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# Introduction to Univariate Subdivision Schemes for Noisy Data

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***Abstract – We introduce and analyse univariate, linear and stationary subdivision schemes for refining noisy data by using geometrical applications. Sub divisions schemes are presented that are based on univariate polynomials. Subdivision is a process of recursively refining discrete data using a set of subdivision rules (called subdivision scheme) which generates a continuous or even smooth limit. Subdivision schemes are multi-resolution methods used in computer-aided geometric design to generate smooth curves or surfaces.***

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## I. INTRODUCTION

Subdivision is a procedure of recursively refining discrete data utilizing a lot of subdivision rules (called subdivision plot) which creates a persistent or even smooth limit. It has various applications, for example, picture recreation, the design of curves and surfaces, shape protection in data and geometric items, the estimation of self-assertive functions, and so on.

Then again, subdivision lies in the centre of multire solution examination (MRA) and wavelet changes, and consequently assumes a focal job in data compression, clamor expulsion, etc. The incredible assortment of applications and in addition the need of enhancing the execution of the existent algorithms prompts the innovation of an extraordinary assortment of subdivision schemes. The fundamental scientific issues to be explored for a subdivision conspire are convergence as the quantity of subdivision steps goes to interminability, the smoothness of the limit objects, and the security of the subdivision procedure and the related multi resolution portrayals.

A subdivision plan produces values related with the vertices of a succession of settled meshes, with a thick association, by rehashed utilization of a lot of neighbourhood refinement rules. These principles decide the values related with a refined work from the values related with the coarser work. The subdivision plan is convergent if the produced values merge consistently to the values of a constant capacity, for any arrangement of beginning values.

Escalated studies have been completed as of late on the speculation of subdivision schemes to treat progressively confused data, for example, complex esteemed data, matrices, sets, curves, and sets of capacities. However, the inquiry how to rough a capacity from its noisy samples by subdivision schemes has stayed open, and it is the motivation behind this paper to address this issue.

A general form of univariate binary subdivision scheme S which maps polygon $f^k = \{f_i^k\}_{i\in\mathbb{Z}}, k \geq 0$ to a refined polygon $f^{k+1} = \{f_i^{k+1}\}_{i\in\mathbb{Z}}$

$$\begin{cases} f_{2i}^{k+1} = \sum_{j\in\mathbb{Z}} \beta_{2j} f_{i-j}^k \\ f_{2i+1}^{k+1} = \sum_{j\in\mathbb{Z}} \beta_{2j+1} f_{i-j}^k \end{cases}$$

where the set $\beta = \{\beta_j : j \in \mathbb{Z}\}$ of coefficients is called mask/stencil of the subdivision scheme. The Laurent polynomial of the mask of $\beta$ scheme (1) is defined as $\beta(z) = \sum_{j\in\mathbb{Z}} \beta_j z^j$. A necessary condition for the uniform convergence of subdivision scheme defined in Equation (1) is $\sum_{j\in\mathbb{Z}} \beta_{2j} = 1, \sum_{j\in\mathbb{Z}} \beta_{2j+1} = 1,$ this is equivalent to $\beta(-1) = 0, \beta(1) = 2$ which implies $\beta(z) = (1+z)b(z), b(1) = 1$

## II. DIFFERENT SUBDIVISION SCHEMES

### LINEAR SCHEMES

We review some results from the theory of linear schemes. We let n = 1, i.e. the points $p_\gamma$ are elements of $\mathbb{R}$. This is no restriction, since convergence and smoothness properties can be treated component wise in the case of scalar linear schemes. In the univariate setting, the (unique) difference operator plays an important role. On grids we look at all the difference operators in the different directions on the grid simultaneously. This allows us to define derived schemes as discrete analogues of the left derivatives.

Let $\Delta_i$ be the operator that maps grid function $(p_\gamma)_{\gamma\in\mathbb{Z}^s}$, to the grid function $(p_\gamma - p_{\gamma-\delta_i})_{\gamma\in\mathbb{Z}^s}$ , where $\delta_i$ denotes the $i-\text{th}$ canonical basis vector in $\mathbb{Z}^s$.

Definition 1

For $p \in l^\infty(\mathbb{Z}^s)$ we define $\Delta_p \in (l^\infty(\mathbb{Z}^s))^s$ as

$\Delta_p : (\Delta_{1p}, \Delta_{2p}, \ldots, \Delta_{sp})^T$ .

For a subdivision scheme to converge, we need that the distance between consecutive points becomes small.

## STATIONERY SCHEMES

Here we talk about just stationary subdivision schemes, and subsequently, we won't compose "stationary" any longer. We signify stationary scheme with letter S, the subdivision procedure ought to be ordinary.

## SOME BASIC DEFINITION

Definition 2.1 A subdivision scheme S is **finite** if there exists an integer $B \geq 0$, such that for every $\alpha \in \mathbb{Z}^s, (v^1)_\alpha$ , is a function of at most $B$ elements of $v^0$.

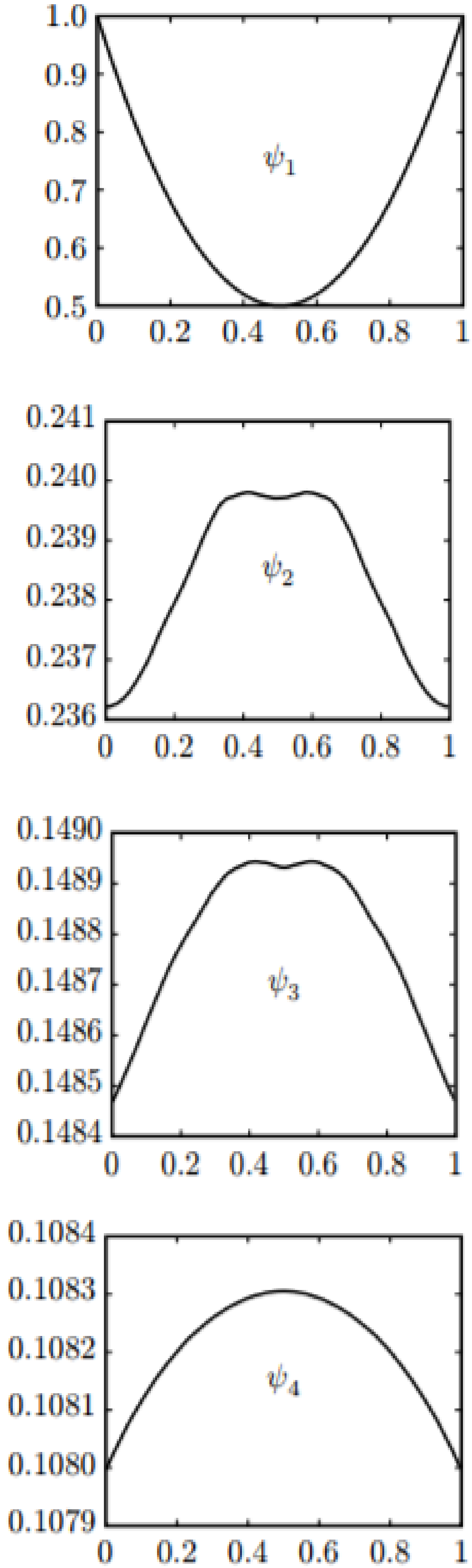
**Definition 1.**1 A subdivision scheme S **reproduces constants** if $S\mathbf{1} = \mathbf{1}$, where $\mathbf{1} \in l_\infty(\mathbb{Z}^s)$ is the constant sequence 1.

**Definition 1.2** A subdivision scheme S is **affine invariant** if $S(av + b\mathbf{1}) = aSv + b\mathbf{1}$ for every $a, b \in \mathbb{R}$ and $v \in l_\infty(\mathbb{Z}^s)$.

Since we are managing stationary subdivision schemes, our dynamical framework is self-governing! A subdivision scheme **S** which repeats constants, can't be asymptotically stable, in such a case that we take $v^0 = 1$ and $\tilde{v}^0 = 0$, at that point for each $j \geq 0$ the separation between $\{(v^j, j)\}$ and $\{(\tilde{v}^j, j)\}$ is 1. It looks like Liapunov soundness yet the primary downside of this elucidation is that, not normal for the Liapunov dependability, the trajectories of "close" initial points remain in a cylinder around the initial trajectory simply after some limited minute, that relies upon the initial sequence and isn't consistently limited. Once more, similarly as with the uniform convergence, if S is move invariant it suffices to take $\Omega = [0,1]^s$, and if S is neighbourhood - we can work with limited sub sequences of $v^0$ and $\tilde{v}^0$.

## III. THE SCHEMES APPLIED TO NOISY DATA

The schemes Sn for n > 1 are designed to deal with noisy data, which is confirmed by the following discussions and experiments. We first introduce a statistical model and then compare the performance of our schemes and an advanced local linear regression method.



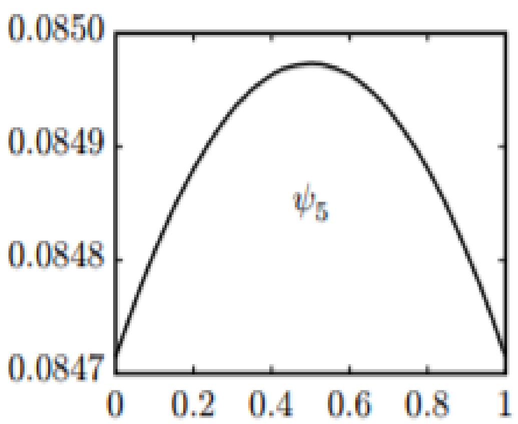



**Figure 1: Plots of the functions ψn for n = 1,..., 5. Note the different scale in each plot.**

## IV. CONCLUSION

In recent years, subdivision schemes have become an important tool in many applications and research areas, including animation, computer graphics, and computer aided geometric design. In this paper we presented application of univariate schemes for subdivision of noisy data. Different schemes which are using geometrical applications for refining data which are univariate, linear and stationary schemes Univariate polynomials

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Novara, P., Romani, L., 2016, Complete characterization of the regions of C 2 and C 3 convergence of combined ternary 4-point subdivision schemes, Applied Mathematics Letters, 62, are taken for the study.

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