Journal of Advances and Scholarly Researches in Allied Education
Vol. XV, Issue No. 5, July-2018, ISSN 2230-7540

The General Equation of Parabola Symmetric about *y* = – *x* Line

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Abstract – The four standard equations of parabola and their symmetry about the coordinate axis is already known result to everyone. As the parabola y^2=4ax is the standard right handed parabola and is symmetric about x axis. The equation of the parabola is always in two variables x and y out of which one variable is linear and the other is quadratic in nature. In this paper, we will derive the general equation of parabola which is symmetric about the line y=-x .

Keywords – Conic Section, Parabola, Types of Parabola, Their Symmetry and Symmetry About y=-x

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INTRODUCTION

Parabola

In mathematics, parabola is roughly U – shaped, symmetric figure in 2 dimensional geometry. It is a plane curve obtained by the locus of points which are at equal distance from fixed point (focus) and fixed line (directrix). There are four standard types of parabola i.e. $y^2 = 4ax$, $y^2 = -4ax$, $x^2 = 4ay$ and $x^2 = -4ay$ .Thus the equation of parabola is in two variables out of which one variable is quadratic and the other one is linear. The parabolas $y^2 = 4ax$ and $y^2 = -4ax$ are symmetric about x-axis and the parabolas $x^2 = 4ay$ and $x^2 = -4ay$ are symmetric about y-axis.

In general, we can write the equation of parabola as $uy^2 + vy + w = x$ which is symmetric about the line parallel to x-axis whereas the parabola $ux^2 + vx + w = y$ are symmetric about the line parallel to y- axis. All these parabolas are not symmetric about $y = -x$ line. So, in this paper, we are going to find the general equation of parabola which is symmetric about line $y = -x$.

History Of Parabola

In (380-320 BC) Menaechmus, a Greek mathematician works on the conic sections and their properties. He studied parabola along with ellipse and hyperbola . He uses the properties of parabola in order to find a cube whose volume is twice that of the given cube . He considered two parabolas $x^2 = y$ and $y^2 = x$ and solve the problem of doubling of volume of cube by finding their intersection.

In (262–190 BCE) Apollonius of Perga had a great contribution in the work on conic sections. He defines the parabola, ellipse and hyperbola and also work on their properties. The elliptical path of the planets and the variation in the speed of moon is explained by him.

In (290- 350 CE), Pappus explains the parabola in terms of focus and directrix i.e. these are the points which are equidistant from the focus and directrix, which we have discussed in this paper.

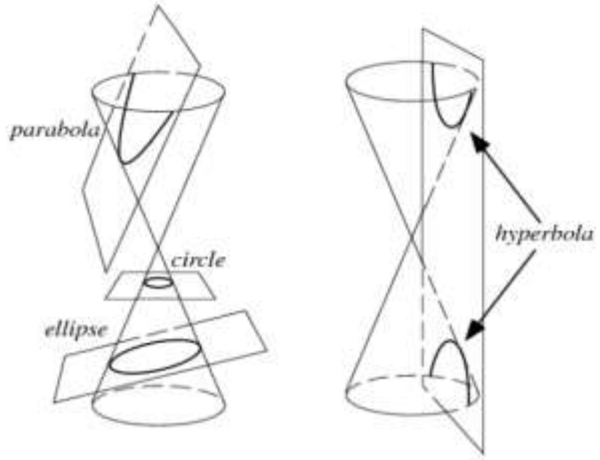
In (1564-1642) Galileo discovered that due to gravity, the path followed by the projectile is parabolic.

In (1623-1662) Pascal worked on conic sections and he considered a parabola as a circle projection.

In (1638-1675) Gregory and in (1643-1727) Newton consider the properties of parabola and discover that the rays of light which travels parallel to the axis of symmetry converges to the focus of the parabola.

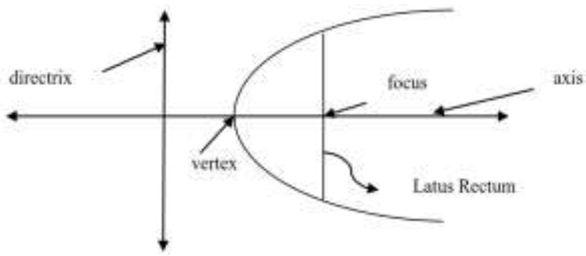
DEFINITIONS

Conic Section – The section of surface generated by the intersection of cone by the plane is known as conics. It may be parabola, ellipse or hyperbola as shown in the figure below. So, the conic section results in parabola if the plane cuts the right circular cone tangentially.



Parabola:- Parabola is a locus of a point which moves such that the ratio of its distance from a fixed point and its perpendicular distance from a fixed line is always same.

The fixed point is known as focus of the parabola, fixed line is directrix of the parabola and the ratio is known as the eccentricity of the parabola, denoted by e. For parabola e = 1

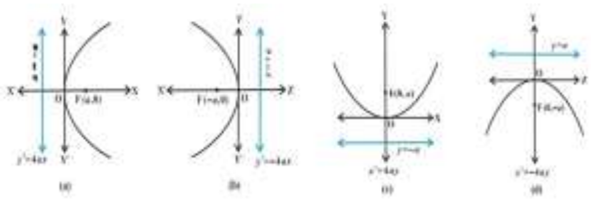


(i) Axis:- The perpendicular line drawn to the fixed line directrix which passes through the focus is called Axis of the parabola.

(ii) Vertex:- It is the point where the axis meets the curve. Here vertex is (0,0) i.e. origin.

(iii) Latus Rectum:- The line joining the two points on the curve parabola is known as chord . And the perpendicular chord to the axis which passes through the focus is known as latus rectum. The total length of the latus rectum is 4a.

In standard four types of parabolas:-



(i) The left handed parabola $y^2 = 4ax$ fig(a) :- Focus is (a,0); vertex is (0,0) and the directrix is x = -a.

(ii) The right handed parabola $y^2 = -4ax$ fig (b) :- Focus is (-a,0); vertex is (0,0) and the directrix is x = a.

(iii) The upward parabola $x^2 = 4ay$ fig (c) :- Focus is (0,a); vertex is (0,0) and the directrix is y = -a.

(iv) The downward parabola $x^2 = -4ay$ fig (d) :- Focus is (0,-a); vertex is (0,0) and the directrix is y = a.

Symmetry of curve:-

When a curve can be divided into two identical mirror images along an axis or imaginary line, then the curve is said to be symmetric about that axis or imaginary line. On folding the figure about the line of symmetry, we get two identical and symmetric halves.

Symmetric about x axis:- The curve $f(x,y)=0$ is symmetric about x- axis if $f(x,y)=f(x,-y)$. Clearly, the parabolas $y^2=4ax$ and $y^2=-4ax$ are symmetric about x-axis.

Symmetric about y- axis:- The curve $f(x,y)=0$ is symmetric about y- axis if $f(x,y)=f(-x,y)$. Clearly, the parabolas $x^2=4ay$ and $x^2=-4ay$ are symmetric about y-axis.

Symmetric about line $y=x$:- The curve $f(x,y)=0$ is symmetric about line $y=x$ if $f(x,y)=f(y,x)$.

Symmetric about line $y=-x$:- The curve $f(x,y)=0$ is symmetric about line $y=-x$ if $f(x,y)=f(-y,-x)$.

Derivation

Let us find a parabola whose focus(the fixed point) is $S(x_1,y_1)$ which lies on line $y=-x$ and the directrix (the fixed line) be $ux+vy+w=0$. Let $P(x,y)$ be any point on the parabola whose equation is to be find.

Now according to the definition of parabola:-

The distance of the point *p(x,y)* from the fixed point $S(x_1,y_1)$ = Perpendicular distance of point *p(x,y)* from the fixed line

$$ux + vy + w = 0$$

$\Rightarrow SP = PM$;

where M is the foot of perpendicular drawn from P on the directrix.

$$\Rightarrow \sqrt{(x-x_1)^2+(y-y_1)^2} = \frac{|ux+vy+w|}{\sqrt{u^2+v^2}}$$

On squaring; we have

$$(x-x_1)^2+(y-y_1)^2=\frac{|ux+vy+w|^2}{u^2+v^2}$$

$$\Rightarrow (u^2+v^2)[(x-x_1)^2+(y-y_1)^2]=(ux+vy+w)^2$$

$$\Rightarrow (u^2+v^2)[x^2+x_1^2-2xx_1+y^2+y_1^2-2yy_1]=u^2x^2+v^2y^2+w^2+2uvxy+2vwy+2uwx$$

$$\Rightarrow u^2x^2+u^2x_1^2-2u^2xx_1+u^2y^2+u^2y_1^2-2u^2yy_1+v^2x^2+v^2x_1^2-2v^2xx_1+v^2y^2+v^2y_1^2-2v^2yy_1=u^2x^2+v^2y^2+w^2+2uvxy+2vwy+2uwx$$

$$\Rightarrow u^2x_1^2-2u^2xx_1+u^2y^2+u^2y_1^2-2u^2yy_1+v^2x^2+v^2x_1^2-2v^2xx_1+v^2y_1^2-2v^2yy_1=w^2+2uvxy+2vwy+2uwx$$

$$\Rightarrow u^2x_1^2-2u^2xx_1+u^2y^2+u^2y_1^2-2u^2yy_1+v^2x^2+v^2x_1^2-2v^2xx_1+v^2y_1^2-2v^2yy_1-w^2-2uvxy-2vwy-2uwx=0 \quad \text{.........(i)}$$

The equation (i) is the general equation of parabola.

Now on substituting v = - u and w = 0 in equation (i); we have

$$\Rightarrow u^2x_1^2-2u^2xx_1+u^2y^2+u^2y_1^2-2u^2yy_1+u^2x^2+u^2x_1^2-2u^2xx_1+u^2y_1^2-2u^2yy_1+2u^2xy=0$$

$$\Rightarrow x_1^2-2xx_1+y^2+y_1^2-2yy_1+x^2+x_1^2-2xx_1+y_1^2-2yy_1+2xy=0$$

$$\Rightarrow 2x_1^2-4xx_1+y^2+2y_1^2-4yy_1+x^2+2xy=0$$

$$\Rightarrow x^2+y^2+2x_1^2+2y_1^2-4xx_1-4yy_1+2xy=0 \quad \text{.......(ii)}$$

This equation (ii) represents a parabola and let it be written as $f(x,y)$

$$f(x,y)=x^2+y^2+2x_1^2+2y_1^2-4xx_1-4yy_1+2xy$$

To check symmetry about line $y=-x$.

Consider

$$f(-y,-x)=(-y)^2+(-x)^2+2x_1^2+2y_1^2-4(-y)x_1-4(-x)y_1+2(-y)(-x)$$
$$=y^2+x^2+2x_1^2+2y_1^2+4yx_1+4xy_1+2yx$$
$$=x^2+y^2+2x_1^2+2y_1^2+4yx_1+4xy_1+2xy \quad \text{..............(iii)}$$

$$\because \text{Point } (x_1,y_1) \text{ lies on } y=-x. \quad \therefore y_1=-x_1$$

Thus equation (iii) becomes

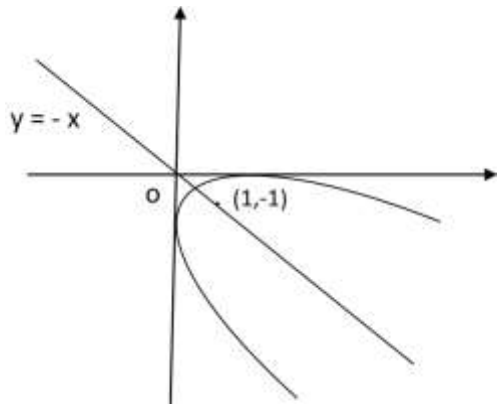
$$f(-y,-x)=x^2+y^2+2x_1^2+2y_1^2+4y(-y_1)+4x(-x_1)+2yx$$
$$=x^2+y^2+2x_1^2+2y_1^2-4xx_1-4yy_1+2xy$$
$$=f(x,y)$$

Thus equation (ii) represents a general equation of parabola which is symmetric about line *y = -x* and whose directrix is *x – y = 0*.

Now consider the focus is (1,-1) the equation (ii) becomes:-

$$x^2+y^2-4x+4y+2xy+4=0$$

which is the equation of the parabola whose directrix is *x – y = 0* and it is symmetric about line y = - x and its figure is as shown below.



And if we take focus as (-1,1) the equation (ii) becomes:-

$$x^2+y^2+4x-4y+2xy+4=0$$

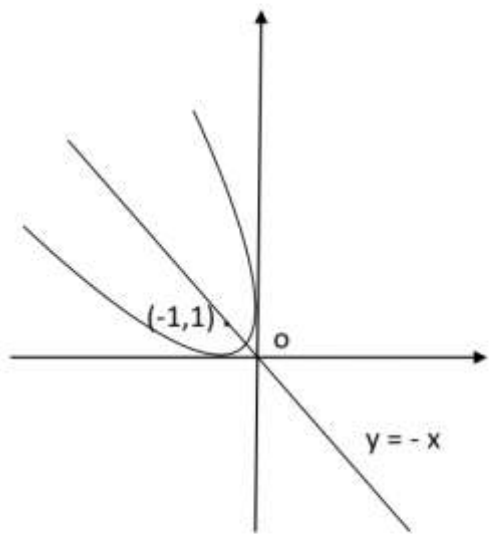
which is again equation of the parabola whose directrix is

$$x-y=0$$

and it is symmetric about line

$$y=-x$$

and its figure is as as given below:-



CONCLUSION

The parabola may have an equation which is quadratic in both x and y and it can be symmetric to line y = - x.

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