

Dominance Contraction Survey on Theory and Applications of Graphs

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Abstract – In this paper we consider the effect of edge contraction on the Dominance Contraction Survey on Theory and Applications of Graphs. In order to reduce the (total) dominance number of a graph, we determine a minimum number of borders that must be contracted. We show that the two numbers for each diagram are at most three. In light of this outcome, graphs are graded and defined by their (total) number of dominance contractions. The next article includes the notion of dominating planar graphs, linked graphs, edge dominance of paths, associated graph cycles and few characteristics. Likewise, we have extended our research on inverse graphic and domain-based applications.

Key Words – Domination, Total Domination, Domination Contraction Number.

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INTRODUCTION

The edge addition and removal in the dominance region have gained a lot of attention. There have been detailed investigations on changes in the conditions of dominance as a result of edge addition and deletion. In addition to addition and elimination, subdivision and contraction are the most critical operations on the graph's edges. It seems, however, that the area of dominance has been subject to subdivision and contraction only recently. Some research the effect of edge subdivision on the number of domination and absolute domination, two of the two most important parameters tested. The domination subdivision number $sd_\mu(G)$ in a diagram is the minimum number of edges to subdivide in order to maximise the domination number (where the edge can be subdivided at the most at once). The first figure was established by Arumugam and Paulraj Joseph [1], which shows that $sd_\alpha(T) < 3$ is at least three vertices for every tree T - and that this top bound is at least three vertices in each graph. However, Haynes and others[5] in 2001 gave the above conjecture a counter-example by showing that $sd_\alpha(G) = 4$ was used for cartesian product $G = K_t - T > 4$. The graph G with $sd_\mu = 5$ was developed later by Swaminathan and Sumathi[9]. [2, 3, 4] includes general limits for subdivision dominance numbers. The total subdivision number sd_{dt} is a parallel definition (G). This number was defined in 2003 by Haynes et al. [\leq] with regard to vertex. They also showed that $sd < 3$ is sufficient for $sd_\mu(G)$, and that $sd < 3$ is sufficient to indicate any T tree. The T -tree with $sd_\beta(T) = 3$ was constructively characterized[8]. [8]. This number can therefore be arbitrarily high in general graphs (see [7]).

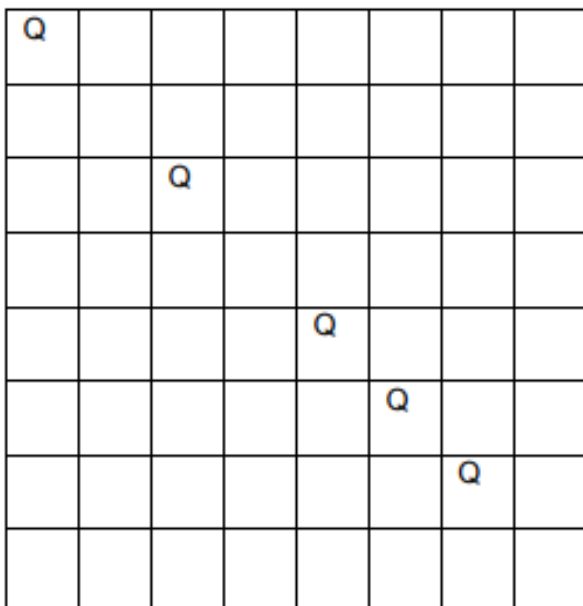
Motivated by recent research into subdivision, we take the edge contraction into account and add similar conceptions, namely the numbers for supremacy and overall dominance. We define the (total) domination number ($ect_\beta(G)$) $ct_\alpha(G)$, as the minimum number of edges to be contract to lower the (total) domination number of graph G with (total) domination number at least (three) two. If the (total) dominance number is (two), then for simplicity, we define ($ct_\beta(G)$) $ct_\mu(G) = 0$. This will explain the importance of (total) dominance contraction number. One aim is to find a minimum (total) dominant set for useful installation in the corresponding graph G . In a facility location problem. If the cost of these facilities is to be cut, the total number of dominant installations must be decreased. In order to do so we must make some adjustments to the edges of G , i.e. add or contract certain edges. Formerly, the number of borders is increased and additional costs are therefore not necessary, whereas the latter is not (in fact, it decreases the number of edges, by Proposition 2.1). The contraction of the edge therefore may be better than the addition of the edging. Then at least one needs to know how many edges to minimise the (total) dominance number need to be contracted. This question is answered by demonstrating that each of graphs G holds $ct_\alpha < 3$ and $ct_\alpha(G) < 3$. An significant comparison with $sd_\alpha(G)$ and sd_μ is made with this result (G). Moreover, we give graph classifications and characterise them according to the two top limits of $ct_\beta(G)$ and $ct_\alpha(G)$.

We want to provide some terminology and notation prior to entering the next section. The contraction numbers of a chart are defined as equal to the

minimum value of the contraction numbers of all components of the chart. Thus only graph $G = (V, E)$ connected to vertex set $V = V(G)$ is taken into account and edges set $E = E$ is considered (G) . To the vertex $v \in V(G)$, the open and closed v districts v must be $N(v)$ and $N[v] = N(v) \cup \{v\}$, respectively. The distance between two vertices x and y is showed with $d(x, y)$. Denote the graph from G by contracting the edge e by G/e . Since edge contraction is a swichage process, i.e. $(G/e)/f = (G/f)/e$, on any of two edges e and f , we may use G/E' to mark a resulting diagram of all edges in E' to build $E(G)$ from G one by one. The reader is referred to as [10] for the terminology and graphic notation which is not given here. In G we say u dominates v if $v \in [u]$, given two vertices u and v . G . G . A subset $D \subseteq V(G)$ is referred to as the dominant set if its vertices dominate any vertex of G . G , denoted by $\beta(G)$, is the minimum cardinality of all dominant sets. A dominant set of $\alpha(G)$ vertices is called shortly a β -set. A complete dominant set T is a dominant set with no isolated vertex in the induced subgraph, which is that each vertex of G has a neighbour in T . The α and α -set complete dominance are similarly defined to $\beta(G)$ and β -set. It is easy to see that β for any unproductive graph is well established.

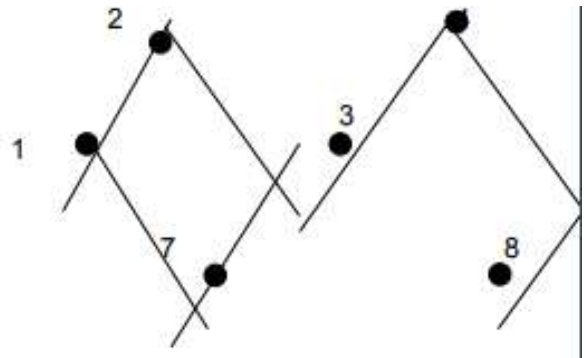
DOMINATIONS IN GRAPHS

The chess freaks in Europe in 1850 thought of finding the smallest number of queens on a chess board with the intention that either a queen charges each of the blocks or a queen inhibits them. This dilemma gave birth to the theory of domination. Five queens were found to be sufficient to tower over the entire chess board (8×8). Here we are showing two of such arrangements of five queens such that all squares are dominated is shown in the following Fig

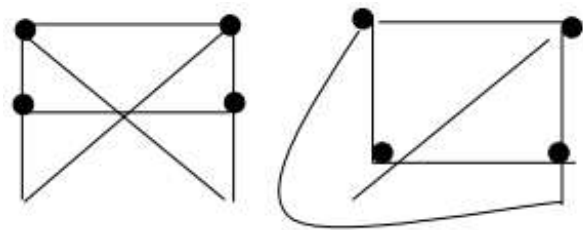


Two arrangements of queens in 8×8 checker board in order to occupy every block by Queens.

Definition 1: In a Graph G of dominating set S where each node of G is either in S or adjoining to any node in S . The domination number $\gamma = \gamma(G)$ is the least cardinality of an arrangement of domination. The Dominating set problem concerns finding a base dominating set.



Planar graph



Non – Planar Graph

For the graph G in Fig. 3, $\{1,3,5\}$ is dominating set of cardinalities 3, $\{3,7,8\}$ is a dominating set of cardinalities four and $\{2,4,7,8\}$ is a dominating set of cardinalities five. Hence the least cardinality is $\gamma(G) = 3$.

Definition 2: Complete dominant known as an arrangement S of nodes in the graph $G(V, E)$, given that each node V is attached to a component S and hence (G) is an overall dominance number. We envisage a PC arrangement for an application in which a group of servers can legally transmit from outside the core group for each PC. Furthermore, in any case, a separate "backup" file server where copy is stored is directly linked to each record server. A small community with this function is a μ collection made up of the framework.

Definition 3: In graph of an edge dominating set $G = (V, E)$ is subset $S \subseteq E$ with the end goal that each arc not in S is adjoining to any one arc in S . An edge dominating set is otherwise known as line dominating set.

Definition 4: A connected dominating set of a graph G is a dominating set D whose induced sub graph is also associated.

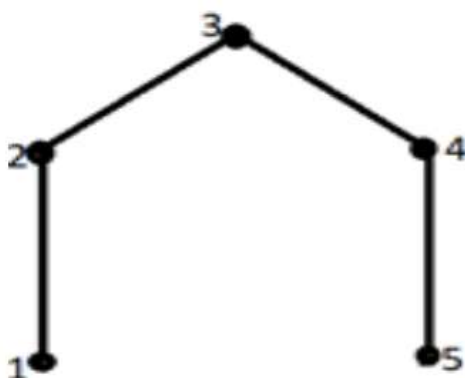
Definition 5: A dominating set S of a graph G is a minimal dominating set of G iff every node v in S fulfills in any of the accompanying two properties:

- (i). node w in $V(G) - S \cap (W) S = \{v\}$.
- (ii). v is adjoining to no node of S .

INVERSE DOMINATIONS IN GRAPHS

The graphics of the domain is based on the passion of a few Chess players in 1850. In 1850. Data bank is the most commonly discussed from the various uses of dominance theory. The communications system consists of site-specific links. The problem is the limited destination arrangements where the senders are set to so that each other site is connected to the plot containing a sender with an immediate mail connection. In other words, the challenge is to find the least dominant figure in this system's graph. The problem for deciding the two disjoint transmission stations of the other set was considered by Kulli and Sigarkanti. This prompted them to define the reverse number of superiority.

Definition <: An arrangement S of nodes in a Graph G is a dominating set if each node not in D is nearby any one node in S . In the event that $V-S$ contains a dominating set say S' of G , at that point S' is known as inverse dominating set concerning S . Example: 1



$D_1 = \{2, 4\}$, $D_2 = \{2, 5\}$ and $D_3 = \{1, 4\}$ are the least dominating sets. Their comparing opposite inverse dominating sets are $D_1^* = \{1, 3, 5\}$, $D_2^* = \{1, 4\}$ and $D_3^* = \{2, 5\}$ respectively. Thus, the domination number of G is $\gamma(G) = 2$ & the inverse domination number of G is $\gamma(G) = 2$. Theorem 3.7: Let d be a positive divisor of a +ve number n . Then a regular graph G on n nodes, for $\gamma(G) = (G) = d$.

Proof: Assume that d is a +ve number and divides $n \geq 1$. i.e.; $n = kd$. Let $V = V_i$, where $V_i = \{v_{i1}, v_{i2}, \dots, v_{id}\}$ for $1 \leq i \leq d$. Let G be the graph with node set as V and every node of V_i is adjoining precisely to one node of V_j , for $j \neq i$. Then $d(v) = k - 1 \forall v \in V$, so G is $k - 1$ regular. Also, each V_i is a γ -set of G .

PRELIMINARIES

Proposition 1: If $E' \subseteq E(G)$, then $|V(G/E')| = |V(G)| - |E'|$ and $|E(G/E')| < |E(G)| - |E'|$.

Proof. Note that contracting an edge decreases the number of vertices by one and the number of edges by at least one. The results follows by induction on $|E'|$.

Lemma 2. Let G be a connected graph.

- 1) If D is a γ -set of G and x, y are two vertices in D , then $\text{cty}(G) < d(x, y)$.
- 2) If T is a γ_t -set of G and x, y, z are three vertices in D such that x, y are adjacent in G , then $\text{cty}(G) < \min\{d(x, z), d(y, z)\}$.

Proof. 1) Let P be a shortest path between x and y and consider $G' = G/E(P)$. We show $\gamma(G') < \gamma(G)$, which implies that $\text{cty}(G) < |E(P)| = d(x, y)$.

Suppose that D is a γ -set of G and v is the contracted vertex in G' . Let $D' = (D \setminus V(P)) \cup \{v\}$. We will show that D' is a dominating set of G' . To this aim, consider $u \in V(G')$.

If $u \in N[v]$, then u is dominated by the contracted vertex v in G' .

If $u \notin N[v]$, then u is also a vertex of G and so D contains a vertex w which dominates u in G . It is clear that $w \notin V(P)$ (otherwise $u \in N[v]$). It follows that $w \in D'$, and w dominates u in G' by the definition of contraction.

Therefore D' is a dominating set of G' and $\gamma(G') < |D'| = |D| - 1 = \gamma(G) - 1$.

- 2) Suppose that T is a γ_t -set of G and $x, y, z \in D$ satisfying $xy \in E(G)$. Assume $d(x, z) > d(y, z)$, without loss of generality. Let P be a shortest path between y and z . Then P does not contain x (otherwise $d(x, z) < d(y, z)$). We will show that $\gamma_t(G/E(P)) < \gamma_t(G)$, which implies $\text{cty}_t(G) < |E(P)| = d(y, z) = \min\{d(x, z), d(y, z)\}$. Denote the contracted vertex in $G/E(P)$ by v and let $T' = (T \setminus V(P)) \cup \{v\}$. In order to prove T' is a total dominating set of G , we need only to consider each vertex $u \in N[x] \cup N[v]$, since x and v are two adjacent vertex in T' . Note that u is also a vertex of G . Then there exists a vertex $w \in T$ such that $uw \in E(G)$. It is clear that $w \notin V(P)$ (otherwise $w \in N[v]$). Hence $w \in T'$ and $uw \in E(G/E(P))$ by the definition of contraction. Therefore T' is a total dominating set of $G/E(P)$, which implies that $\gamma_t(G) < |T'| = |T| - 1 = \gamma_t(G) - 1$.

Now we investigate the relationship between the (total) dominating sets in the original graph and the (total) dominating sets in the contracted graph.

Lemma 3 (Contraction Lemma) Let G be a connected graph.

- 1) If D is a dominating set of G and E is a subset of $E(G[D])$, then G/E' contains a dominating set D' such that $|D'| = |D| - |E'|$.
- 2) If T is a total dominating set of G and E' is a subset of $E(G[T])$ such that $G[T]/E'$ contains no isolated vertex, then G/E' contains a total dominating set T' such that $|T'| = |T| - |E'|$.

Proof. 1) We will show that $D' = V(G[D]/E')$ is a dominating set of G/E' . Then $|D'| = |D| - |E'|$ by Proposition 1.

Let S be the set of all contracted vertices in G/E' . Since $E' \subseteq E(G[D])$, then $S \subseteq D'$. Hence we need only to consider such vertex v that $v \notin N[s]$ for any $s \in S$. In that case, v is also a vertex of G . Thus D contains a vertex $u \in D$ such that $u = v$ or $uv \in E(G)$. It is easy to observe that u is not incident with any edge of E' ; otherwise $u \in S$, a contradiction to the assumption that $v \notin N[s]$ for any $s \in S$. Therefore u lies in D' . If $u = v$ then we are done. Otherwise u remains adjacent to v in G/E' . Finally we conclude that D' is a dominating set of G/E' .

- 2) Since T is a dominating set of G , then by 1), T' is a dominating set of G/E' and $|T'| = |T| - |E'|$. Moreover, T' is a total dominating set, since $G[T'] = G[T]/E'$ contains no isolated vertex.

Lemma 4 (Expansion Lemma) Let G be a connected graph.

- 1) If E' is a subset of $E(G)$ and D' is a dominating set of $G' = G/E'$, then G has a dominating set D such that $G'[D']$ is a spanning subgraph of $G[D]/F$ where $F \subseteq E(G[D])$ and $|F| = |E'|$. As a consequence, $|D| = |D'| + |E'|$ and $|E(G[D])| > |E(G'[D'])| + |E'|$.
- 2) If E' is a subset of $E(G)$ and T' is a total dominating set of $G' = G/E'$, then G has a total dominating set T such that $G'[T']$ is a spanning subgraph of $G[T]/F$ where $F \subseteq E(G[T])$ and $|F| = |E'|$. As a consequence, $|T| = |T'| + |E'|$ and $|E(G[T])| > |E(G'[T'])| + |E'|$.

Proof. 1) By induction on $k = |E'|$. First consider $k = 1$. Suppose that $E' = \{xy\}$ and v is the contracted vertex in $G' = G/xy$. We distinguish two cases.

Case 1. $v \in D'$. Let $D = (D' \setminus \{v\}) \cup \{x, y\}$. Then $G[D]/xy = G'[D']$. In order to show that D is a dominating set of G , we need only to consider such vertex u that $u \notin N[x] \cup N[y]$. Since u is also a vertex of G/xy , then D' contains a vertex w which dominates u . It is easy to see $w \in D$ since $w \neq v$ (otherwise $u \in N[x] \cup N[y]$). If u

$= w$ then we are done. Otherwise $uw \in E(G')$, which implies $uw \in E(G)$. Thus D is a dominating set of G .

Case 2. $v \notin D'$. Then D' contains a vertex z such that $vz \in E(G')$. Assume $xz \in E(G)$, without loss of generality. Let $D = D' \cup \{x\}$. Then $G'[D']$ is a spanning subgraph of $G[D]/xz$. To show that D is a dominating set of G , we need only to consider each vertex $u \notin \{x, y\}$ of G . Since u is also a vertex of G' , then there exists a vertex $w \in D' \subseteq D$ such that $u = w$ or $uw \in E(G')$. If $u = w$ then we are done. Otherwise $uw \in E(G')$. Since $v \notin D'$, then $w \neq v$, which implies $uw \in E(G)$. Therefore D is a dominating set of G .

Now consider $k > 2$ and let D' be a dominating set of $G' = G/E'$. By the induction hypothesis, the result holds for $E'' = E' \setminus \{e\}$ where $e \in E'$. Note that $G/E' = (G/e)/E''$. Then $G'' = G/e$ has a dominating set D'' such that

$G'[D']$ is a spanning subgraph of $G''[D'']/F''$, $|F''| = |E''|$. (1)

Applying the result of $k = 1$ to $G'' = G/e$ we have that G has a dominating set D such that

$G''[D'']$ is a spanning subgraph of $G[D]/F'$, $|F'| = 1$. (2)

Combining (1) and (2) yields that G has a dominating set D such that $G'[D']$ is a spanning subgraph of $(G[D]/F'')/F' = G[D]/F$ where $F = F' \cup F''$. It is easy to see that $F'' \subseteq E(G''[D'']) \setminus E(G'[D'])$ and $F' \subseteq E(G[D]) \setminus E(G''[D''])$, which implies that $|F| = |F''| + |F'| = |E''| + 1 = |E'|$. It follows from Proposition 1 that

$$|D| = |V(G[D])| = |V(G[D]/F)| + |F| = |V(G'[D'])| + |E'| = |D'| + |E'|$$

and

$$|E(G[D])| > |E(G[D]/F)| + |F| > |E(G'[D'])| + |E'|.$$

- 2) Let T' be a total dominating set of $G' = G/E'$. Then T' is a dominating set. By 1), G has a dominating set T such that $G'[T']$ is a spanning subgraph of $G[T]/F$ where $F \subseteq E(G[T])$ and $|F| = |E'|$. If $G[T]$ has an isolated vertex, then this vertex remains isolated in $G[T]/F$. Hence $G'[T']$ contains an isolated vertex, since $G'[T']$ is a spanning subgraph of $G[T]/F$. That contradicts the hypothesis that T' is a total dominating set of G' .

DOMINATION CONTRACTION NUMBER

In this section we consider domination contraction number of a graph. Let us begin with some simple examples.

Proposition 1 For a path P_n and a cycle C_n on n vertices, $\gamma(P_n) = \gamma(C_n) = \lceil n/3 \rceil$.

Proposition 2 For a path P_n and a cycle C_n on $n > 4$ vertices, $\text{cty}(P_n) = \text{cty}(C_n) = i$, where $n = 3k + i$, $1 < i < 3$.

Theorem 1 For a connected graph G , $\text{cty}(G) < 3$.

Proof. Let D be a γ -set of G . If $|D| = 1$ then $\text{cty}(G) = 0$ by the definition. Assume $|D| > 2$ below. Choose two vertices x, y in D such that $d(x, y)$ is as small as possible. We claim that $d(x, y) < 3$, which implies $\text{cty}(G) < 3$.

Suppose to the contrary that $d(x, y) = k > 4$. Let $P = xv_1v_2 \dots v_{k-1}y$ be a shortest path between x and y . Then neither x nor y can dominate v_2 since P is a shortest path. Thus there exists a vertex z in $D \setminus \{x, y\}$ such that $v_2 \in N[z]$. It follows that

$$d(x, z) \leq d(x, v_2) + d(v_2, z) \leq 2 + 1 = 3,$$

a contradiction to the choice of x, y .

Next we determine when a graph has domination contraction number 0, 1, 2 or 3.

Lemma 1 For a connected graph G , $\text{cty}(G) = 0$ if and only if G admits a star as its spanning tree.

Proof. It is clear that $\gamma(G) = 1$ if and only if G has a vertex joined to all other vertices in G , i.e. G admits a star as its spanning tree.

Lemma 2 For a connected graph G , $\text{cty}(G) = 1$ if and only if there exists a γ -set D which is not independent.

Proof. If D is a γ -set which is not independent, then there exist two adjacent vertices x, y in D . By Lemma 2.2, $\text{cty}(G) < d(x, y) = 1$. Clearly $\text{cty}(G) \neq 0$, since $\gamma(G) = |D| > 2$. Hence $\text{cty}(G) = 1$.

Conversely, assume $\text{cty}(G) = 1$. Then there exists an edge xy such that $\gamma(G/xy) = \gamma(G) - 1$. Let D' be a γ -set of $G' = G/xy$. By the Expansion Lemma, G has a dominating set D such that $|D| = |D'| + 1$ and $|E(G[D])| > |E(G'[D'])| + 1 > 1$. Then D is a γ -set of G since $|D'| = \gamma(G') = \gamma(G) - 1$, and D is not independent.

By Lemma 2, a connected graph G has $\text{cty}(G) \leq 1$ if and only if every γ -set of G is independent. As a consequence, $\gamma(G) = i(G)$, where $i(G)$ is the independent domination number of G , defined as the minimum cardinality of all maximal independent sets in G . However, we cannot conclude from $\gamma(G) = i(G)$ that $\text{cty}(G) \neq 1$. We go forward to characterize graphs with contraction domination number 2. For convenience, we call a dominating set D of G with $|D| = \gamma(G) + 1$ a $(\gamma + 1)$ -set.

CONCLUSION

There is an important comparison of the general constraints to the numbers of dominance and absolute domination with those in subdivision numbers. These limitations allow graphs, as we have done to be clearly graded and characterised. We also have to concentrate on certain groups of graphs, like trees, and offer positive characteristics according to their numbers. The processes of graphs such as graphic products are also desirable for consideration. On the other hand, it is worth studying further the relationship between contraction numbers and subdivision numbers.

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