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Certain Investigation on Euler Matrix Method for Linear Second-Order Partial Differential Equations with Various Conditions

Binod Kumar Tiwari[1]* Dr. Pratibha Yadav[2]

[1] Research Scholar, Department of Mathematics, Magadh University, Bodhgaya, Bihar

[2] Associate Professor, Department of Mathematics, College of Commerce, Patna, M.U. Bodhgaya, Bihar

Abstract – In this essay we are studying the nature of a linear secondary differential equation system with different requirements for the Euler matrix. This is the intention study is to apply the Euler matrix method to linear second order partial differential equations under the most general conditions. Error analysis of the method is presented. By using the residual correction procedure, the absolute error may be estimated. The effectiveness of the method is illustrated in numerical examples. Numerical results are overlapped with the theoretical results. Some important results are also discussed.

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1. INTRODUCTION:

There are some well-known numerical methods such as finite difference methods, finite element methods, polynomial approximate methods, spectral methods, Galerkin, and collocation methods to numerically solve PDEs [1-2]. However, recently various approximate methods are discussed in the literature which as the Transform differential process, Legendre-wavelet method. Chebyshev-tau method and Form of Adomian breaking down. In this article, we have developed a matrix method dependent on method Euler polynomials. The method was given by error estimation and error analysis.

Let Ω be a rectangular region $\Omega=\{(x,y):0\le x,y\le b\le\infty\}$ and $\partial\Omega$ is the boundary of Ω. In general form, for all $(x,y)\in\Omega$, linear Partial equations differential with variable coefficients follow as,

$$P(x,y)\frac{\partial^2 u}{\partial x^2}+Q(x,y)\frac{\partial^2 u}{\partial x\partial y}+R(x,y)\frac{\partial^2 u}{\partial y^2}+S(x,y)\frac{\partial u}{\partial x}+$$

$$T(x,y)\frac{\partial u}{\partial y}+V(x,y)u=G(x,y) \qquad (1.1)$$

In this article, we take into account (1.1): conditions in three complicated form [12].

Case 1: Conditions defined at the points x = α_k and y = β_k, where $\alpha_r,\beta_k\in\partial\Omega$

$$\sum_{k=1}^{t}\sum_{i=0}^{1}\sum_{j=0}^{1}a_{i,j}^{k}u^{(i,j)}(\alpha_k,\beta_k)=\lambda_k$$

Case 2 : Conditions defined at the points y = y_k, where $y_k\in\partial\Omega$

$$\sum_{k=1}^{p}\sum_{i=0}^{1}\sum_{j=0}^{1}b_{i,j}^{k}(x)u^{(i,j)}(x,y_k)=g_k(x)$$

Case 3 : Conditions defined at the points

$$x=\eta_k,\ \text{where}\ \eta_k\in\partial\Omega$$

$$\sum_{k=1}^{m}\sum_{i=0}^{1}\sum_{j=0}^{1}c_{i,j}^{k}(y)u^{(i,j)}(\eta_k,y)=h_k(y)$$

Here

$$P(x,y),Q(x,y)R(x,y),S(x,y),t(x,y),V(x,y)\text{and } g(x,y)$$

are functions defined in Ω.

2. DEFINITIONS AND LEMMAS

Euler Polynomials

Euler numbers and polynomials are very useful in classical analysis and numerical mathematics. In many respects, they are closely linked to theory of Bemoulli polynomials and numbers. Euler polynomials and number are summarized as follows [14-17]. The classical Euler polynomials $E_n(x)$ is usually defined

$$\frac{2e^{xt}}{e^t+1}=\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!}, |t|<\pi$$

Definition 2.1, Here M The linear is the space of n real matrices, by $I_n \in M$ the identity matrix and S the subspace of all symmetric matrices in M.A. linear functional L on M It is said that it is "positive" if $L(A)>0$ for any $A\in S$ and $A>0$.

Definition 2.2: A couple real-valued functions (f, g) defined on $[t_0,\infty)$ is being named a averaging pair if.

(i) f is nonnegative and locally integrable on $[t_0,\infty)$ satisfying $\int_{t_0}^{\infty} f(s)ds \neq 0$;

(ii) g>0 is absolutely continuous on every compact subinterval of (t_0,∞); and

(iii) for $0\le k<1$

$$\lim_{t\to\infty}\int_{t_0}^{t} f(s)\left[\left(\int_{t_0}^{s} g(u)f^2(u)du\right)^{-1}\left(\int_{t_0}^{s} f(u)du\right)^{k}\right]ds=\infty$$

Definition 2.3 : Let L be a positive linear functional and B = B(t) a real valued matrix function which is invertible for each $t\in[t_0,\infty)$. A quartet of real-valued functions (f, g, L, B) defined on $[t_0,\infty)$ is a generalized averaging quartet if the conditions (i) and (ii) in Definition 2.2 and the following condition (iii) hold (iii) for $0\le k<1$.

$$\lim_{t\to\infty}\int_{t_0}^{t} f(s)\left[\left(\int_{t_0}^{s} g(u)f^2(u)L(B(u))du\right)^{-1}\left(\int_{t_0}^{s} f(u)du\right)^{k}\right]ds=\infty$$

Lemma 2.4:

(I) let conditions in Definition 2.3 hold: then

$$\int_{t_0}^{\infty} f(s)ds=\infty$$

(II) Let $c\in C([t_0,\infty),R)$ and $\int_{t_0}^{\infty} f(s)ds=\infty$, then

$$\lim_{t\to\infty}\left[\left(\int_{t_0}^{\infty} f(s)ds\right)^{-1}\int_{t_0}^{\infty} f(s)c(s)ds\right]ds=\infty$$

Implies

$$\lim_{t\to\infty}\left[\left(\int_{\tau}^{\infty} f(s)ds\right)^{-1}\int_{\tau}^{\infty} f(s)c(s)ds\right]ds=\infty,\ t\ge\tau\ge t_0$$

Lemma 2.5: [36] Let L be a positive linear functional on M. Then, for any $A;B\in S$, We got

$$(L[A^TB])^2\le L[A^TA]L[B^TB]$$

Lemma 2.6: Let L be a positive linear functional on M. For any $R\in M, B\in S$ and $B>0$, So for everyone $v\in(C(t_0,\infty),(0,\infty))$

$$L\left[\frac{1}{v}R^TBR\right]\ge(vL[B^{-1}])^{-1}(L[R])^2$$

Lemma 2.7: Let X(t) be a nontrivial prepared solution of (1.1) and det $X(t)\neq 0$ for $t_0\ge 0$. So for everyone $a\in C^1((t_0,\infty),(0\infty))$ then matrix function.

$$W(t)=a(t)r(t)P(t)\psi(X(t))K(X'(t))G^{-1}(X(t)) \qquad (2.1)$$

Satisfies the equation

$$W'(t)=\frac{a'(t)}{a(t)}W(t)-\frac{p(t)}{r(t)}R(t)P^{-1}(t)W(t)-a(t)Q(t)F(X'(t))$$

$$-\frac{W(t)G'(X(t))X'(t)K^{-1}(X'(t))\psi^{-1}(X(t))p^{-1}(t)W(t)}{a(t)r(t)} \qquad (2.2)$$

3. MAIN RESULTS

There are some well-known Numerical approaches such as methods of final differences finite element methods, polynomial approximate methods, spectral methods, Galerkin, and collocation methods to numerically solve PDEs (1.1)

Theorem 3.1: Assume that all conditions stated in Section 1 are satisfied; suppose for any solution X (t) for (1.1) $G'(X(t))X'(t)K^{-1}(X'(t))^{-1}(X(t))>0$ for t > t_0, and P(t) and R(t) are commutative with $G'(X(t))X'(t)K^{-1}(X'(t))^{-1}(X(t))$ Suppose further that a function occurs a $\in C^1([t_0,\infty),(0,\infty)$ and a generalized averaging quartet.

$$(f, ar, L: P(t)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(x(t)))^{-1}),$$

Where L is positive linear functional on M, satisfying

$$\lim_{x\to\infty} L\left[\Xi_{t_0}^{t}\,|(t_0,t)\right]=\infty \qquad (3.1)$$

And the matrix J defined by

$$J(t_0,t)=\frac{1}{2}(a'(t)r(t)\ell_n - a(t)p(t)R(t)p^{-1}(t))P(t)(\psi(t))$$

$$\times K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1}$$

$$\int_{t_1}^{t}\left[a(s)Q(s)F(X'(s))-\frac{(a'(s)r(s)I_n - a(s)p(s)R(s)p^{-1}(s))}{4a(s)r(s)}\right.$$

$$\left.P(s)\psi(X(s))K(X'(s))(X'(s))^{-1}(G'(X(s)))^{-1}\right]d(s) \quad (3.2)$$

and $\Xi_{t_0}^{t}:M\rightarrow M$ is the linear operator defined by

$$\Xi_{t_0}^{t}U(t)=\left(\int_{t_0}^{t}f(s)ds\right)^{-1}\int_{t_0}^{t}(s)U(s)ds \quad (3.3)$$

Then every prepared Oscillatory solution of (1.1) no $[t_0,\infty)$

Proof: Suppose the Theorem 3.1 is not true and X (t) is any nontrivial prepared result of (1.1) in $[t_0,\infty)$ which is nonoscillatory. Suppose without lack of generality that $X(t)\neq 0, t\geq t_1\geq t_0$. Then by Lemma 2.7. W(t) is symmetric and satisfies the Riccati equation (2.2). That is,

$$W'(t)=\frac{a'(t)}{a(t)}W(t)-\frac{p(t)}{r(t)}R(t)p^{-1}(t)W(t)-a(t)Q(t)F(X'(t))$$

$$-\frac{W(t)G'(X(t)X'(t)K^{-1}(X'(t)\psi^{-1}(X(t))P^{-1}(r)W(t)}{a(t)r(t)} \quad (3.4)$$

Integrating both sides of (3.4) for t_1 to 1, we obtain

W(t)

$$=W(t)$$

$$=W(t_1)+\int_{t_1}^{t}\left[\frac{a'(t)}{a(t)}W(t)-\frac{p(t)}{r(t)}R(t)P^{-1}(t)W(t)-a(t)Q(t)F(X'(t))\right.$$

$$\left.-\frac{W(t)G'(X(t)X'(t)K^{-1}(X'(t)\psi^{-1}(X(t))P^{-1}(t)W(t)}{a(t)r(t)}\right]ds \quad (3.5)$$

Now use of previous lemma and integrate Then we know what contradicts the fact (f, ar, L, $P(t)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1}$ Is a generalized averaging quartet

Corollary 3.2 : in case the above conditions hold and

$$G'(X)(t))X'(t)K^{-1}(X'(t))\psi^{-1}(X(t))P^{-1}(t)\geq A>0$$

And

$$F(X'(t))\geq B>0 \quad t\in[t_0\infty)$$

where A, B $\in$S are constant positive definite matrices, and A is commutative with P(t) and R(t). Suppose further that there exist an averaging pair (f, ar), where $a\in C^1([t_0\infty),(0,\infty)$ and L is a linear functional positive on M satisfying (3.1), where

$$J(t_0,t)=\frac{1}{2}(a'(t)r(t)I_n - a(t)p(t)R(t)P^{-1}(t))A^{-1}$$

$$+\int_{t_1}^{t}\left[a(s)Q(s)F(X'(s))-\frac{(a'(s)r(s)I_n - a(s)p(s)R(s)P^{-1}(s))^2}{4a(s)r(s)}A^{-1}\right]d(s)$$

and $\Xi_{t_0}^{t}:S\rightarrow S$ is the linear operator defined by (3.3). Then any prepared solution of oscillatory (1.1) on $[t_0,\infty)$.

Remark 3.2. Theorem 3.1 and Corollary 3.2 are improvement and generalize of Theorem 3.1 and Corollary 3.1 by Yang [56]. In fact, Theorem 3.1 in [56] is not applicable if we choose such that

$$\lim_{t\to\infty}\frac{ds}{a(s)r(s)L[P(s)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t))^{-1}]}<\infty$$

Or

$$P(t)\neq R(t)$$

Remark 3.4 : Theorem 3.1 is improvement and generalize of Theorem 3.1 by Xu and Zhu [53]. In fact, Theorem 3.1 in [53] is not applicable if we choose such that

$$\lim_{t\to\infty}\int_{t_0}^{t}\frac{ds}{a(s)r(s)L[P(s)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1}]}<\infty$$

Remark 3.5 : Theorem 3.1 and Corollary 3.2 are improved and generalize to Theorem 3.1 and Corollary 3.1 by Yang and Tang[59]. In fact, Theorem 3.1 in [57] is not applicable if we choose such that P(t)≠R(t). But when P(t) = R(t), $\psi(X(t))=I_n$ and $K(X'(t))=X'(t)$ in Theorem 3.1 and Corollary 3.2 give Theorem 3.1 and Corollary 3.2 in [57], respectively. Also, when $G'(X(t))>0$ and P(t) > 0 in Theorem 3.1 [57], the outcome of these positive definite matrices is not necessarily positive definite.

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Corresponding Author

Binod Kumar Tiwari*

Research Scholar, Department of Mathematics, Magadh University, Bodhgaya, Bihar