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Study on Asymptotic Classification of Finite Dimensional Nonlinear SDES

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Abstract – Given that our fundamental supposition which will ensure the stability of the basic deterministic equation is the dissipative condition, in this part we explore how the outcomes can be reached out to the finite– dimensional case. Generally, we can demonstrate analogs of the fundamental outcomes concerning a characterization of asymptotic stability (under frail conditions on f) and an order of the asymptotic conduct (under solid mean– returning conditions a long way from the equilibrium). In this Article, we studied about the Asymptotic Classification Of Finite Dimensional Nonlinear SDES.

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I. INTRODUCTION

Where the hidden deterministic ODE has a novel internationally stable equilibrium at zero. In this part, we try to broaden our outcomes to the finite– dimensional case, expecting that the outcomes on limited dimensional relative equations can be of help. Similarly we will work with a d– dimensional framework, so the noise intensity will be a ceaseless d × r matrix– esteemed function and B r– dimensional standard Brownian motion. *f* ought to be a function from $\mathbb{R}^d$ to $\mathbb{R}^d$, and be nonstop with the goal that solutions of the SDE can exist. In any case, it is essential to ask how we should catch sensibly the suspicion that x = 0 is an interesting all inclusive stable equilibrium arrangement of (1.1).

$$dX(t) = -f\big(X(t)\big)dt + \sigma(t)dB\,(t) \qquad (1.1)$$

As to uniqueness, we should ask for that f(x) = 0 if and just if x = 0. Worldwide steadiness is anyway harder to portray, and all in all even deterministic research has focused on giving adequate conditions under which all solutions of

$$x'(t) = -f(x(t)) \qquad (1.2)$$

obey x(t) → 0 as t → ∞. One popular assumption in the stochastic literature is the so called dissipative condition

$$\langle x, f(x)\rangle > 0 \text{ for all } x \neq 0,$$

and it is easy to see that this yields x(t) → 0 as t → ∞, by demonstrating that the Liapunov function V(x(t)) = $\|x(t)\|_2^2$ is diminishing on directions. It is additionally certain that the dissipative condition makes x = 0 the extraordinary equilibrium, for if there were another at $x^* \neq 0$, at that point

We have

$$0 = \langle x^*, 0\rangle = \langle x^*, f(x^*)\rangle > 0$$

A logical inconsistency. We see likewise that in the one– dimensional case, the condition xf(x) > 0 for $x \neq 0$, which portrays the presence of a remarkable and all around stable equilibrium, is nothing other than the dissipative condition.

The examination of good adequate conditions on f which ensure worldwide steadiness for the conventional equation (1.2) shapes a considerable assemblage of work, and as opposed to endeavoring to follow this, we specify the first commitments of Olech and Hartman in a progression of papers in the 1960s. In Hartman, worldwide strength is guarantee by

$$[J(x)]_{ij} = \frac{\partial f_i}{\partial x_j}(x) \text{ is such that } H(x) := \frac{1}{2}(J(x) + J(x)^T) \text{ is negative definite} \qquad (1.3)$$

In the two–dimensional case, Olech proves that

$$\text{trace} J(x) \le 0 \text{ and } |f(x)| \ge \phi > 0 \text{ for } |x| \ge x^* \qquad (1.4)$$

suffice. The second of these conditions is weakened in Hartman and Olech to

$$|x||f(x)| > K \text{ for all } |x| \geq M, \text{ or } \int_0^\infty \inf_{\|x\|=\rho} |f(x)|\, d\rho = +\infty \qquad (1.5)$$

and the first of Olech's assumptions is modified to

$$\alpha(x) \leq 0, \text{ where } \alpha(x) = \max_{1\leq i<j\leq d}\{\lambda_i(x) + \lambda_j(x)\} \qquad (1.6)$$

and the λ(x's) are Eigen values of H(x). The nearby asymptotic stability of the equilibrium is additionally expected. In the 1970's Brock and Scheinkman shown that some of Olech and Hartman's conditions can be concluded from Liapunov contemplations. Specifically, they demonstrate that a portion of the conditions utilized as a part of suggest the dissipative condition. This is specifically compelling to us, as our way to deal with understanding the stability and boundedness of solutions might be viewed as a Liapunov– like approach. A later paper of Gasull, LLibre and Sotomayor thinks about the connections between these conditions and worldwide stability. As the section builds up, the connection between these current conditions and the conditions we will require are drawn out.

For the situation where we demonstrate stability, we have discovered that it is never again enough to expect just the worldwide stability condition that did the trick in the scalar case. Rather, our confirmation requires that *f* comply

$$\liminf_{x\to\infty} \inf_{|y|=x} \langle y, f(y)\rangle > 0.$$

It is interesting to see that this condition infers the primary condition in (1.5). Besides, we estimate that in the limited dimensional stochastic case, it might be important for the function f to give some negligible quality of mean inversion at infinity, on the grounds that the stochastic piece of the equation can be transient (as in its standard can develop to infinity as t → ∞). It is sensible to relegate the source of this issue to the short life of the stochastic bother in the limited dimensional part, on the grounds that in the scalar case, where no extra condition on f is required, the annoyance $\int_0^t \sigma(s)dB(s)$ being a time– changed one–dimensional Brownian motion, is intermittent. To give some inspiration in the matter of why we expect some additional condition on f within the sight of an aggregately transient bother, we review the deterministic outcomes in Chapter 1, and compose the differential equation in the integral shape

$$x(t) = \xi - \int_0^t f(x(s))\, ds + \int_0^t g(s)\, ds, \quad t \geq 0. \qquad (1.7)$$

In the case when g(t) → 0 but $\int_0^t g(s)\, ds = +\infty$, we have shown that unless f has enough strength to counteract the cumulative perturbation R t 0 g(s) ds, it is possible that x(t) → ∞ as t → ∞. If one writes the stochastic equation in integral form

$$X(t) = \xi - \int_0^t f(X(s))\, ds + \int_0^t \sigma(s)\, dB(s), \quad t \geq 0,$$

we can figure that when the total perturbation $\int_0^t \sigma(s)dB(s)$ isn't joined (which happens when $\sigma \notin L^2([0,\infty);\mathbb{R}^{d\times r}))$ some insignificant quality in *f* is expected to shield the arrangement from getting away to infinity. There is another motivation to trust that the similarity with the deterministic equation here is advocated. For the situation when g is in L^1 (0, ∞) and the total perturbation $\int_0^t g(s)ds$ unites, that the arrangement of (1.7) obeys x(t) → 0 as t → ∞ utilizing just the worldwide stability condition x*f*(x) > 0 for x ≠ 0, which is nothing other than the dissipative condition in one measurement. In this part, an immediate simple of this outcome in the stochastic case is demonstrated. It can be demonstrated that when f obeys just the dissipative condition, and $\sigma \notin L^2([0,\infty);\mathbb{R}^{d\times r}))$ (so that the cumulative stochastic perturbation $\int_0^t \sigma(s)dB(s)$ converges), then X (t) → 0 as t → ∞ a.s.

II. SUFFICIENT CONDITIONS FOR ASYMPTOTIC BEHAVIOUR

The functions Σ_i determine the asymptotic behavior of X. Let N ⊆ {1, 2. . . d} be defined by

$$N = \{i \in \{1,2,\ldots,d\} : \sigma_i \notin L^2(0,\infty)\}. \qquad (1.8)$$

Where σ_i is defined. Note that if *i* ∉ N, then σ_i ∈ L^2 (0, ∞) and we immediately have that Y_i(t) → 0 as t → ∞ a.s

Theorem 1. Suppose that f satisfies (1.12), (1.2.4) and (1.22). Suppose that σ obeys and $\sigma \notin L^2([0,\infty);\mathbb{R}^{d\times r}$. Let X be the solution of (1.11). Let N be the set defined in (1.8) and Σ_i be defined by (2.3.2) for each i ∈ N.

a) If Σ_i (t) → 0 as t → ∞ for each *i* ∈ N, then X obeys (1.11).

b) If X obeys (1.11), then lim $\inf_{t\to\infty}$ Σ_i (t) = 0 for each *i* ∈ N.

c) If lim $\inf_{t\to\infty}$ Σ_i (t) > 0 for some *i* ∈ N, then P[$\lim_{t\to\infty}$ X(t) = 0] = 0

d) If $\lim_{t\to\infty} \Sigma_i$ (t) = ∞ for some *i* ∈ N then $\lim\sup_{t\to\infty}$ ‖X (t) ‖ = ∞ a.s.

An interesting fact of this outcome is that it is pointless for σ(t) → 0 as t → ∞ all together for solutions of (1.11) to comply (1.11). Truth be told, we can even have $\lim\sup_{t\to\infty} \|\sigma(t)\|_F^2$ = ∞ and still have X(t) → 0 as t → ∞ a.s..

Note that the condition

$$\lim_{t\to\infty} \|\sigma(t)\|_F^2 \log t = 0 \qquad (1.9)$$

Implies that Σ_i (t) → 0 as t → ∞ for every I ∈ N, and for I ∉ N despite everything it implies that Y_i (t) → 0 as t → ∞. Additionally take note of that the condition

$$\lim_{t\to\infty} \sigma_i^2(t)\log t = +\infty \text{ for some } i \in \{1,\ldots,d\} \qquad (1.10)$$

Implies that Σ_i(t) → ∞ as t → ∞, and at long last that the condition

$$\liminf_{t\to\infty} \sigma_i^2(t)\log t > 0$$

implies that lim $\inf_{t\to\infty}$ Σ_i(t) > 0. The following outcome is in this way an easy result of Theorem 1.8

Theorem 2. Suppose that *f* satisfies (1.12), (1.2.4) and (1.22). Suppose that σ obeys. Let X be the solution of (1.11)

i. If for all *i* ∈ {1, . . . , d} σ_i obeys limt→∞ σ 2 i (t) log t = 0, then X obeys (1.11)

ii, If there is *i* ∈ {1. . . d} such that σ_i obeys lim $\inf_{t\to\infty}$ σ 2 *i* (t) log t ∈ (0, ∞), then $\mathbb{P}$[$\lim_{t\to\infty}$ X(t) = 0] = 0.

iii. If there is *i* ∈ {1. . . d} such that σ_i obeys $\lim_{t\to\infty}$ σ^2 *i* (t) log t = ∞, then lim $\sup_{t\to\infty}$ ‖X(t)‖ = ∞ a.s.

Chan and Williams have demonstrated for the situation when t↦σ 2(t) is diminishing, that Y complies if and just if σ complies. Consequently, our last outcome is an end product of this perception and of Theorem 1.21. It can likewise be reasoned from Theorem 2

Theorem 3. Suppose that f satisfies (1.12), (1.2.4) and (1.22). Suppose that σ obeys and $\|\sigma\|_F^2$ is decreasing. Let X be the solution of (1.11). Then the following are equivalent:

A. σ obeys limt→∞ $\|\sigma\|_F^2$log t = 0;

B. $\lim_{t\to\infty}$ X(t, ξ) = 0 with positive probability for some ξ ∈ $\mathbb{R}^d$.

C. $\lim_{t\to\infty}$ X(t, ξ) = 0 a.s. for each ξ ∈$\mathbb{R}^d$.

Another outcome a similar way, however with a marginally weaker monotonicity theory is the accompanying.

Theorem 4. Suppose that f satisfies (1.12), (1.13) and (1.22). Suppose that σ obeys and that $(\int_n^{n+1} \|\sigma(s)\|_F^2\, ds)_{n\geq 0}$ is non–increasing. Let X be the solution of (1.11). Then the following are equivalent:

A. σ obeys $\lim_{n\to\infty}$ $\int_n^{n+1} \|\sigma(s)\|_F^2\, ds)$log n = 0;

B. $\lim_{t\to\infty}$ X(t, ξ) = 0 with positive probability for some ξ ∈ $\mathbb{R}^d$;

C. $\text{Lim}_{t\to\infty}$ X(t, ξ) = 0 a.s. for each ξ ∈ $\mathbb{R}^d$.

2.1 Set–up of the problem and main results

Given these general contemplations, we currently abridge the issue to be examined in exact terms, and blueprint the fundamental aftereffects of the part. Let d and r be whole numbers. We settle a complete separated likelihood space (Ω, F,(F(t))t≥0, P). Let B be a standard r– dimensional Brownian motion which is adjusted to $(F(t))_{t\geq 0}$. We think about the stochastic differential equation

$$dX(t) = -f(X(t))\,dt + \sigma(t)\,dB(t), \quad t\geq 0; \quad X(0) = \xi \in \mathbb{R}^d. \qquad (1.11)$$

We suppose that

$$f \in C(\mathbb{R}^d;\mathbb{R}^d); \quad \langle x, f(x)\rangle > 0, \quad x\neq 0; \quad f(0) = 0, \qquad (1.12)$$

and that σ complies. To rearrange the presence and uniqueness of an exceptional continuous adjusted arrangement of (1.11) on [0, ∞), we accept that *f* : R d → R d is locally Lipschitz continuous. See e.g., .Hereinafter, we allude to this exceptional continuous and adjusted process as the arrangement of (1.11). For the situation when σ is indistinguishably zero, it takes after under the speculation (1.12) that the arrangement x of equation (1.2.5)

$$x'(t) = -f(x(t)), \quad t > 0; \quad x(0) = \xi,$$

Obeys

$$\lim_{t\to\infty} x(t;\xi) = 0 \text{ for all } \xi \in \mathbb{R}^d. \qquad (1.13)$$

Clearly x(t) = 0 for all t ≥ 0 if ξ = 0. The question naturally arises: if the solution x of (1.13) obeys (1.10), under what conditions on f and σ does the solution X of (1.11) obey

$$\lim_{t\to\infty} X(t,\xi) = 0, \quad \text{a.s. for each } \xi \in \mathbb{R}^d. \qquad (1.14)$$

We showed under the scalar version of condition (1.12) that the solution X of (1.11) obeys (1.11) if and only if σ obeys

$$S_{\text{scalar}}(\epsilon)\sum_{n=0}^{\infty}\left\{1-\Phi\left(\frac{\epsilon}{\sqrt{\int_n^{n+1}\sigma^2(s)\,ds}}\right)\right\} < +\infty, \quad \text{for every } \epsilon > 0, \qquad (1.15)$$

where Φ is the appropriation function of a standardized typical random variable. Comparing integral conditions were created moreover. In this section, we demonstrate that a relating condition on σ likewise does the trick. Truth be told, that if f complies (1.12) and is locally Lipschitz continuous, and σ is additionally continuous, at that point the arrangement X of (1.11) complies (1.11) if and just if the condition,

$$S(\epsilon) = \sum_{n=0}^{\infty}\left\{1-\Phi\left(\frac{\epsilon}{\sqrt{\int_n^{n+1}\|\sigma(s)\|_F^2(s)\,ds}}\right)\right\} < +\infty, \quad \text{for every } \epsilon > 0, \qquad (1.16)$$

provided that *f* obeys

There exists φ > 0 such that $\phi := \lim_{x\to\infty} inf\ inf^{*}_{y=x}\langle y, f(y)\rangle$

A condition weaker than, however like, (1.12). As in the scalar case, along these lines, we see that the condition that ensures the stability of the linear equation when annoyed by σ gets the job done likewise for every single nonlinear equation for which f complies (1.14) For the situation when (1.14) isn't expected, it can at present be demonstrated that if (1.13) does not hold, at that point

$$\mathbb{P}[X(t,\xi) \to 0 \text{ as } t \to \infty] = 0 \text{ for each } \xi \in \mathbb{R}^d. \qquad (1.17)$$

Likewise, if (1.13) holds, the main possible limiting conduct of solutions are that X(t) → 0 as t → ∞ or ‖X(t)‖ → ∞ as t → ∞. For the situation when σ ∈ L^2 (0, ∞), X complies (1.11) with no further conditions on *f*. The other real outcome in the section gives a complete characterization of the asymptotic conduct of solutions of (1.11) under a reinforcing of (1.14), to be specific

$$\liminf_{r\to\infty}\inf_{\|x\|=r}\frac{\langle x, f(x)\rangle}{\|x\|} = +\infty, \qquad (1.18)$$

which is an immediate simple of the condition expected to give an arrangement of solutions of (1.11) in the scalar case. We demonstrate that solutions of (1.11) are either (a) joined to zero with likelihood one (b) limited, not concurrent to zero, but rather approach zero subjectively close endlessly frequently with likelihood one or (c) are unbounded with likelihood one. Plausibility (a) happens when S(ϵ) is limited for all ϵ (b) happens when S(ϵ) is limited for some , however endless for others, and (c) happens when S(ϵ) is unbounded for all . In this way, this outcome is specifically comparable which applies to linear stochastic differential equations whose basic deterministic part is internationally steady. Despite the fact that the condition (1.13) is essential and adequate for X to comply (1.11), it might end up being somewhat awkward for use in a few circumstances. Thus we find some sharp adequate conditions for X to comply (1.11). In the event that f complies (1.12) and is locally Lipschitz continuous, and σ is continuous however isn't square integrable, in light of the fact that σ_{ij} isn't square integrable for j ∈ J_i , at that point

$$\lim_{t\to\infty}\int_0^t e^{-2(t-s)}\sum_{l\in J_i}\sigma_{il}^2(s)\,ds\cdot\log\log\left(\int_0^t e^{2s}\sum_{l\in J_i}\sigma_{il}^2(s)\,ds\right) = 0, \qquad (1.19)$$

suggests that the arrangement X of (1.11) complies (1.11). We additionally build up banter brings about the situation when $t \mapsto \|\sigma(t)\|_F^2$ monotone, and show that the condition (1.16) is difficult to unwind on the off chance that we expect X to comply (1.11). The primary outcomes are demonstrated by demonstrating that the stability of (1.11) is personally associated with the stability of a linear SDE with a similar dispersion coefficient. The stability of the linear SDE can be portrayed by misusing the way that an unequivocal answer for the equation can be composed down, and that the arrangement is a Gaussian procedure.

III. STATEMENT AND DISCUSSION OF MAIN RESULTS

We begin by demonstrating that solutions of (1.11) will turn out to be subjectively huge at whatever point the dissemination coefficient is with the end goal that solutions of the relating relative equation have a similar property. Besides, if solutions are limited yet not concurrent to zero, at that point solutions of (1.11) don't join to zero.

Theorem 5. Suppose that f satisfies. Suppose that σ obeys and let S. Let X be the solution of (1.11).

(A) Suppose that S obeys. Then

$$\limsup_{t\to\infty}\|X(t)\| = +\infty, \quad a.s.$$

(B) Suppose that S obeys. Then there is a deterministic c3 > 0 such that

$$\limsup_{t\to\infty} \|X(t)\| \geq c_3, \quad a.s.$$

We demonstrate that its solutions can either tend to zero or their modulus keeps an eye on infinity if and just if solutions of a linear equation with a similar dissemination tend to zero.

Theorem 6. Suppose that f satisfies (1.12) and (1.2.4). Suppose σ obeys. Let X be the solution of (1.11), and Y the solution. Then there exist a.s. events Ω1 and Ω2 such that

$$\{\omega : \lim_{t\to\infty} X(t,\omega) = 0\} \subseteq \{\omega : \lim_{t\to\infty} Y(t,\omega) = 0\} \cap \Omega_1, \tag{1.20}$$

$$\{\omega : \lim_{t\to\infty} Y(t,\omega) = 0\} \subseteq \{\omega : \lim_{t\to\infty} X(t,\omega) = 0\} \cup \{\omega : \lim_{t\to\infty} \|X(t,\omega)\| = \infty\} \cap \Omega_2. \tag{1.21}$$

At the point when taken in conjunction, we see that the condition verges on portraying the joining of solutions of (1.11) to zero, dependent upon the likelihood that ‖X(t)‖ → ∞ as t → ∞ being eliminated.

Theorem 7. Suppose that f satisfies (1.12) and (1.2.4). Suppose σ obeys. Let X be the solution of (1.11). Let Φ be given.

I. If σ obeys (2.2.6), then for each ξ ∈ $\mathbb{R}^d$,

$$\left\{\lim_{t\to\infty} \| X(t,\xi) \| = \infty\right\} \cup \left\{\lim_{t\to\infty} \| X(t,\xi) \| = 0\right\} \quad is\ an\ a.s.event$$

II. If X(t, ξ) → 0 with positive probability for some ξ ∈ $\mathbb{R}^d$, then σ obeys.

Proof. To demonstrate part (I), implies that Y (t) → 0 as t → ∞ a.s. Hypothesis 1.21 at that point implies that the occasion {lim $_{t\to\infty}$ ‖X(t, ξ)‖ = ∞} ∪ {lim $_{t\to\infty}$ X(t, ξ) = 0} is a.s. To indicate part (ii), by theory and Theorem 2, we see that $\mathbb{P}$[Y (t) → 0 as t → ∞] > 0. It takes after that σ complies. Part (I) of Theorem 1is inadmissible, as it doesn't decide out the likelihood that ‖X(t)‖ → ∞ as t → ∞ with positive likelihood. On the off chance that further limitations are forced on f and σ, notwithstanding, it is possible to reason that X(t, ξ) → 0 as t → ∞ a.s. In the scalar case, it was appeared in Appleby and Rodkina that no such extra conditions are required.

Our first outcome toward this path imposes an additional condition on σ, however not on f. We take note of that when σ ∈ L [2] ([0, ∞); $\mathbb{R}^{d\times r}$), Y complies and that X complies (1.11). Be that as it may, we can't matter specifically the semimartingale union hypothesis of Lipster– Shiryaev straight forwardly to the non–negative semirtingale $\|X\|^2$, in light of the fact that it isn't ensured that $\mathbb{E}[\|X(t)\|^2]$ for all t ≥ 0. The proof of the accompanying hypothesis, which is conceded to the following segment, utilizes the thoughts of [Theorem 7] intensely, be that as it may

Theorem 8. Suppose that f satisfies (1.12) and (1.14). Suppose also that σ obeys and σ ∈ L 2 ([0, ∞); R d×r). Let X be the solution of (1.11), and Y the solution. Then X obeys (1.11) and lim $_{t\to\infty}$ Y (t) = 0 a.s.

It can be seen from Theorem 1.23 that it just stays to demonstrate Theorem 1.21 for the situation when $\sigma \notin L^2([0,\infty);\mathbb{R}^{d\times r}$ under an extra limitation on f (however no additional condition on σ) we can give essential and adequate conditions regarding σ for which X tends to zero a.s

Theorem 9. Suppose f obeys (1.11) and in addition to (1.12), obeys

$$\lim_{r\to\infty} inf \underset{\|x\|=r}{inf} \langle x, f(x)\rangle > 0 \tag{1.22}$$

Assume that σ complies. Let X be the arrangement of (1.11). Let θ be characterized and let Φ be given. At that point the accompanying are proportionate:

A. S obeys;

B. Lim $_{t\to\infty}$ X(t, ξ) = 0 with positive probability for some ξ ∈ $\mathbb{R}^d$.

C. Lim $_{t\to\infty}$ X(t, ξ) = 0 a.s. for each ξ ∈ $\mathbb{R}^d$

Notice that no monotonicity conditions are required on $\|\sigma\|_F^2$ all together for this outcome to hold. The condition (1.22) was not required to demonstrate a practically equivalent to bring about the scalar case. In any case, the condition is weaker than the condition (1.12) which was required in the scalar case to secure the stability of solutions of (1.11)

There is one last outcome in this area. It gives a complete portrayal of the asymptotic conduct of solutions of (1.11) under a reinforcing of (1.22), to be specific

$$\lim_{r\to\infty} inf \underset{\|x\|=r}{inf} \frac{\langle x, f(x)\rangle}{\|x\|} \tag{1.23}$$

(1.23) is an immediate simple of the condition expected to give a characterization of solutions of (1.11) in the scalar case. The accompanying outcome is in this way an immediate speculation of a scalar outcome from to limited dimensions

Theorem 10. Suppose f obeys (1.2.4), (1.12), and (1.23). Suppose that σ obeys. Let X be the solution of

(1.11). Let θ be defined and let Φ be given. Then the following are equivalent:

A. If S obeys then $\lim_{t\to\infty}$ X(t) = 0, a.s. for each ξ ∈ $\mathbb{R}^d$

B. If S obeys, then there exists deterministic 0 < c1 ≤ c2 < +∞ such that

$$c_1 \leq \lim_{t\to\infty} sup \parallel X(t) \parallel \leq c_2, \quad a.s., for\ each\ \xi \epsilon R^d$$

Moreover,

$$\lim_{t\to\infty} inf \parallel X(t) \parallel = 0$$

A. If S obeys, then lim $\sup_{t\to\infty}$ ‖X(t)‖ = +∞ a.s., for each ξ ∈ $\mathbb{R}^d$

IV. CONCLUSION

By worldwide properties we allude to properties of the first body being referred to and its pictures under direct changes while the neighborhood properties relate to the structure of lower dimensional areas and projections of the body, i.e., to the straight structure of a normed space in the soul of functional analysis. In the two theories we are keen on the asymptotic behavior, as the measurement develops to in detail, of the significant amounts. Just as the fundamental and sufficient condition, we additionally investigate the simple sufficient conditions and the associations between the conditions which portray the different classes of long–run behavior.

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