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Flow between Parallel Plates for Non-Newtonian Fluid

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Abstract – The present paper provides the non-Newtonian fluid flow between two fixed parallel horizontal plates. A mathematical model is developed to describe the fluid motion. The fluid is assumed to depend exponentially on viscosity. The governing equations are non-dimensionalized and the steady state equations are solved numerically using shooting method technique. The effect of the nature of the non-constant viscosity of a function of the space variable y shows much influence on the velocity fields.

Key Words: Fluid Flow, Blood Flow, Viscous Flow, Steady State, Momentum Equation.

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1. INTRODUCTION

Various researchers have focused their mind towards non-Newtonian fluid with various perspectives. Ronald [1] explained about the field of rheology concerns the deformation and flow behaviour of fluids. The prefix ‘rheo’ is for Greek word and refers to something that flows because of the particulate nature of blood. He expected the rheological behaviour of the blood to be somewhat more complex than a simple fluid such as water. He mentions that in order to understand the flow behaviour of blood, one must first define the relationship between shear stress and shear rate.

Ishikawa et al. [2] found that the non-Newtonian pulsatile flow through a stenosed tube is different from Newtonian flow. The non-Newtonian properly strengthens the peaks of wall shear stress and wall pressure, weakens the strength of the vortex and reduces the vortex size and separated region. Therefore, they concluded that non-Newtonian flow is more stable than Newtonian flow.

Hazem [3] studied the two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid with heat generation in the presence of uniform suction of blowing. He concluded that the effect of the suction velocity on the shear at wall depends on the value of the non-Newtonian parameter.

Makinde [4] investigates the effects of hematocrit variation on the flow stability. He computes the hemodynamics analysis in large blood vessels and concluded that an increase in hematocrit towards the central core region of the artery has a stabilizing effect on the flow.

In this pepper we investigate the velocity profile of non-Newtonian viscous flow between two parallel plates using shooting method technique.

2. MATHEMATICAL MODEL

We consider an incompressible viscous flow between two parallel plates.

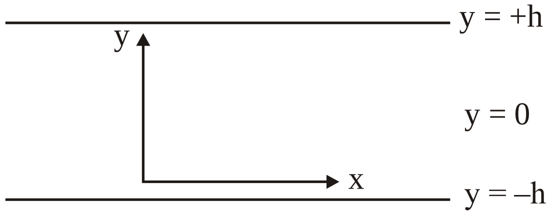

Fig. 1

Governing equation :

$$\rho\left(\frac{\partial u}{\partial t}\right) = \frac{-\partial p}{\partial x} + \frac{\partial}{\partial y}\left\{\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial y}\right\} \qquad \text{.....(2.1)}$$

With the boundary condition

$$u(h) = u(-h) = 0 \qquad \text{.....(2.2)}$$

where

ρ – density

p – pressure

μ – viscosity

h – height of the channel

t – time

u – velocity components on x

v_0 – velocity components on y

3. NON-DIMENSIONALIZATION

Let $\phi = \frac{u}{v_0}, \quad \bar{y} = \frac{y}{h}$

$$\bar{t} = \frac{t}{t_0}, \quad \bar{\mu} = \frac{\mu}{\mu_0} \tag{3.1}$$

Then, from equation (2.1), (2.2) & (3.1), we have

$$\frac{\partial \phi}{\partial t} = T + a\frac{\partial}{\partial y}\left\{\left(\mu_0 \bar{\mu}\frac{\partial \phi}{\partial y}\right) + \frac{\partial \phi}{\partial y}\right\} \tag{3.2}$$

Where

$$T = -\frac{t_0}{\rho v_0}\frac{\partial p}{\partial x}, \quad a = \frac{t_0}{\rho h^2}$$

and taking $\mu_0 = 1$

4. STEADY CASE

We assume that the fluid properties and the variables of this flow are independent of time i.e. $\frac{d}{dt} = 0$

Therefore,

$$T + a\frac{d}{dy}\left\{\left(\bar{\mu}\frac{\partial \phi}{\partial y}\right) + \frac{\partial \phi}{\partial y}\right\} = 0 \tag{4.1}$$

CASE I

Let the viscosity $\mu = \mu_0 e^{\frac{\beta}{1+y^2}}$

Where □ is the viscosity variation parameter.

Then, equation (4.1) becomes

$$\phi'' = 2y\frac{\beta(1+y^2)^{-2}e^{\beta(1+y^2)^{-1}}\phi' - \frac{T}{a}}{e^{\beta(1+y^2)^{-1}} + 1} \tag{4.2}$$

and

$$\phi(-1) = \phi(1) = 0 \tag{4.3}$$

CASE II

Let the viscosity $\mu = \mu_0 \operatorname{sech} y, (\beta = 1)$ then equations (4.1) becomes.

$$\phi'' = \frac{(\operatorname{sech} y \tanh y)\phi' - \frac{T}{a}}{1 + \operatorname{sech} y} \tag{4.4}$$

and

$$\phi(-1) = \phi(1) = 0 \tag{4.5}$$

5. EXISTENCE AND UNIQUENESS OF SOLUTION

Definition : Let f(x, y) be a function of two variables defined over a set $D \subset R^2$. It is said that the function f(x, y) satisfies a lipschitz condition in variable y if the constant L > 0 exists such that the following property holds.

$$| f(x, y_1) - f(x, y_2 | \le L | y_1 - y_2 |$$

Whenever $(x, y_1), (x, y_2) \in D$ The constant L is called the Lipschitz constant for f.

Theorem : [W.R.Derrik, (1976]

The equations (4.2) and (4.4) above which satisfy the initial conditions (4.3) and (4.5) has a unique solution $\phi'(-1)$.

Proof :

The solution of the equation (4.2) and (4.4) are :

$$\phi'' = \frac{2y\beta(1+y^2)^{-2}e^{\beta(1+y^2)^{-1}}\phi' - \frac{T}{a}}{e^{\beta(1+y^2)^{-1}} + 1}$$

and

$$\phi'' = \frac{(\operatorname{sech} y \tanh y)\phi' - \frac{T}{a}}{1 + \operatorname{sech} y}$$

$$\phi(-1)=\phi(1)=0$$

Let $\phi_1 = y$

$$\phi_2 = \phi$$

$$\phi_3 = \phi'$$

then the equation (4.2) becomes.

$$\begin{bmatrix}\phi_1'\\ \phi_2'\\ \phi_3'\end{bmatrix} = \begin{bmatrix} 1 \\ \phi_3 \\ \dfrac{2\phi_1\beta(1+\phi_1^2)^{-2}e^{\beta(1+\phi_1{}^2)^{-1}}\phi_3 - T/a}{e^{\beta(1+\phi_1^2)^{-}}+1}\end{bmatrix}$$

also, equation (4.4) becomes

$$\begin{bmatrix}\phi_1'\\ \phi_2'\\ \phi_3'\end{bmatrix} = \begin{bmatrix} 1 \\ \phi_3 \\ \dfrac{(\operatorname{sech}\phi_1\tanh\phi_1)\phi_3 - T/a}{1+\operatorname{sech}\phi_1}\end{bmatrix}$$

subject to initial condition

$$\begin{bmatrix}\phi_1(-1)\\ \phi_2(-1)\\ \phi_3(-1)\end{bmatrix} = \begin{bmatrix}-1\\ 0\\ k\end{bmatrix}$$

Where k are guessed such that $\phi_2(-1)=0$

therefore,

$$\begin{bmatrix}\phi_1'\\ \phi_2'\\ \phi_3'\end{bmatrix} = \begin{bmatrix}f_1(\phi_1,\phi_2,\phi_3)\\ f_2(\phi_1,\phi_2,\phi_3)\\ f_3(\phi_1,\phi_2,\phi_3)\end{bmatrix}$$

The $\operatorname{Max}\dfrac{\partial f_i}{\partial \phi_j} \le L,$

For i = 1, 2, 3 & j = 1, 2, 3 are Lipschitz conditions and bounded. Hence by existence theorem the solution is unique.

6. RESULTS

We solve equation (4.2) and (4.4) numerically, using shooting method and the result is shown below graphically.

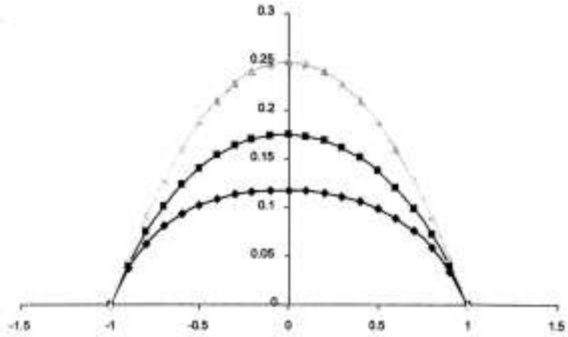

Fig. 2 : Velocity Profile,
$T/a = 1, \ldots\ldots, \beta = 0,\ \beta = 1,\ \beta = 2$

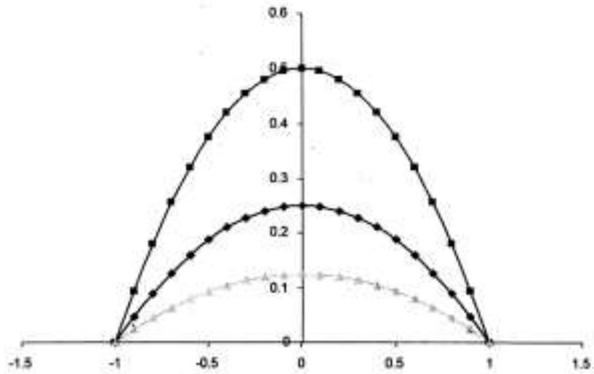

Fig. 3 : Velocity Profile,
$\beta = I,\ \dfrac{T}{a} = 0.5, \dfrac{T}{a} = 1, \dfrac{T}{a} = 2$

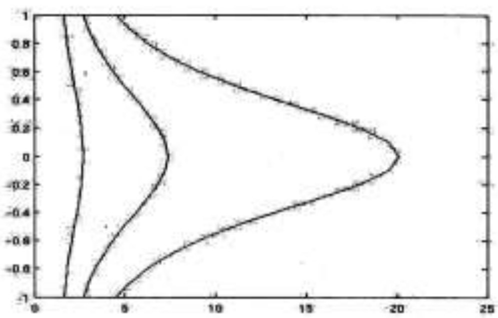

Fig. 4: Viscosity variation : □ = 1.0, 2.0, 3.0, in the case I above.

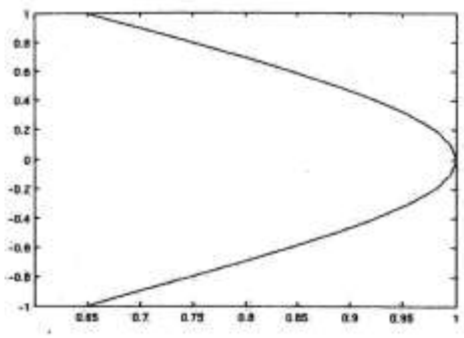

Fig. 5 : Viscosity variation : □ = 1, in the case II above.

7. CONCLUSION

The one-dimensional flow of an incompressible non-Newtonian fluid between two parallel plates is presented. We proved the existence and the uniqueness of the steady state solution. The numerical results show that the velocity profile increases when either the viscosity or pressure component increases. Fig. 2 shows the viscosity behaviour, as the viscosity increases the velocity increases. At a constant viscosity the velocity profile also increases as any other fluid components (pressure and density) increases as shown in fig. 3. At low viscosity the fluid flow form a pulsatile, which is the nature of blood flow in an artery. As a result of that, this model can be use to describe the blood viscosity flow mechanics.

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