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An Analysis on Fixed Point Theorem and Its Application in Fuzzy Metric Space

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***Abstract** – The principle point of this examination is to introduce the idea of general Mann and general Ishikawa write twofold sequences iterations with blunders to approximate fixed points. We demonstrate that the general Mann compose twofold sequence iteration process with blunders meets strongly to a fortuitous event point of two continuous pseudo-contractive mappings, each of which maps a bounded shut arched nonempty subset of a genuine Hilbert space into itself. In addition, we talk about equality from the S, T-secure qualities point of view under specific confinements between the general Mann write twofold sequence iteration process with mistakes and the general Ishikawa iterations with blunders. An application is additionally given to help our thought utilizing good compose mappings. In this paper, we manage iterative methods for approximation of fixed points and their applications. We initially talk about fixed point theorems for a non-expansive mapping or a group of non-expansive mappings.*

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INTRODUCTION

Some traditional fixed point theorems for single esteemed non-extensive mappings have been connected with multivalued mappings. The essential outcomes toward this way were developed by Markin (1968) in a Hilbert space setting and by Browder (1976) for spaces having a feebly persistent duality mapping.

Lami Dozo (1973) summed up these outcomes to a Banach space satisfying Opial's condition. By using Edelstein's method of asymptotic core interests. Lim (1974) got a fixed point theory for a multi-esteemed non-broad self-mapping in a reliably raised Banach space. Kirk and Massa (1990) gave a growth of Lim's theory showing the nearness of a fixed point in a Banach space for which the asymptotic point of convergence of a limited grouping in a close limited raised subset is nonempty and limited.

Various request remain open about the nearness of fixed points for multivalued non-broad mappings when the Banach space satisfies geometric properties which ensure the nearness of a fixed point in the single esteemed case, for instance, if X is a reliably bended space.

The beginning stage of fixed point theory lies in the method of dynamic approximations used for exhibiting nearness of arrangements of differential equations introduced independently by Joseph Liouville in 1837 and Charles Emile Picard in 1890. Regardless, formally it was started in the beginning of twentieth century as a basic bit of examination. The pondering of this conventional theory is the leading work of the impressive Polish mathematician Stefan Banach disseminated in 1922 which gives a significant method to find the fixed points of a guide.

In any case, on chronicled point of view, the genuine set up result in fixed point theory is a direct result of L. E. J. Brouwer given in 1912. The watched Banach withdrawal standard (BCP) states that "a compression mapping on a whole metric space has an exceptional fixed point". Banach used contracting mapto get this key outcome. The Brouwer fixed point theory is of phenomenal centrality in the numerical treatment of equations. It accurately communicates that "a nonstop guide on a close unit ball in Rn has a fixed point". A basic extension of this is the Schauder's fixed point speculation of 1930 communicating "a persistent guide on an angled preservationist subspace of a Banach space has a fixed point". These adulated comes about have been used, summed up and connected in various courses by a couple of mathematicians, analysts, budgetary specialists for single esteemed and multivalued mappings under different contractive conditions in various spaces. Kannan showed a fixed point theory for the maps not by any means nonstop. This was another crucial progression in fixed point theory. In this way unique outcomes identifying with fixed points, essential fixed points, chance occasion points, et cetera have been explored for maps satisfying unmistakable contractive conditions in different settings.

The examination of fixed points of multivalued mappings was begun by Nadler (1969) and nonsweeping mapping by Markin (1973) using the possibility of Hausdorff metric. Actually, Nadler showed that a multivalued withdrawal has a fixed point in an aggregate metric space. Ćirić (2009) summed up Nadler's outcome to multivalued semi constriction maps.

Thusly, it got extraordinary thought in pertinent arithmetic and was extended and summed up on various settings. Further, cream fixed point theory for nonlinear single-esteemed and multivalued maps is another headway in the zone of multivalued examination. For a genuine progression of the hybrid fixed point theory.

In this way fixed point theory has been broadly inspected, summed up and enhanced in different methodologies, for instance, metric, topological and orchestrate theoretic. This progress in fixed point theory widened the uses of different fixed point brings about various domains, for instance, the nearness theory of differential and basic equations, dynamic programming, fractal and anarchy theory, discrete elements, masses elements, differential contemplations, system examination, break number-crunching, advancement and diversion theory, variational disparities and control theory, flexibility and pliancy theory and different requests of numerical sciences.

Fixed point theory has reliably been stimulating in itself and its applications in new domains. Starting at now, it has found new and hot regions of activity. The beginning of fixed point theory in software engineering enhances its propriety in different areas. On account of the presence of convenient and speedy computational devices, another horizon has been given to fixed point theory. The fixed point equations are handled by methods for some iterative methodology. In context of their strong applications, it is of wonderful excitement to know whether these iterative methodologies are numerically consistent or not. The examination of quality of iterative procedures acknowledges a praised put in significant arithmetic in view of wild direct of capacities in discrete elements, fractal designs and diverse other numerical figurings where PC writing computer programs is incorporated. This kind of issue for certified esteemed capacities was first discussed by M. Urabe (1956) in 1956. The main outcome on the dauntlessness of iterative methodologies on metric spaces is a direct result of Alexander M. Ostrowski. This outcome was extended to multivalued managers by Singh and Chadha (1995). Czerwik et al (2002) extended this to the setting of summed up metric spaces.

FIXED POINTS BY GIVEN ITERATIVE SCHEMES WITH APPLICATIONS

Over the latest couple of decades examinations of fixed points by some iterative designs have pulled in various mathematicians. With the present quick changes in fixed point theory, there has been a restored energy for iterative designs. The properties of emphasess between the kind of arrangements and kind of managers have not been completely thought about and are presently under talk. The theory of heads has included a central place in present day examine using iterative designs because of its assurance of titanic utility in fixed point theory and its applications. There are different papers that have considered fixed points by some iterative designs. It is to some degree charming to observe that the kind of chairmen accept a significant part in examinations of fixed points.

The Mann iterative arrangement was composed in 1953, and it is used to get joining to a fixed point for a few classes of mappings. Considering fixed point emphasis methods with botches begins from convenient numerical computations. This subject of research expect a basic part in the robustness issue of fixed point emphasess. In 1995, Liu (1995) began an examination of fixed point emphasess with botches. A couple of makers have exhibited some fixed point theorems for Mann-type cycles with botches using a couple of classes of mappings.

Suppose that *H* is a real Hilbert space and *A* is a nonlinear mapping of *H* into itself. The map *A* is said to be accretive if $\forall x, y \in D(A)$, we have that

$$\langle Ax - Ay, x - y \rangle \geq 0, \tag{1}$$

and it is said to be strongly accretive if $A - kI$ is accretive, where $k \in (0,1)$ is a constant and I denotes the identity operator on *H.*

The map *A* is said to be ϕ-strongly accretive if $\forall x, y \in E,$ exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that $\langle Ax - Ay, x - y \rangle \geq \phi(\|x - y\|)\|x - y\|,$ and it is called uniformly accretive if there exists a strictly increasing function $\psi : [0,\infty) \to [0,\infty)$ with $\psi(0) = 0$ such that

$$\langle Ax - Ay, x - y \rangle \geq \psi(\|x - y\|).$$

Let $N(A) = \{x^* \in H : Ax^* = 0\}$ denote the null space (set of zero) of *A.* If $N(A) \neq \phi$ and (1) holds for all $x \in D(A)$ and $y \in N(A),$ then *A* is said to be quasi-accretive. The notions of strongly, ϕ-strongly, uniformly quasi-accretive are similarly defined. *A* is said to be ϕ-accretive if $\forall r > 0$ the operator $(I + rA)$ is subjective. Closely related to the class of accretive maps is the class of pseudo-contractive maps.

A map $T : H \to H$ is said to be pseudo-contractive if $\forall x, y \in D(T)$ we have that

$$\langle (I-T)x-(I-T)y, x-y\rangle \geq 0, \tag{2}$$

Observe that T is pseudo-contractive if and only if $A=(I-T)$ is accretive.

A mapping $T:H\to H$ is called Lipschitzian (or L-Lipschitzian) if there exists $L>0$ such that

$$\|Tx-Ty\| \leq L\|x-y\|, \quad \forall x,y\in H.$$

In the sequel we use $L>1$.

FIXED POINT THEOREMS WITHIN COMPLETE METRIC SPACES

The STUDY in fixed point theory has generally created in three principle bearings: generalization of conditions which guarantee presence, and, if conceivable, uniqueness, of fixed points; examination of the character of the sequence of iterates $\{T^n x\}_{n=0}^{\infty}$, where $T:X\to X$, X a total metric space, is the map under thought; investigation of the topological properties of the arrangement of fixed points, at whatever point T has more than one fixed point. This note treats just a few parts of the first and second inquiry, along a line followed by numerous different creators.

More precisely we consider maps $T:X\to X$, which satisfy conditions of the type $d(Tx,Ty)\leqslant \varphi(d(x,y))+\psi(d(Tx,x))+\chi(d(Ty,y))$ for each $x,y\in X$, what's more, for these mappings we demonstrate, under appropriate speculations, presence and uniqueness of fixed points.

Through all the examination, X indicates a total metric space and $T, T:X\to X$, an asymptotically regular mapping; i.e., a function satisfying $\lim_n d(T^n x, T^{n+1}x)=0$ for each $x\in X$.

Furthermore, we suppose that there exist three functions $\varphi,\psi,\chi,$ from $[0,+\infty[$ into $[0,+\infty[$, which satisfy the assumptions:

(I_1) $\varphi(r)<r \quad \text{if } r>0,$

(I_2) there exists $\lim_{r\to\bar{r}+} \varphi(r) \leqslant \varphi(\bar{r}) \quad \text{for each } \bar{r}\in[0,+\infty[$

(I_3) $\psi(0)=\chi(0)=0.$

Moreover, we suppose that *T,* satisfy the inequality

(I_4) $d(Tx,Ty)\leqslant \varphi(d(x,y))+\psi(d(Tx,x))+\chi(d(Ty,y))$ for each $x,y\in X$.

Lemma. Under the above assumptions on X and T and if, in addition, ψ and χ are continuous at $r=0,$ then, for each $x\in X,$ there exists $z\in X$ such that $\{T^n x\}_{n=0}^{\infty}$ converges to z.

Proof. Suppose that there exists $x\in X$ such that the sequence of iterates is not a Cauchy sequence. Then, following there exist $\varepsilon>0, \{m(j)\}_{j=0}^{\infty}, \{n(j)\}_{j=0}^{\infty}$ which satisfy the conditions

$$m(j)>n(j) \quad \text{for each } j\in N \tag{3}$$

$$\lim_j n(j) = +\infty \tag{4}$$

$$d(T^{m(j)}x, T^{n(j)}x) \geqslant \varepsilon \tag{5}$$

$$d(T^{m(j)-1}x, T^{n(j)}x) < \varepsilon. \tag{6}$$

Then, we have

$$\varepsilon \leqslant d(T^{m(j)}x, T^{n(j)}x) \leqslant d(T^{m(j)}x, T^{m(j)-1}x)+d(T^{m(j)-1}x, T^{n(j)}x) < \varepsilon + d(T^{m(j)}x, T^{m(j)-1}x)$$

which implies

$$\lim_j d(T^{m(j)}x, T^{n(j)}x) = \varepsilon. \tag{7}$$

On the other hand

$$d(T^{m(j)}x, T^{n(j)}x) \leqslant d(T^{m(j)}x, T^{m(j)+1}x)+d(T^{n(j)}x, T^{n(j)+1}x)+d(T^{m(j)+1}x, T^{n(j)+1}x) \leqslant d(T^{m(j)}x, T^{m(j)+1}x)+d(T^{n(j)}x, T^{n(j)+1}x)+\varphi(d(T^{m(j)}x, T^{n(j)}x)) + \psi(d(T^{m(j)+1}x, T^{m(j)}x))+\chi(d(T^{n(j)+1}x, T^{n(j)}x))$$

that is

$$d(T^{m(j)}x, T^{n(j)}x)-\varphi(d(T^{m(j)}x, T^{n(j)}x)) \leqslant d(T^{m(j)}x, T^{m(j)+1}x)+d(T^{n(j)}x, T^{n(j)+1}x)+\psi(d(T^{m(j)+1}x, T^{m(j)}x))+\chi(d(T^{n(j)+1}x, T^{n(j)}x))$$

and, letting $j\to+\infty$ $\quad \varepsilon - \lim_j \varphi(d(T^{m(j)}x, T^{n(j)}x)) \leqslant 0.$

FIXED POINT THEOREM ON FUZZY METRIC SPACES

FUZZY metric space is a generalization of metric space. The study on uncertainty and on randomness began to explore with the concept of fuzziness in mathematics. Fuzzy set is used in fuzzy metric space, which is initiated by Lofti. A. Zadeh. After that Kramosil and Michalek[4] introduced the concept of fuzzy metric space. A very important notion of fuzzy metric space with continuous tnorm is laid by Georage and Veeramani. Grabiec extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Compatible mapping is generalized from commutatively mappings by Jungck. After that Jungck and Rhodes [3] initiated the notion of weak compatible and proved that compatible maps are weakly compatible but converse is not true. A common E.A property is the generalization of the concept of non compatibility is introduced under strict contractive conditions by Aamri and El. Moutawakil.

In present time, Fuzzy set theory and Fuzzy logic is not only active field of research in mathematics but also in other field of engineering, medicine, communication, physics, biology etc. are field in which the applicability of fuzzy theory was accepted. Since, Many authors regarding the theory of fuzzy sets and its applications have developed a lot of literature.

In this paper, we prove a new fixed point theorem on fuzzy metric space by using above results. We also give an example which satisfies our main result in the paper.

CONCLUSION

In this examination, we have displayed the most critical fixed point theorems in the expository investigation of problems in the connected science: Banach's and Schauder's fixed point theorem. This postulation shows an efficient and exhaustive investigation of fixed point theory in fuzzy metric and normed linear spaces. We have followed the fuzzy metric spaces as characterized by I. Kramosil and J. Michalek. We have utilized fuzzy normed linear spaces as characterized by T. Sack and S. K. Samanta however presented in a somewhat unique manner.

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