

Study of Cascade System under Reliability Theory for Different Distributions

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Abstract – In this paper we have considered diverse distribution to acquire the system reliability of a n -course system. The assessment of reliability is considered primarily for course systems. For this reason, a few distributions are considered specifically, exponential, Weibull, gamma, Rayleigh, Lindley, uniform and two-point distribution. At that point some notable distributions in particular, exponential, gamma and Weibull distribution are utilized to assess the reliability of n -course system. The numerical values $R(1)$, $R(2)$, $R(3)$ and R_3 have likewise been processed and given in unthinkable structures to some particular values of the parameters.

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I. INTRODUCTION

Reliability theory is an entrenched logical order with its own principles and methods for tackling its issues. Probability theory and numerical statistics assume a noteworthy job in the vast majority of the issues in reliability theory. Truth be told, reliability is frequently characterized as far as probability. Standby redundancy is a notable method to expand the reliability of a system. In standby redundancy, it is accepted that a component, replacing a fizzled component works precisely in a similar environment, for example it faces a similar stress. Yet, it may not be fundamentally so. A n -course system is an uncommon kind of n -standby system in which another component faces k (called a weakening variable) times the stress on the former component. The motivation behind this proposal is to gauge the reliability of different course models utilizing diverse stress-strength (S-S) distributions.

Reliability theory was perceived as a logical order during the 1950s, spent significant time in the 1960s, integrated into hazard evaluation during the 1970s, incorporated in system analysis with the broad methodological advancements and useful applications in 1990s. Since its beginning it has three principal errands: system portrayal and demonstrating, system model measurement, uncertainty displaying and evaluation. By and by because of the expanded intricacy of the systems these errands have turned out to be additionally testing.

Reliability theory is an entrenched logical control with its very own principles and methods for taking care of its issues. Probability theory and scientific statistics assume a noteworthy job in the greater part of the

issues in reliability theory. Truth be told, reliability is frequently characterized as far as probability. The interference theory which is a fundamental piece of reliability theory, has obtained a significant spot in investigation of reliability of the systems. In it, reliability of a system is considered from the association of strength of the system, state X , and the stress taking a shot at it, state Y , which is the sole reason for its failure; where X and Y both are thought to be random factors. Here, reliability, state R , of the system is characterized as $R = P(X < Y)$. Further, standby redundancy is a notable strategy to expand the reliability of a system. In standby redundancy, it is expected that a component, replacing a fizzled component works precisely in a similar environment, for example it faces a similar stress. Be that as it may, by and by it may not be fundamentally so. A n -course system is a unique kind of n -standby system in which another component faces K (called a weakening element) times the stress on the first component. The reliability of a system and its other reliability attributes of a course system can be communicated as certain functions of the parameters of the distributions of strength (X) and stress (Y) and the constriction factor (K). We gauge these parameters and substitute these values in the articulations for reliability to get their appraisals of reliability. On the off chance that the assessments of parameters utilized here are most extreme probability estimators, at that point from the invariance property of MLE's, the relating estimators of reliability are likewise MLE's. There exists broad writing for estimation of single component stress-strength system.

By models here we mean numerical models, specifically probabilistic models. The numerical

models utilized in reliability theory can be extensively isolated into three gatherings: time-dependent models, stress–strength (S–S) models or interference models and time dependent S–S models. For time dependent models stress isn't mulled over though for S–S models time isn't considered; however, it doesn't imply that the other factor is absent; it is just ignored or its impact is unimportant. In time dependent S–S models both time and stresses are taken into contemplations.

The word 'stress' and 'strength' utilized in the reliability theory in a more extensive sense, relevant much of the time well past the conventional, mechanical or basic systems. In reliability theory by 'stress' we mean any office which will in general produce failure of a component, a gadget or a material. The term organization might be a mechanical burden, environmental hazard, electric voltage and so forth. The 'strength' speaks to an organization opposing failure of the system and it is estimated by the mean stress required to cause the failure of the system. The Interference theory depends on the way that when the strength of a component or a gadget or a material is not exactly the stress forced on it, the failure happens. Here the reliability R , of a component (or system) is characterized as the probability that the strength of the component, state X (a r.v), isn't not exactly the stress, say Y (another r.v), on it. Emblematically,

$$R = P(X \geq Y)$$

The S-S models are likewise called interference models in light of the fact that here the reliability can be spoken to as far as interference zone among stress and strength densities. When the particular distributions of stress and strength are known (or assessed), one can acquire reliability of a system by utilizing condition

On the off chance that $f(x)$ and $g(y)$ are the densities of X and Y individually then from

$$R = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(x) dx \right] g(y) dy$$

$$= \int_{-\infty}^{\infty} \bar{F}(y) g(y) dy$$

$$R = \int_{-\infty}^{\infty} \left[\int_{-\infty}^x g(y) dy \right] f(x) dx$$

Or

$$= \int_{-\infty}^{\infty} G(x) f(x) dx$$

where $F(x) = 1 - \bar{F}(x)$, $F(X)$ and $G(Y)$ are the cumulative distribution functions of X and Y , separately.

The distribution of stress and strength which are regularly utilized in reliability theory are exponential, gamma, normal, Weibull, log-normal, Rayleigh and Lindley distributions. Various distributions are utilized in various situations.

II. CASCADE SYSTEM

Cascade reliability model is an uncommon kind of Stress-Strength model. Reliability might be utilized as a proportion of the system's success in giving its function appropriately. Mathematically, reliability $R(t)$ is that a system will be successful in the interval $(0, t)$. In the event that $R(t) = P(T > t)$ for $t > 0$, where T is a random variable signifying the failure time.

Cascade system were first created and concentrated by Pandit and Sriwastav. Numerous creators, for example, Kapur and Lamberson, Bhowal, Hanagal to give some examples have examined interference models in reliability without taking time into thought. They have contemplated single effect systems. A n -Cascade system is an exceptional kind of n -standby system. In a Cascade System the stresses on consequent components are constricted by a factor ' k ', called attenuation factor. Attenuation factor is commonly thought to be a steady for every one of the components or to be a parameter having diverse fixed values for various components. In any case, an attenuation factor may likewise be a random variable. The vast majority of the dialogs of interference models accept that the parameters of stress and strength distributions are constants. In any case, by and large this supposition may not be valid and the parameters might be accepted themselves (parameters) to be random variables. For instance, arrangements corrosive action might be exceptionally affected by variety in its temperature and thus the distribution of stress (corrosive action) may have distinctive parametric values which shift randomly with temperature or as such, the stress parameter might be taken as a random variable.

III. MATHEMATICAL FORMULATION

Give us a chance to consider a system with n components working under the effect of stresses. Let X_i and Z_i be the lower and upper strengths, separately, on the i th component and Y_i be the stress on it, $i = 1, 2, \dots, n$. In cascade system after each failure the stress is altered by a factor k [2] with the end goal that

$$Y_2 = kY_1, Y_3 = kY_2 = k^2Y_1, \dots, Y_i = k^{i-1}Y_1 \text{ etc.}$$

where Y_1 is the stress on the principal component. Clearly once the distribution of Y_1 is indicated the distribution of $Y_2, Y_3, \dots, Y_n$ are consequently determined.

The i th component works if the stress $k^{i-1}Y_1$ lie in the interval (X_i, Z_i) . At whatever point a stress falls outside these two limits, the component fails and another from standby replaces the failed component and the system keeps on working. The system fails just if all the n components in cascade fail. It is additionally expected that every one of the components work independently. At that point the reliability, R_n , of the system is given by

$$R_n = R(1) + R(2) + \dots + R(n),$$

where $R(r)$ is the marginal reliability due to the r th component. Here we consider $X_1, X_2, \dots, X_n$ and $Z_1, Z_2, \dots, Z_n$ are i.i.d random variables and let $f(x)$, $h(z)$ be the probability density function of X, Z and $g(y_1)$ be the pdf of Y_1 .

Now we have,

$$\begin{aligned} R(1) &= P(X < Y_1 < Z) \\ &= P(Y_1 > X) - P(Y_1 > X, Y_1 > Z) \\ &= \int_{-\infty}^{\infty} F(y_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(y_1)H(y_1)g(y_1)dy_1. \end{aligned}$$

where $F(x)$ and $H(z)$ are c.d.f's of X and Z respectively

$$\begin{aligned} R(2) &= P(X < Y_1 < Z)^c \cdot P(X < kY_1 < Z) \\ &= [1 - R(1)][P(kY_1 > X) - P(kY_1 > X, kY_1 > Z)] \\ &= [1 - R(1)] \left[\int_{-\infty}^{\infty} F(ky_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(ky_1)H(ky_1)g(y_1)dy_1 \right] \end{aligned}$$

Similarly

$$\begin{aligned} R(3) &= [1 - R(1)][1 - R(2)] \\ &\times \left[\int_{-\infty}^{\infty} F(k^2y_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(k^2y_1)H(k^2y_1)g(y_1)dy_1 \right] \end{aligned}$$

In general, we get

$$\begin{aligned} R(r) &= [1 - R(1)][1 - R(2)] \dots [1 - R(r-1)] \\ &\times \left[\int_{-\infty}^{\infty} F(k^{r-1}y_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(k^{r-1}y_1)H(k^{r-1}y_1)g(y_1)dy_1 \right] \end{aligned}$$

IV. STRESS-STRENGTH FOLLOWS SPECIFIC DISTRIBUTIONS

At the point when Stress-Strength pursue specific distributions we can assess the articulation (2.5) to get $R(r)$ and in this way acquire the system reliability. In the accompanying five sub-sections we expect distinctive specific distributions of all the Stress-Strength included and acquire articulations of system reliability.

Stress-Strength follows Exponential Distributions

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which pursues exponential distributions with methods $1/\lambda$ and $1/\gamma$ and the p.d.f of Y_1 be exponential density with parameter μ i.e.

$$\begin{aligned} f(x, \lambda) &= \lambda e^{-\lambda x}, & x > 0, \lambda > 0; \\ h(z, \gamma) &= \gamma e^{-\gamma z}, & z > 0, \gamma > 0; \\ g(y_1, \mu) &= \mu e^{-\mu y_1}, & y_1 > 0, \mu > 0 \end{aligned}$$

then from above mathematical formulation we have

$$\begin{aligned} R(1) &= \frac{\mu}{\mu + \gamma} - \frac{\mu}{\mu + \gamma + \lambda}, \\ R(2) &= [1 - R(1)] \left[\frac{\mu}{\mu + \gamma k} - \frac{\mu}{\mu + \gamma k + \lambda k} \right], \\ R(3) &= [1 - R(1)][1 - R(2)] \left[\frac{\mu}{\mu + \gamma k^2} - \frac{\mu}{\mu + \gamma k^2 + \lambda k^2} \right]. \end{aligned}$$

In general,

$$\begin{aligned} R(r) &= [1 - R(1)][1 - R(2)] \dots [1 - R(r-1)] \\ &\times \left[\frac{\mu}{\mu + \gamma k^{r-1}} - \frac{\mu}{\mu + \gamma k^{r-1} + \lambda k^{r-1}} \right]. \end{aligned}$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in (2.1) we can obtain R_n , the reliability of the system.

Stress-Strength follows Rayleigh Distributions

Give the strengths of the n components a chance to be i.i.d with p.d.f $f(x)$ and $h(z)$ which pursues rayleigh distributions with parameters σ_1 and σ_3 , and the p.d.f of Y_1 be rayleigh density with parameter σ_2 for example

$$\begin{aligned} f(x, \sigma_1) &= \frac{x}{\sigma_1^2} e^{-\frac{x^2}{2\sigma_1^2}}, & x > 0, \sigma_1 > 0; \\ h(z, \sigma_3) &= \frac{z}{\sigma_3^2} e^{-\frac{z^2}{2\sigma_3^2}}, & z > 0, \sigma_3 > 0; \end{aligned}$$

$$g(y_1, \sigma_2) = \frac{y_1}{\sigma_2^2} e^{-\frac{y_1^2}{2\sigma_2^2}}, \quad y_1 > 0, \sigma_2 > 0$$

then from above mathematical formulation we have

$$R(1) = \frac{\sigma_3^2}{\sigma_2^2 + \sigma_3^2} - \frac{\sigma_1^2 \sigma_3^2}{\sigma_1^2 \sigma_3^2 + \sigma_2^2 \sigma_3^2 + \sigma_1^2 \sigma_2^2},$$

$$R(2) = [1 - R(1)] \left[\frac{\sigma_3^2}{k^2 \sigma_2^2 + \sigma_3^2} - \frac{\sigma_1^2 \sigma_3^2}{\sigma_1^2 \sigma_3^2 + k^2 \sigma_2^2 \sigma_3^2 + k^2 \sigma_1^2 \sigma_2^2} \right],$$

$$R(3) = [1 - R(1)][1 - R(2)] \left[\frac{\gamma_4 \alpha_3^3 + \alpha_3^3}{\alpha_3^3} - \frac{\alpha_3^1 \alpha_3^3 + \gamma_4 \alpha_3^3 \alpha_3^3 + \gamma_4 \alpha_3^1 \alpha_3^3}{\alpha_3^1 \alpha_3^3} \right]$$

In general,

$$R(r) = [1 - R(1)][1 - R(2)] \dots [1 - R(r - 1)]$$

$$\times \left[\frac{\sigma_3^2}{k^{2r-2} \sigma_2^2 + \sigma_3^2} - \frac{\sigma_1^2 \sigma_3^2}{\sigma_1^2 \sigma_3^2 + k^{2r-2} \sigma_2^2 \sigma_3^2 + k^{2r-2} \sigma_1^2 \sigma_2^2} \right]$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in first equation we can get R_n , the reliability of the system.

Stress-Strength pursues Lindley Distributions

Give the strengths of the n components a chance to be i.i.d with p.d.f $f(x)$ and $h(z)$ which pursues Lindley distributions with parameters θ and γ the p.d.f of Y_1 be Lindley density with parameter μ for example

$$f(x, \theta) = \frac{\theta^2}{(1 + \theta)} (1 + x) e^{-\theta x}, \quad x > 0, \theta > 0;$$

$$h(z, \gamma) = \frac{\gamma^2}{(1 + \gamma)} (1 + z) e^{-\gamma z}, \quad z > 0, \gamma > 0;$$

$$g(y_1, \mu) = \frac{\mu^2}{(1 + \mu)} (1 + y_1) e^{-\mu y_1}, \quad y_1 > 0, \mu > 0$$

then from above mathematical formulation we have

$$\begin{aligned} R(1) = & \frac{\mu^2}{1 + \mu} \left[\frac{1}{\mu + \gamma} + \frac{1}{(\mu + \gamma)^2} + \frac{\gamma}{(1 + \gamma)(\mu + \gamma)^2} + \frac{2\gamma}{(1 + \gamma)(\mu + \gamma)^3} \right. \\ & - \frac{1}{\theta + \gamma + \mu} - \frac{1}{(\theta + \gamma + \mu)^2} - \frac{\theta}{(1 + \theta)(\theta + \gamma + \mu)^2} \\ & - \frac{\gamma}{(1 + \gamma)(\theta + \gamma + \mu)^2} - \frac{2\theta}{(1 + \theta)(\theta + \gamma + \mu)^3} \\ & - \frac{2\gamma}{(1 + \gamma)(\theta + \gamma + \mu)^3} - \frac{2\theta\gamma}{(1 + \theta)(1 + \gamma)(\theta + \gamma + \mu)^3} \\ & \left. - \frac{6\theta\gamma}{(1 + \theta)(1 + \gamma)(\theta + \gamma + \mu)^4} \right] \end{aligned}$$

$$\begin{aligned} R(2) = & [1 - R(1)] \left[\frac{\mu^2}{1 + \mu} \left\{ \frac{1}{\mu + \gamma k} + \frac{1}{(\mu + \gamma k)^2} + \frac{\gamma k}{(1 + \gamma)(\mu + \gamma k)^2} \right. \right. \\ & + \frac{2\gamma k}{(1 + \gamma)(\mu + \gamma k)^3} - \frac{1}{\theta k + \gamma k + \mu} - \frac{1}{(\theta k + \gamma k + \mu)^2} \\ & - \frac{\theta k}{(1 + \theta)(\theta k + \gamma k + \mu)^2} - \frac{\gamma k}{(1 + \gamma)(\theta k + \gamma k + \mu)^2} \\ & - \frac{2\theta k}{(1 + \theta)(\theta k + \gamma k + \mu)^3} - \frac{2\gamma k}{(1 + \gamma)(\theta k + \gamma k + \mu)^3} \\ & \left. \left. - \frac{2\theta\gamma k^2}{(1 + \theta)(1 + \gamma)(\theta k + \gamma k + \mu)^3} \right\} \right] \end{aligned}$$

$$\begin{aligned} R(3) = & [1 - R(1)][1 - R(2)] \left[\frac{\mu^2}{1 + \mu} \left\{ \frac{1}{\mu + \gamma k^2} + \frac{1}{(\mu + \gamma k^2)^2} \right. \right. \\ & + \frac{\gamma k^2}{(1 + \gamma)(\mu + \gamma k^2)^2} + \frac{2\gamma k^2}{(1 + \gamma)(\mu + \gamma k^2)^3} - \frac{1}{\theta k^2 + \gamma k^2 + \mu} \\ & - \frac{1}{(\theta k^2 + \gamma k^2 + \mu)^2} - \frac{\theta k^2}{(1 + \theta)(\theta k^2 + \gamma k^2 + \mu)^2} \\ & - \frac{\gamma k^2}{(1 + \gamma)(\theta k^2 + \gamma k^2 + \mu)^2} - \frac{2\theta k^2}{(1 + \theta)(\theta k^2 + \gamma k^2 + \mu)^3} \\ & - \frac{2\gamma k^2}{(1 + \gamma)(\theta k^2 + \gamma k^2 + \mu)^3} - \frac{2\theta\gamma k^4}{(1 + \theta)(1 + \gamma)(\theta k^2 + \gamma k^2 + \mu)^3} \\ & \left. \left. - \frac{6\theta\gamma k^4}{(1 + \theta)(1 + \gamma)(\theta k^2 + \gamma k^2 + \mu)^4} \right\} \right] \end{aligned}$$

In general,

$$\begin{aligned} R(r) = & [1 - R(1)][1 - R(2)] \dots [1 - R(r - 1)] \left[\frac{\mu^2}{1 + \mu} \left\{ \frac{1}{\mu + \gamma k^{r-1}} \right. \right. \\ & + \frac{1}{(\mu + \gamma k^{r-1})^2} + \frac{\gamma k^{r-1}}{(1 + \gamma)(\mu + \gamma k^{r-1})^2} + \frac{2\gamma k^{r-1}}{(1 + \gamma)(\mu + \gamma k^{r-1})^3} \\ & - \frac{1}{\theta k^{r-1} + \gamma k^{r-1} + \mu} - \frac{1}{(\theta k^{r-1} + \gamma k^{r-1} + \mu)^2} \\ & - \frac{\theta k^{r-1}}{(1 + \theta)(\theta k^{r-1} + \gamma k^{r-1} + \mu)^2} - \frac{\gamma k^{r-1}}{(1 + \gamma)(\theta k^{r-1} + \gamma k^{r-1} + \mu)^2} \\ & - \frac{2\theta k^{r-1}}{(1 + \theta)(\theta k^{r-1} + \gamma k^{r-1} + \mu)^3} - \frac{2\gamma k^{r-1}}{(1 + \gamma)(\theta k^{r-1} + \gamma k^{r-1} + \mu)^3} \\ & \left. \left. - \frac{2\theta\gamma k^{2(r-1)}}{(1 + \theta)(1 + \gamma)(\theta k^{r-1} + \gamma k^{r-1} + \mu)^3} \right\} \right] \end{aligned}$$

$$-\frac{6\theta\gamma k^{2(r-1)}}{(1+\theta)(1+\gamma)(\theta k^{r-1} + \gamma k^{r-1} + \mu)^4} \Bigg\} \Bigg]$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in first equation we can obtain R_n , the reliability of the system.

Both Strength follows One Parameter Exponential and Stress follows Lindley Distributions

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows one parameter exponential with means $1/\lambda$ and $1/\theta$ and the p.d.f of Y_1 be Lindley density with parameter μ i.e.

$$f(x, \lambda) = \lambda e^{-\lambda x}, \quad x > 0, \lambda > 0$$

$$h(z, \theta) = \theta e^{-\theta z}, \quad z > 0, \theta > 0;$$

$$g(y_1, \mu) = \frac{\mu^2}{(1+\mu)}(1+y_1)e^{-\mu y_1}, \quad y_1 > 0, \mu > 0$$

Then from above we have

$$R(1) = \frac{\mu^2}{1+\mu} \left[\frac{1}{\theta+\mu} + \frac{1}{(\theta+\mu)^2} - \frac{1}{\theta+\mu+\lambda} - \frac{1}{(\theta+\mu+\lambda)^2} \right]$$

$$R(2) = [1 - R(1)] \left[\frac{\mu^2}{1+\mu} \left\{ \frac{1}{\theta k + \mu} + \frac{1}{(\theta k + \mu)^2} - \frac{1}{\theta k + \lambda k + \mu} - \frac{1}{(\theta k + \lambda k + \mu)^2} \right\} \right],$$

$$R(3) = [1 - R(1)][1 - R(2)] \left[\frac{\mu^2}{1+\mu} \left\{ \frac{1}{\theta k^2 + \mu} + \frac{1}{(\theta k^2 + \mu)^2} - \frac{1}{\theta k^2 + \lambda k^2 + \mu} - \frac{1}{(\theta k^2 + \lambda k^2 + \mu)^2} \right\} \right].$$

In general,

$$R(r) = [1 - R(1)][1 - R(2)] \dots [1 - R(r-1)] \left[\frac{\mu^2}{1+\mu} \left\{ \frac{1}{\theta k^{r-1} + \mu} + \frac{1}{(\theta k^{r-1} + \mu)^2} - \frac{1}{\theta k^{r-1} + \lambda k^{r-1} + \mu} - \frac{1}{(\theta k^{r-1} + \lambda k^{r-1} + \mu)^2} \right\} \right].$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in (2.1) we can obtain R_n , the reliability of the system.

Both Strength follows One Parameter Exponential and Stress follows Two Parameter Gamma

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows one parameter

exponential with means $\frac{1}{\lambda}$ and $\frac{1}{\theta}$ and $g(y_1)$ be two parameters gamma densities with degrees of freedom γ and μ respectively and unit scale parameters i.e.

$$f(x, \lambda) = \lambda e^{-\lambda x}, \quad x > 0, \lambda > 0;$$

$$h(z, \theta) = \theta e^{-\theta z}, \quad z > 0, \theta > 0$$

$$g(y_1, \gamma, \mu) = \frac{1}{\gamma^\mu \Gamma(\mu)} y_1^{\mu-1} e^{-\frac{y_1}{\gamma}}; \quad y_1 > 0, \gamma, \mu > 0$$

Then from above we have

$$R(1) = \frac{1}{(1+\theta\gamma)^\mu} - \frac{1}{(1+\theta\gamma+\lambda\gamma)^\mu},$$

$$R(2) = [1 - R(1)] \left[\frac{1}{(1+\theta\gamma k)^\mu} - \frac{1}{(1+\theta\gamma k + \lambda\gamma k)^\mu} \right],$$

$$R(3) = [1 - R(1)][1 - R(2)] \left[\frac{1}{(1+\theta\gamma k^2)^\mu} - \frac{1}{(1+\theta\gamma k^2 + \lambda\gamma k^2)^\mu} \right].$$

In general,

$$R(r) = [1 - R(1)][1 - R(2)] \dots [1 - R(r-1)]$$

$$\times \left[\frac{1}{(1+\theta\gamma k^{r-1})^\mu} - \frac{1}{(1+\theta\gamma k^{r-1} + \lambda\gamma k^{r-1})^\mu} \right].$$

IV. NUMERICAL EVALUATION

For some particular values of the parameters engaged with the expressions of $R(r)$, $r = 1, 2, 3$ we assess the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 for the over five cases from their expressions got in the last section.

Table 1. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength are exponential variates

μ	γ	λ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	0.3	0.3	0.1	0.1442	0.0235	0.0025	0.1702
1	0.5	0.5	0.1	0.1667	0.0361	0.0040	0.2067
1	0.7	0.7	0.1	0.1716	0.0475	0.0054	0.2245
2	0.3	0.3	0.2	0.1003	0.0247	0.0052	0.1302
2	0.5	0.5	0.2	0.1333	0.0375	0.0081	0.1789
2	0.7	0.7	0.2	0.1525	0.0486	0.0108	0.2120
3	0.3	0.3	0.3	0.0758	0.0254	0.0079	0.1090
3	0.5	0.5	0.3	0.1071	0.0387	0.0123	0.1581
3	0.7	0.7	0.3	0.1290	0.0500	0.0163	0.1953

From the Table 1, we see that in the event that the strength parameter λ and γ builds, at that point the system reliability R_3 increment. At the point when the stress parameter μ builds $R(1)$ diminishes however $R(2)$ and $R(3)$ increments. For example, if $\mu = 1$, $R(1) = 0.1442$ and if $\mu = 2$, $R(1) = 0.1003$. As

a rule we see that when γ , λ builds then $R(2)$ and $R(3)$ will likewise increments. at the point when the attenuation factor k expands then the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 diminishes.

Table 2. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength are Rayleigh variates

σ_1	σ_2	σ_3	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	3	3	0.1	0.4091	0.0479	0.0005	0.4575
1	5	5	0.1	0.4630	0.1055	0.0012	0.5697
1	7	7	0.1	0.4804	0.1681	0.0021	0.6506
2	3	3	0.2	0.2647	0.0563	0.0025	0.3235
2	5	5	0.2	0.3788	0.1158	0.0054	0.5000
2	7	7	0.2	0.4258	0.1756	0.0090	0.6144
3	3	3	0.3	0.1667	0.0583	0.0062	0.2312
3	5	5	0.3	0.2907	0.1241	0.0135	0.4256
3	7	7	0.3	0.3657	0.1805	0.0216	0.5678

From the Table 2, plainly with some set of values of the parameters on the off chance that σ_1 expands, at that point the system reliability decline. i.e., if $\sigma_1 = 1$, $R_3 = 0.4575$ and if $\sigma_1 = 2$, $R_3 = 0.3235$. Be that as it may, on the off chance that the stress parameter σ_2 and strength parameter σ_3 builds, at that point $R(1)$, $R(2)$ and $R(3)$ likewise increments. Here likewise observe that when the attenuation factor k expands then the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 diminishes.

Table 3. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength are Lindley variates

μ	θ	γ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	3	4	2	0.0553	0.0279	0.0139	0.0972
1	5	6	2	0.0404	0.0200	0.0099	0.0703
1	7	8	2	0.0311	0.0154	0.0076	0.0541
2	3	4	3	0.1000	0.0415	0.0148	0.1564
2	5	6	3	0.0829	0.0312	0.0108	0.1248
2	7	8	3	0.0683	0.0246	0.0084	0.1013
3	3	4	4	0.1217	0.0481	0.0135	0.1833
3	5	6	4	0.1111	0.0366	0.0098	0.1575
3	7	8	4	0.0964	0.0293	0.0077	0.1334

From the Table 3, unmistakably when the strength parameters θ and γ expands then the system reliability R_3 and marginal reliabilities $R(1)$, $R(2)$, $R(3)$ diminishes. In any case, if the stress parameter μ and attenuation factor k builds reliabilities likewise increments

Table 4. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength follows one parameter exponential and stress follows Lindley distributions

μ	θ	γ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	2	3	2	0.1250	0.0616	0.0302	0.2168
1	3	4	2	0.0859	0.0421	0.0206	0.1487
1	4	5	2	0.0650	0.0318	0.0156	0.1124
2	2	3	4	0.1990	0.0667	0.0179	0.2836
2	3	4	4	0.1554	0.0474	0.0124	0.2152
2	4	5	4	0.1270	0.0368	0.0095	0.1734
3	2	3	6	0.2236	0.0697	0.0130	0.3063
3	3	4	6	0.1900	0.0495	0.0089	0.2484
3	4	5	6	0.1642	0.0387	0.0068	0.2097

From the Table 4, unmistakably with some set of values of the parameters on the off chance that θ and λ builds, at that point the system reliability diminishes and $R(1)$, $R(2)$ and $R(3)$ likewise diminishes. Here likewise observe that when the attenuation factor k and stress parameter μ builds then the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 increments.

Table 5. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength follows one parameter exponential and stress follows Lindley distributions

μ	γ	λ	θ	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.1	0.2	0.3	0.4	2	0.0054	0.0096	0.0159	0.0308
0.2	0.3	0.4	0.5	2	0.0191	0.0310	0.0042	0.0944
0.3	0.4	0.5	0.6	2	0.0411	0.0590	0.0720	0.1721
0.1	0.2	0.3	0.4	3	0.0054	0.0131	0.0250	0.0435
0.2	0.3	0.4	0.5	3	0.0191	0.0395	0.0578	0.1165
0.3	0.4	0.5	0.6	3	0.0411	0.0700	0.0799	0.1910
0.1	0.2	0.3	0.4	4	0.0054	0.0160	0.0311	0.0525
0.2	0.3	0.4	0.5	4	0.0191	0.0456	0.0628	0.1275
0.3	0.4	0.5	0.6	4	0.0411	0.0765	0.0799	0.1955

From the organized value of Table 5, we see that when the strength parameters λ and θ and stress parameters μ and γ increments marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 increments with consistent value of k . At the point when the attenuation factor k increments there are noteworthy increment in the values of $R(2)$, $R(3)$ and R_3 yet no huge distinction in the values of $R(1)$.

V. GRAPHICAL REPRESENTATIONS

A few graphs are plotted in Fig. 1(a), Fig. 1(b), Fig. 2(a), Fig. 2(b), Fig. 3(a), Fig.3(b), Fig. 4(a), Fig. 4(b), Fig. 5(a) and Fig. 5(b) taking various parameters along the flat pivot and the relating reliabilities along the vertical hub for various parametric values. Fig. 1(a) - Fig. 1(b) connotes

that reliabilities increment consistently with increasing γ . These graphs might be utilized to peruse the middle of the road values straightforwardly. In Fig. 2(a) - Fig. 2(b) it is seen that graphs of $R(1)$, $R(2)$, $R(3)$ and R_3 against σ_2 for fix values of σ_1 and k are plotted for various values of σ_3 . From these graphs it is seen that if the stress parameter σ_2 and strength parameter σ_3 increment, $R(1)$, $R(2)$, $R(3)$ and R_3 additionally increments. Thus it is likewise observed that in Fig. 3(a) - Fig. 3(b) and Fig. 4(a) - Fig. 4(b), reliabilities are diminishes with expanding values of their parameters. In any case, in Fig. 5(a) - Fig. 5(b) taking the attenuation factor k along the level hub and the relating reliability along the vertical pivot for various values of $\mu, \gamma, \lambda, \theta$, it is to be seen that reliability is expanding with k .

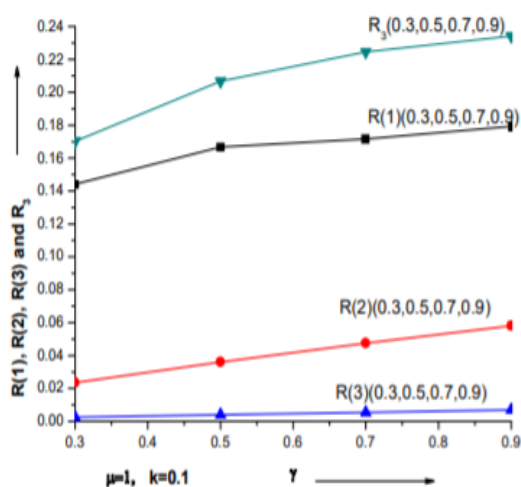


Fig. 1(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of λ for Stress-Strength follows Exponential Distributions

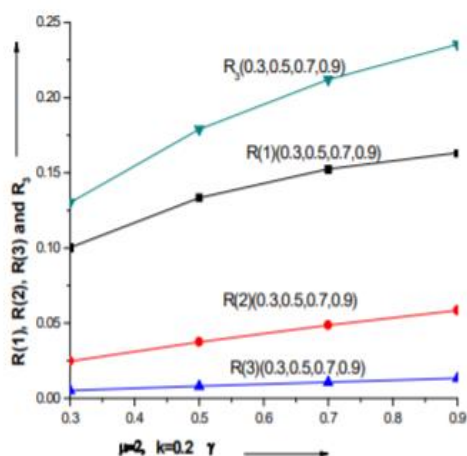


Fig. 1b Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of λ for Stress-Strength follows Exponential Distributions

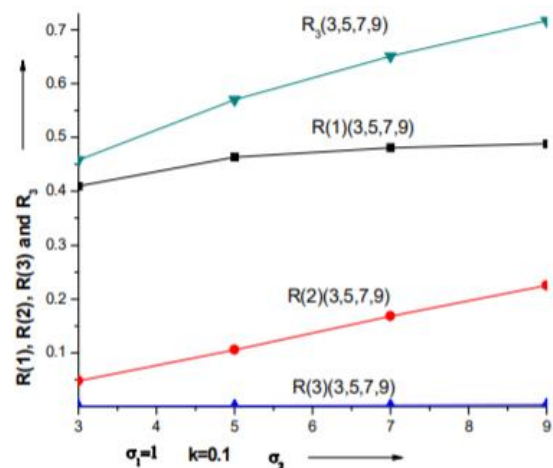


Fig. 2(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of σ_3 for Stress-Strength follows Rayleigh Distributions

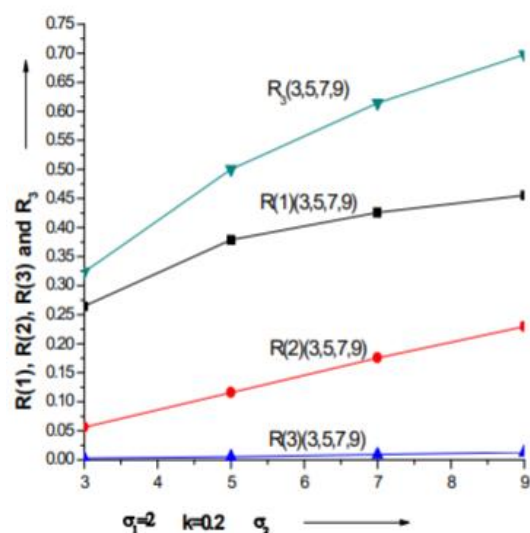


Fig. 2(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of σ_3 for Stress-Strength follows Rayleigh Distributions

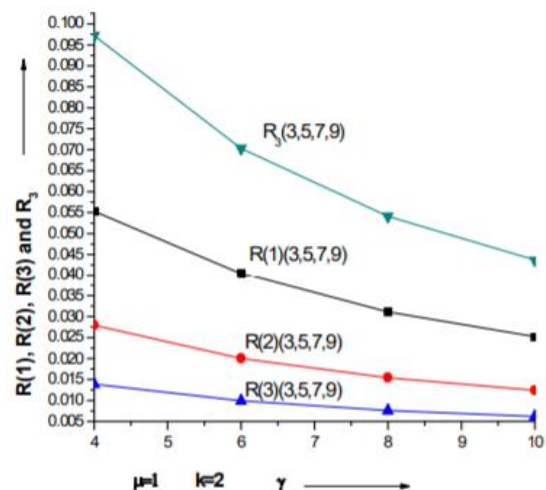


Fig. 3(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for Stress-Strength follows Lindley Distributions

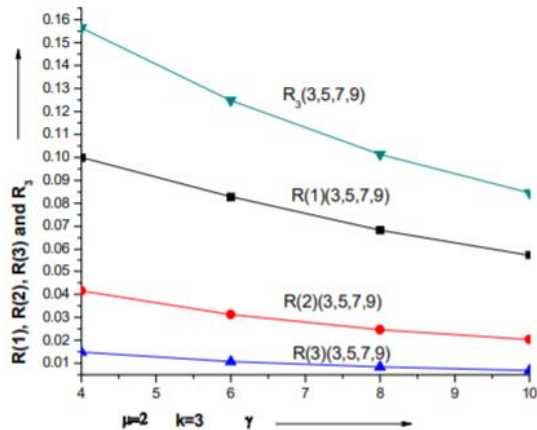


Fig. 3(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for Stress-Strength follows Lindley Distributions

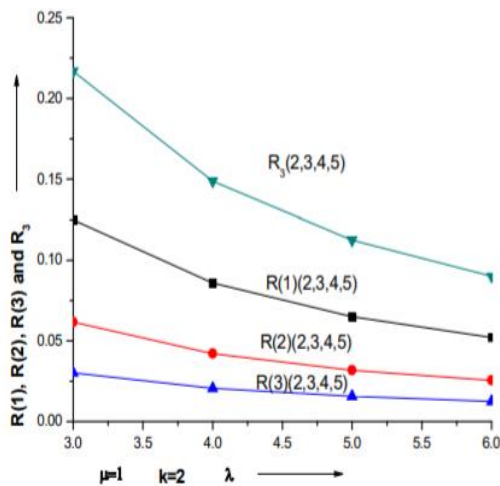


Fig. 4(a) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for Both Strength follows One-Parameter Exponential and Stress follows Lindley Distributions

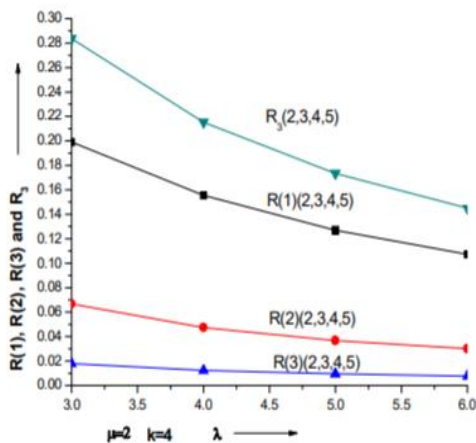


Fig. 4(b) Graph of $R(1)$, $R(2)$, $R(3)$ and R_3 for different fixed values of θ for Both Strength follows One-Parameter Exponential and Stress follows Lindley Distributions

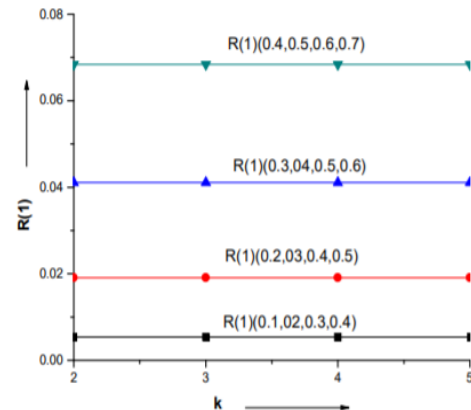


Fig. 5(a) Graph of $R(1)$ for different fixed values of $\mu, \gamma, \lambda, \theta$ for Both Strength follows One-Parameter Exponential and Stress follows Two-Parameter Gamma Distributions

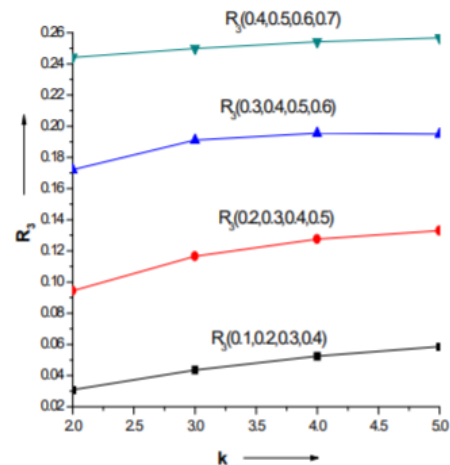


Fig. 5(b) Graph of $R(1)$ for different fixed values of $\mu, \gamma, \lambda, \theta$ for Both Strength follows One-Parameter Exponential and Stress follows Two-Parameter Gamma Distributions

VII. CONCLUSION

In our investigation, diverse cascade models are considered to assess the system reliability. For this estimation, a few distributions viz. exponential, gamma, Weibull, Rayleigh, Lindley, two-point and uniform distribution are considered.

In stress-strength model the reliability of a component is characterized as the probability that its strength isn't not exactly the stress taking a shot at it. Be that as it may, sometimes a component can work just when the stress Y on it isn't just not exactly certain values, state Z , however should be more prominent than some other value, say X , for

example stress is inside sure limits, where X and Z are recognized as lower and upper strengths. This supposition has been followed in our investigation, where reliability of n - cascade system under such procedure has been gotten utilizing distinctive stress-strength distribution. It has been seen from the numerical values of the reliability that when the attenuation factor increments, marginal reliabilities and system reliability diminishes. The outcomes may fluctuate from distribution to distribution.

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