ the magnitude of the attenuation coefficient found to be higher than that of $0=75^0$ increasing values of the plate thickness.

10. CONCLUSIONS

The Lamb waves propagation in a thermally conducting elastic homogeneous, transversely isotropic plate in the presence of non-viscous fluid layers on its both sides, with varying temperature has been studied in the frame of reference of non-classical theories of thermoelasticity. It is found that there exists a coupling of three types of waves in the solid plate and two mechanical waves in each liquid layer due to mechanical stresses. It is noticed that for the symmetrical and skew-symmetrical modes, the profiles of the phase velocity show distinguishable results at the long-wavelengths and attains the Rayleigh waves velocity with increasing wavenumbers. The effect of the liquid temperature and thickness of the fluid layers has also been detected significantly on attenuation coefficient and distribution curves in the considered material plate.

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