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A Numerical Discussion about Willem De Sitter Cosmological Model

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Abstract – In this paper we will investigate a numerical discussion about Williem sitter cosmological model filled with an electro magnetized with Nambu strings in general relativity. Here we assume that in electromagnetic field tensor E_{ij}, E_{23} is the only non-vanishing component. Under the assumption that the expansion θ in the model is proportional to the shear σ which leads to N = M^n (where M and N are functions of time only). Also we will discuss the cosmological model physical behavior.

Keywords – Numerical, Fundamental, Einstein Static, Model, Equations etc.

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1. INTRODUCTION

The Dutch astronomer Willem de Sitter (1872-1934) gave important contributions to the rise of relativistic Cosmology. The debate from 1916 to 1918 between de Sitter and Albert Einstein (1879-1955) is a fundamental chapter in the history of the scientific view of the universe. In fact, during such a debate both Einstein and de Sitter formulated their own mathematical expressions for the metric of the universe as a whole.

Einstein static model of the universe

In the fall of 1916 Einstein debated with de Sitter on the problem of suit- able boundary conditions at infinity. According to the Principle of Relativity, Einstein tried to obtain values for the *gμv*'s at infinity that was invariant for all transformations. He avoided this difficulty by replacing such boundary conditions with the condition of closure, introducing a "finite and yet unbounded universe".[4] Einstein proposed a spherical model of the universe, in which the matter was uniformly and homogenously distributed.

This static solution had line element:

$$ds^2 = dx_4^2 - g_{\alpha\beta}dx_\alpha dx_\beta; \qquad (1)$$

$$g_{\alpha\beta} = -\left[\delta_{\alpha\beta} + \frac{x_\alpha x_\beta}{R^2-(x_1^2+x_2^2+x_3^2)}\right]. \qquad (2)$$

R was the radius of curvature of the three-space (*x*1*, x*2*, x*3), that was every- where orthogonal to the time dimension *x*4.

This model fully achieved the relativity of inertia. There was not any independent property of space which claimed to the origin of inertia, so the latter was entirely produced by masses in the universe. The condition of spatial closure ensured that both the gravitational potential and the hypothetical average density of ponderable matter remained constant *in space*.

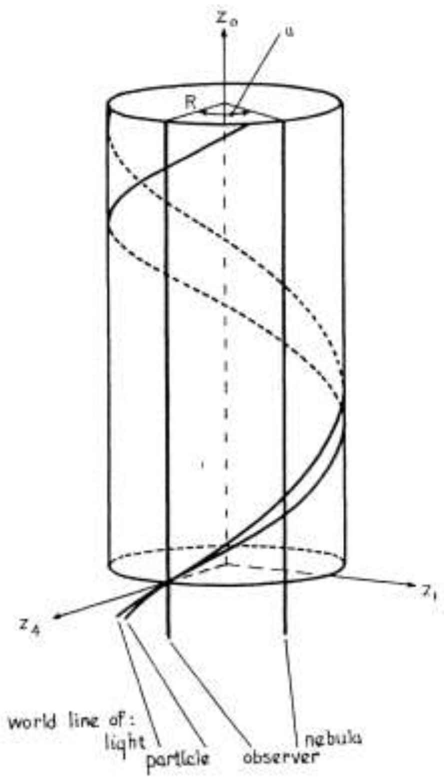

Figure 1: Einstein's "cylindrical" universe. One spatial dimension is disregarded. The vertical axis is along the direction of time [from *Robertson 1933*, p. 70].

Einstein modified his field equations accounting for the supposed static nature of the universe, i.e. to preserve the gravitational potential and the density of matter constant *in time*. He inserted the so-called *cosmological term*, namely the fundamental tensor $g\mu\nu$ multiplied by $-\lambda$, a universal but unknown constant:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -kT_{\mu\nu} \tag{3}$$

In this way field equations expressed the observational evidence of the static equilibrium of the universe.

The new constant λ, the radius of the universe R, and the mean density of "world matter" ρ were strictly connected:

$$\lambda = \frac{kpc^2}{2} = \frac{1}{R^2} \tag{4}$$

In Einstein model the metric of the universe could be given as a solution of relativistic field equations with the cosmological term. Both the general covariance and the laws of conservation of momentum and energy were still satisfied.

3. DE SITTER "EMPTY" MODEL OF THE UNIVERSE

Right after Einstein model appeared, de Sitter proposed his own solution of field equations. The Dutch astronomer admired Einstein conception of the universe "as a contradiction-free chain of reasoning",[7] and gave a different solution also maintaining the λ-term. However de Sitter preferred the original relativistic theory of gravitation, "without the undeterminable λ, which is just philosophically and not physically desirable".[8]

3.1 The "mathematical postulate of relativity of inertia"

De Sitter approached the cosmological problem in a different way. It was mainly Paul Ehrenfest (1830-1933) who suggested him a mathematical conception of inertia[9], which led de Sitter to propose a finite and "empty" universe.

De Sitter proposed a distinction between the "world matter" and the "ordinary matter". The former was hypothetically distributed through space with density $\rho 0$. The latter corresponded to observable objects as planets and stars, i.e. to locally condensed world matter with density $\rho 1$. By this assumption, de Sitter pointed out that "inertia is produced by the whole of world matter, and gravitation by its local deviations from homogeneity".[13] Neglecting all pressures and internal forces, and supposing all matter to be at rest, the energy-momentum tensor became:

$$T_{44} = (p_o + p_1)c^2 g_{44}\,. \tag{5}$$

De Sitter made the hypothesis to neglect gravitation on large-scale, and to take $\rho 0$ constant.

According to de Sitter, the three-dimensional finite world proposed by the Feinstein seed the "material relativity requirement",[14] or equivalently the "material postulate of relativity of inertia".[15]

The Dutch astronomer pointed out that the relativistic field equations were "the fundamental ones"[17]: the postulate that at infinity all $g\mu\nu$'s were invariant for all transformations was more important than the "Machian" postulate of inertia introduced by Einstein. In fact in Einstein model, for the hypothetical value $R = \infty$, the whole of $g\mu\nu$'s degenerated to

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This set of values was invariant for all transformations for which, at infinity, $t0 = t$. In other words, in Einstein cylindrical world it was possible to find systems of reference in which the $g\mu\nu$'s only depended on the space-variables, and not on the "time". However the "time" of such a systems had "a separate position",[18] because it was "the same always and everywhere".[19] For such a reason, according to de Sitter, the time coordinate in Einstein model was nothing else than an absolute time, and there the world matter took "the place of the absolute space in Newton's theory, or of the inertial system".[20] De Sitter proposed that the potentials should have degenerate at infinity to the values:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

According to him, "if at infinity all $g\mu\nu$'s were *zero*, then we could truly say that the *whole* of inertia, as well as gravitation, is thus produced. This is the reasoning which has led to the postulate that at infinity all $g\mu\nu$'s shall be zero".[21] De Sitter called this requirement the "mathematical relativity condition",[22] or the "mathematical postulate of relativity of inertia".[23] In fact, such a condition corresponded to the possibility that "the world as a whole can perform random motions without us (within the world) being able to observe it"[24]: "the postulate of the invariance of the $g\mu\nu$'s at infinity - de Sitter stated - has no real physical meaning. It is purely mathematical"[25]

3.2 A universe without "world matter"

In a letter to Einstein[26] de Sitter proposed his own solution of the metric of the universe as a whole, actually the second relativistic model in modern Cosmology.

The Dutch astronomer considered field equations with the λ-term and without matter, i.e. by assuming $\rho 0 = 0$:

$$R_{\mu v} - \frac{1}{2} g_{\mu v} R - \lambda g_{\mu v} = 0. \qquad (6)$$

These equations could be satisfied by the $g\mu v$'s given by the metric:

$$ds^2 = \frac{-dx^2 - dy^2 - dz^2 + c^2 dt^2}{\left[1 - \frac{\lambda}{12}(c^2t^2 - x^2 - y^2 - z^2)\right]^2}. \qquad (7)$$

The coordinates (x, y, z, t) could have infinite values, on condition that $g\mu v$'s were null at infinity. Such a condition was equivalent to the finiteness of the world in natural (proper) measure. In fact the length of any semi-axis in natural measure was:

$$L_a = \int_0^{\infty} \sqrt{-g_{\alpha\alpha}}\, dx_{\alpha}. \qquad (8)$$

A finite world (i.e. a finite value of $L\alpha$) necessary implied $g\alpha\alpha = 0$ for $x\alpha \to \infty$, and vice versa.[27]

De Sitter pointed out that in his model no world matter was necessary, and the insertion of the λ-term satisfied the mathematical postulate of relativity of inertia. In this system there was not any universal time, nor any difference between the "time" and the other coordinates: none of these coordinates had any physical meaning.[28] The cosmological constant determined the value of the curvature radius R:

$$\lambda = \frac{3}{R^2} \qquad (9)$$

by using an imaginary "time"-coordinate $\xi 4 = ict$, the geometry of de Sitter world was that of a 4-dimensional hyper-sphere which could be described in a 5-dimensional Euclidean space:

$$R^2 = \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \xi_5^2\,. \qquad (10)$$

In hyper-spherical coordinates the metric of such a *four-dimensional world* resulted:

$$ds^2 = -R^2\{dw^2 + \sin^2 w[d\zeta^2 + \sin^2\zeta(d\psi^2 + \sin^2\psi d\theta^2)]\} \qquad (11)$$

where $0 \le \theta \le 2\pi$; $0 \le \psi, \zeta, \omega \le \pi$. Equivalently, by replacing the imaginary "time"-coordinate $\xi 4$ with a real time-coordinate ($\xi 4 \to i\xi 4$), the geometry of de Sitter world corresponded to a 4-dimensional hyperboloid in a 4+1- dimensional Minkowski space-time:

$$R^1 = \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \xi_5^2. \qquad (12)$$

By pseudo-spherical coordinates (with $i\omega^O = \omega$), the metric of space-time resulted:

$$ds^2 = -R^2\{d\dot\omega^2 + \sinh^2\dot\omega[d\zeta^2 + \sin^2\zeta(d\psi^2 + \sin^2\psi d\theta^2)]\} \qquad (13)$$

where $0 \le \theta \le 2\pi$; $0 \le \psi, \zeta \le \pi$; $-\infty < \omega 0 < +\infty$.

The potentials in the hyper-spherical coordinate system were:

$$g_{\mu v} = -\left[\delta_{\mu v} + \frac{x_\mu x_v}{R^2 - (x_1^2 + x_2^2 + x_3^2 + x_4^2)}\right]. \qquad (14)$$

Thus the metric proposed by de Sitter,

$$ds^2 = \frac{-dx^2 - dy^2 - dz^2 + c^2 dz^2}{\left[1 - \frac{\lambda}{12}(c^2t^2 - x^2 - y^2 - z^2)\right]^2}, \qquad (15)$$

"If a single test particle - de Sitter wrote to Einstein - existed in the world, that is, there were *no* sun and stars, etc., it would have inertia".[30] Essentially, in the universe proposed by de Sitter a suitable metric was obtained without any "physical" masses.[31] Such forms of matter as stars and nebula were to be regarded as "test particles" in a fixed background metric, which curvature was determined by the cosmological constant.[32]

3.2.1 Einstein criticism

Einstein acknowledged de Sitter's solution to be "very interesting",[35] but "must have been disappointed",[34] and tried to discard this anti-Machian solution: "I cannot grant - Einstein wrote to de Sitter - your solution any physical possibility".[35] In fact, the cosmological term took a fundamental role in de Sitter model in order to involve a sort of spatial (and not material) origin of inertia. "The $g\mu v$ field – Einstein replied to de Sitter - should *be fully determined by matter, and not be able to exist without the latter* ".[36]

At first Einstein objected that the hyperboloid surface

$$1 - \frac{\lambda}{12}(c^2t^2 - x^2 - y^2 + z^2) = 0 \qquad (16)$$

was a singularity. On this surface there was a discontinuity, because the $g44$ term "jumped" [37] from $+\infty$ to $-\infty$, and $g\alpha\alpha$'s from $-\infty$ to $+\infty$. Such a surface lied in the physically finite, but it was not possible to assume infinite values for the potentials, because of the supposed static nature of the universe and the small velocities measured on stars.[38] Moreover, the four-dimensional continuum proposed by de Sitter did not have the property that all its points were equivalent.[39] In fact it had a preferred point, i.e. the center of the conic section

$$1 + (c^2t^2 - x^2 - y^2 - z^2) = 0 \qquad (17)$$

De Sitter replied that the hyper-surface involved a finite natural spatial dis- tance and an infinite natural temporal distance. Thus the discontinuity was only apparent, and this problem was "not interesting".[40] Also the supposed preferred point was later shown to be a geometrical consequence of that choice of coordinates, and not a true physical aspect.[41] "My four-dimensional world - de Sitter remarked to Einstein - also has the λ-term, but no *world matter*".[42]

3.2.2 Elliptical geometry

In order better to compare his own model with Einstein solution, de Sitter proposed another expression of the metric.[43] By using spherical polar coordinates, he represented the hyperboloid universe (system *B*) as the Einstein universe (system *A*), i.e. as 3-dimensional hyper-spheres embedded in a 4-dimensional Euclidean space:

$$ds_A^2 = -dr^2 - R^2 \sin^2\frac{r}{R}(d\psi^2 + \sin^2\psi d\theta^2) + c^2dt^2, \qquad (18)$$

$$ds_B^2 = -dr^2 - R^2 \sin^2\frac{r}{R}(d\psi^2 + \sin^2\psi d\theta^2) + \cos^2\frac{r}{R}c^2dt^2. \qquad (19)$$

De Sitter acknowledged Einstein remark to be correct, but gave a different interpretation. According to the Dutch astronomer, such a remark involved a *philosophical*, and not a *physical* requirement.[52] In fact, the "equator" at $r = \pi R$ was at a finite distance in space, but was physically inaccessible.[53] The velocity of a material particle became zero for $r = \pi R$. Thus a material particle which was on the polar line on the origin could have no velocity, nor energy. "All these results - de Sitter stated - sound very strange and paradoxical. They are, of course, all due to the fact that $g44$ becomes zero for $r = \pi R$. We can say that on the polar line the four-dimensional time-space is reduced to the three-dimensional space: *there is no time*, and consequently no motion".[54] The time needed by a ray of light, or by a material particle, to travel by any point to the equator was infinite. Thus the singularity at $r = \pi R$ could never affect any physical experiment.[55]

4. CONCLUSION

"At the present time - de Sitter wrote in 1920 - the choice between the systems *A* and *B* is purely a matter of taste. There is no physical criterion as yet available to decide between them".[59] However de Sitter noticed that these systems differed in their physical consequences. In fact, in de Sitter world a particle at rest would not have remained at rest unless it was at the origin. This mass test would have escaped far away because of the presence of the cosmological constant. Thus the de Sitter system to all appearances was static, and required a positive radial velocity for distant objects. This effect of recession was known as "de Sitter effect". At that time, and during the Twenties, this effect appeared to be connected in some manner with the first red-shift observations of many nebula. The interest in de Sitter effect survived until 1930, when truly non-static theoretical models of the universe were proposed to explain the red-shift problem and the astronomical evidences of a cosmic recession .[60]

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