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An Analysis of Fixed Point Theorems in Some Metric Spaces: A Review

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Abstract – The present investigation is fundamentally worried about a few new sorts of fixed point theorems in various spaces, for example, cone metric spaces and fuzzy metric spaces. By utilizing these acquired fixed point theorems, we at that point demonstrate the presence and uniqueness of the solutions to two classes of two-point customary differential equation problems.

The principle point of this examination is to introduce the idea of general Mann and general Ishikawa write twofold sequences iterations with blunders to approximate fixed points. We demonstrate that the general Mann compose twofold sequence iteration process with blunders meets strongly to a fortuitous event point of two continuous pseudo-contractive mappings, each of which maps a bounded shut arched nonempty subset of a genuine Hilbert space into itself. In addition, we talk about equality from the S, T-secure qualities point of view under specific confinements between the general Mann write twofold sequence iteration process with mistakes and the general Ishikawa iterations with blunders. An application is additionally given to help our thought utilizing good compose mappings.

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INTRODUCTION

The starting point of fixed point theory lies in the method of progressive approximations utilized for demonstrating presence of solutions of differential equations presented autonomously by Joseph Liouville in 1837 and Charles Emile Picard in 1890. In any case, formally it was begun in the start of twentieth century as an essential piece of investigation. The deliberation of this traditional theory is the spearheading work of the considerable Polish mathematician Stefan Banach distributed in 1922 which gives a valuable method to locate the fixed points of a map.

Notwithstanding, on chronicled point of view, the real established result in fixed point theory is because of L. E. J. Brouwer given in 1912. The observed Banach contraction principle (BCP) states that "a contraction mapping on an entire metric space has a one of a kind fixed point". Banach utilized contracting mapto get this key result. The Brouwer fixed point hypothesis is of extraordinary significance in the numerical treatment of equations. It precisely expresses that "a continuous map on a shut unit ball in Rn has a fixed point". These praised results have been utilized, generalized and reached out in different routes by a few mathematicians, researchers, financial experts for single valued and multivalued mappings under various contractive conditions in different spaces. Kannan demonstrated a fixed point hypothesis for the maps not really continuous. This was another vital advancement in fixed point theory.

In this manner fixed point theory has been widely examined, generalized and improved in various methodologies, for example, metric, topological and arrange theoretic. This headway in fixed point theory broadened the applications of various fixed point results in different territories, for example, the presence theory of differential and integral equations, dynamic programming, fractal and mayhem theory, discrete dynamics, populace dynamics, differential considerations, framework investigation, interim arithmetic, optimization and game theory, variational inequalities and control theory, elasticity and plasticity theory and other various orders of numerical sciences.

As a rule of handy utility, the mapping under thought might not have an exact fixed point because of some tight limitations on the space or the map. Further, there may emerge some functional circumstances where the presence of a fixed point isn't entirely required yet an approximate fixed point is all that could possibly be needed. The theory of approximate fixed points assumes an imperative part in such circumstances. Approximate fixed point property for different sorts of mappings has been an unmistakable territory of research throughout the previous couple of decades. A traditional best approximation hypothesis was presented by Fan in 1969. Subsequently, a few creators, including Reich

(1978), Prolla (1983), Sehgal and Singh (1989), have determined augmentations of

Fan's hypothesis in numerous ways. Approximate fixed points of continuous maps have been considered by Rafi and Salami (2006) and numerous others. Tijs et al (2003) contemplated approximate fixed point theorems for contraction and non-far reaching maps by weakening the conditions on the spaces. Branzei et al (2003) additionally stretched out these results to multi functions in Banach spaces. Singh and Prasad (2002) demonstrated an approximate fixed point hypothesis for semi contraction in metric spaces.

Fixed point theory has dependably been energizing in itself and its applications in new territories. As of now, it has discovered new and hot territories of action. The start of fixed point theory in computer science improves its appropriateness in various domains. Because of the appearance of expedient and quick computational tools, another skyline has been given to fixed point theory. The fixed point equations are tackled by methods for some iterative methodology. In perspective of their solid applications, it is of awesome enthusiasm to know whether these iterative methodology are numerically steady or not. The investigation of strength of iterative strategies appreciates a commended put in relevant mathematics because of tumultuous conduct of functions in discrete dynamics, fractal graphics and different other numerical calculations where computer programming is included. This sort of problem for genuine valued functions was first talked about by M. Urabe (1956) in 1956. The principal result on the steadiness of iterative strategies on metric spaces is because of Alexander M. Ostrowski. This result was stretched out to multivalued administrators by Singh and Chadha (1995). Czerwik et al (2002) stretched out this to the setting of generalized metric spaces.

The theory of the fixed point has imperative applications in fields, for example, differential equations, equilibrium problems, variational imbalance, optimization problems, maxmin problems and so on which has pulled in numerous researchers' consideration and turned into a hotly debated issue in mathematics and connected mathematics field for a long time. A few researchers have demonstrated the fixed point hypothesis in halfway request metric space, and connected them to demonstrate the presence and uniqueness of the solution to the two-point standard differential equation problems. Motivated by the current advance in this fields, we will consider in the present examination the presence and uniqueness of the fixed point for some special mappings in cone metric spaces and fuzzy metric spaces and additionally their applications to the following two-point conventional differential equations.

BASIC CONCEPTS

We begin with the essential ideas identified with metric spaces, fixed points and distinctive contraction conditions.

Definition 1. Give X a chance to be a non-purge set together with a separation function $d: X \times X \to R_+$. The function *d* is said to be a *metric* iff for all $x, y, z \in X$, the following conditions are satisfied (i) $d(x, y) \geq 0$ and $d(x, y) = 0$ *if* $x = y$,

(ii) $d(x, y) = d(y, x)$,

(iii) $d(x, z) \leq d(x, y) + d(y, z)$.

The match (X, d) is known as a metric space.

Definition 2. A sequence in a metric space is a Cauchy sequence if for every $\varepsilon > 0$, there exists $n_0 \in N$ such that $d(x_n, x_m) < \varepsilon$, for all $n, m > n_0$.

Definition 3. A metric space (X, d) is called finished, if each Cauchy sequence meets in it.

Definition 4. The diameter of a set A denoted by is $\delta(A)$ defined as $\delta(A) = \sup\{d(a, b) : a, b \in A\}$. It implies that width of the set is the slightest upper bound of the separations between the points of the set A. In the event that the width is limited, i.e. $\delta(A) < \infty$, Then A is bounded.

Definition 5. Let (X, d) be a metric space and $T: X \to X$. Then T has a fixed point if there is a $x \in X$ such that $Tx = x$. The point *x* is called a fixed point of *T.*

Definition 6. Let (X, *d)* be a metric space and $T, S: X \to X$. Then *x* is called a coincidence (respectively, common fixed) point of *T* and *S,* if $x \in X$ such that $Tx = Sx$ (respectively, $x = Tx = Sx$).

In a few circumstances an approximate solution of the problem is all that could possibly be needed, so we need to discover approximate fixed point rather than fixed point.

Definition 7. Let (X, d) be a metric space and $T: X \to X$. an element $x_0 \in X$ is called an approximate fixed point (or ε- fixed point) of *T* if $d(Tx_0, x_0) < \varepsilon$, when $\varepsilon > 0$.

Approximate fixed point of the function exists if the characterized mapping has approximate fixed point property.

Definition 8. A map T is said to fulfill approximate fixed point property (AFPP) if for each $\varepsilon > 0,\ Fix_\varepsilon(T) \neq \phi,$ where $Fix_\varepsilon(T)$ is the set of all approximate fixed point of $T.$

The following condition ensures the presence of approximate fixed points.

Definition 9. A map $T: X \to X$ is said to be asymptotically regular if for any $x \in X,$ $\lim_{n\to\infty} d(T^n x, T^{n+1}x) = 0.$

Definition 10. Let (X, d) be a metric space. A mapping $T: X \to X$ is called

(i) Lipschitzian (or L- Lipschitzian) if there exists $L > 0$ such that for all $x,\ y \in X,\ d(Tx, Ty) \le L \cdot d(x, y),$

(ii) Banach contraction if *T* is α- Lipschitzian with $\alpha \in [0, 1)$ and for all $x,\ y \in X,\ d(Tx, Ty) \le \alpha \cdot d(x, y),$

(iii) Non-expansive if T is 1-Lipschitzian and for all $x,\ y \in X,\ \ d(Tx, Ty) \le d(x, y),$

(iv) Contractive if $d(Tx, Ty) < d(x, y),$ for all $x, y \in X,\ x \neq y,$

(v) Isometry if $d(Tx, Ty) = d(x, y),$ for all $x, y \in X.$

Examples 1. (i) Let $T: [1/2, 2] \to [1/2, 2],$ be defined as $T(x) = 1/x,$ then *T* is 4-Lipschitzian with $Fix\ (T) = \{1\},$ where $Fix(T)$ denotes fixed point of the mapping *T.*

(ii) $T: R \to R,\ T(x) = x/2 + 3,\ \ x \in R.$ Obviously *T* is a Banach contraction and $Fix(T) = \{6\}.$

(iii) $Tx = 1 - x,\ x \in R$ is non-expansive and $Fix(T) = \{1/2\}.$

(iv) $T: [1, \infty] \to [1, \infty],\ T(x) = x + 1/x,$ is contractive and $Fix(T) = \phi.$

(v) $T(x) = x + 2,$ Then $Fix(T) = \phi$ is isometry.

The observed Banach contraction principle (BC'P) is the least complex and one of the most flexible and key results in the fixed point theory. It produces approximation of any coveted precision as well as decides from the earlier and a-posteriori mistake gauges.

Theorem 1, Let (X, d) be a total metric space and $T: X \to X$ satisfies (be).

At that point T has a one of a kind fixed point p. i.e. $Tp = p$ And $\lim_{n\to\infty} T^n x = p.$

Moreover, we have a-priori error estimate $d(T^n x,\ p) \le \frac{\alpha^n}{1-\alpha} d(Tx, x)$ and the a-posteriori error estimate

$$d(T^n x,\ p) \le \frac{\alpha}{1-\alpha} d(T^n x,\ T^{n-1}x).$$

$$d(T^n(x),\ p) \le \frac{\alpha}{1-\alpha} d(T^n(x),\ T^{n-1}(x)).$$

There are different generalizations of the commended contraction mapping principle. These generalizations are acquired either by weakening the contraction condition of the map by giving an adequately rich structure to the space with a specific end goal to repay the unwinding of the contraction condition or by broadening the structure of the space or sometime consolidating both the methodologies.

Kannari was the first to propose a fixed point hypothesis for a discontinuous map. He precisely demonstrated the following.

Theorem 2 (Kannan Contraction Theorem). Let *T* be a self map on a complete metric space (X, d) satisfying the following condition (usually called Kannan contraction), for all $x, y \in X$ and some $\beta \in [0, 1/2),$

$$d(Tx, Ty) \le \beta[d(x, Tx) + d(y, Ty)]$$

Then *T* has a unique fixed point.

Kannan's hypothesis propelled various augmentations and generalizations of the BC'P and his own particular fixed point hypothesis on different settings. Chatterjea demonstrated a fixed point hypothesis for discontinuous mapping fulfilling a condition which is really a sort of double of Kannan mapping.

Definition 11. Let (X, d) be a metric space. A mapping $T: X \to X$ is called Chatterjea contraction if for all $x, y \in X$ and some $\gamma \in [0, 1/2)$,

$$d(Tx, Ty) \le \gamma[d(x, Ty) + d(y, Tx)]$$

Zamfirescu obtained some interesting results by combining Banach, Kannan and Chatterjea contraction conditions.

Definition 12. Let (X, d) be a metric space. A mapping $T: X \to X$ is called Zamfirescu contraction if for all $x, y \in X$ and some $\alpha \in [0, 1)$, $\beta, \gamma \in [0, \frac{1}{2})$, satisfies at least one of the following conditions.

(i) $$d(Tx, Ty) \le \alpha d(x, y)$$

(ii) $$d(Tx, Ty) \le \beta[d(x, Tx) + d(y, Ty)]$$

(iii) $$d(Tx, Ty) \le \gamma[d(x, Ty) + d(y, Tx)]$$

Among different generalizations of the Banach contraction, the Ciric contraction (additionally called as semi contraction) is thought to be the most general one.

Definition 13. Let (X, d) be a metric space. A mapping $T: X \to X$ is called quasi contraction if

$$d(Tx, Ty) \le k.\max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\},$$

for some $0 \le k < 1$ and all *x*, *y* in *X.*

Definition 14. A mapping $T: X \to X$ is called weak or almost contraction if there exist $\alpha \in (0, 1)$ and $L \ge 0$ such that for all $x, y \in X$, $d(Tx, Ty) \le \alpha\, d(x, y) + L\, d(y, Tx).$

It is important to note that any mapping satisfying Banach, Kannan. Chatterjea. Zamfirescu, or Ciric (with constant k in(0.1/2)) type conditions are a weak or almost contraction, Jungck extended Banach contraction in the following manner.

Definition 15. Let (X, d) be a metric space and $T, S: X \to X.$ The following condition is known as Junsck contraction.

$d(Tx, Ty) \le \alpha \cdot d(Sx, Sy),$ for some $\alpha \in [0, 1)$ and all $x, y \in X.$

In spite of the fact that this condition was known to Singh and Kulshrestlia however Jungck was credited for introducing useful confirmation with respect to the presence of a typical fixed point of driving maps. Facilitate different fixed point and normal fixed point results are explored in the writing for the maps fulfilling diverse conditions. We review some of them as follows.

Definition 16. Let (X, d) be a metric space and $T, S: X \to X.$ The mappings *T* and *S* are

(i) commuting, if $TSx = STx$ for all $x \in X,$

(ii) weakly commuting, if $d(TSx, STx) \le d(Tx, Sx)$ for all $x \in X,$

(iii) compatible, if $\lim_{n\to\infty} d(TSx_n, STx_n) = 0,$ whenever $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t,$ for some $t \in X,$

(iv) weakly compatible, if they commute at their coincidence points; i.e.. if $Tu = Su$ for some u in X, then $TSu = STu,$

(v) satisfying the property *(E. A.)*, if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t,$ for some $t \in X.$

Notice that weakly driving mappings are perfect and good mappings are weakly perfect yet the opposite need not be valid. It is anything but difficult to see that two non-compatible mappings fulfill the property (E. A.). Fixed point theory for multivalued mappings is a characteristic generalization of the theory of single valued mappings.

FIXED POINT THEOREMS IN COMPLETE METRIC SPACES

The STUDY in fixed point theory has generally created in three principle bearings: generalization of conditions which guarantee presence, and, if conceivable, uniqueness, of fixed points; examination of the character of the sequence of iterates $\{T^n x\}_{n=0}^{\infty},$ where $T: X \to X$, X a total metric space, is the map under thought; investigation of the topological properties of the arrangement of fixed points, at whatever point T has more than one fixed point. This note treats just a few parts of the first and second inquiry, along a line followed by numerous different creators.

More precisely we consider maps $T: X \to X,$ which satisfy conditions of the type $d(Tx, Ty) \leqslant \varphi(d(x, y)) + \psi(d(Tx, x)) + \chi(d(Ty, y))$ for each $x, y \in X,$ what's more, for these mappings we demonstrate, under appropriate speculations, presence and uniqueness of fixed points.

1. Through all the examination, X indicates a total metric space and $T, T: X \to X,$ an asymptotically regular mapping; i.e., a function satisfying $\lim_n d(T^n x, T^{n+1} x) = 0$ for each $x \in X.$

Furthermore, we suppose that there exist three functions $\varphi, \psi, \chi,$ from $[0, +\infty[$ into $[0, +\infty[,$ which satisfy the assumptions:

(II) $\varphi(r) < r \quad \text{if } r > 0,$

(I2) there exists $\lim_{r \to \bar{r}+} \varphi(r) \leqslant \varphi(\bar{r}) \quad \text{for each } \bar{r} \in [0, +\infty[$

(I3) $\psi(0) = \chi(0) = 0.$

Moreover, we suppose that T, satisfy the inequality

(I4) $d(Tx, Ty) \leqslant \varphi(d(x, y)) + \psi(d(Tx, x)) + \chi(d(Ty, y))$ for each $x, y \in X.$

Lemma. Under the above assumptions on X and T and if, in addition, ψ and χ are continuous at $r = 0,$ then, for each $x \in X,$ there exists $z \in X$ such that $\{T^n x\}_{n=0}^{\infty}$ converges to $z.$

Proof. Suppose that there exists $x \in X$ such that the sequence of iterates is not a Cauchy sequence. Then, following there exist $\varepsilon > 0, \{m(j)\}_{j=0}^{\infty}, \{n(j)\}_{j=0}^{\infty}$ which satisfy the conditions

$$m(j) > n(j) \quad \text{for each } j \in N \tag{1.10}$$

$$\lim_j n(j) = +\infty \tag{1.11}$$

$$d(T^{m(j)}x, T^{n(j)}x) \geqslant \varepsilon \tag{1.12}$$

$$d(T^{m(j)-1}x, T^{n(j)}x) < \varepsilon. \tag{1.13}$$

Then, we have $\varepsilon \leqslant d(T^{m(j)}x, T^{n(j)}x) \leqslant d(T^{m(j)}x, T^{m(j)-1}x) + d(T^{m(j)-1}x, T^{n(j)}x) < \varepsilon + d(T^{m(j)}x, T^{m(j)-1}x)$ which implies

$$\lim_j d(T^{m(j)}x, T^{n(j)}x) = \varepsilon. \tag{1.14}$$

On the other hand

$$d(T^{m(j)}x, T^{n(j)}x) \leqslant d(T^{m(j)}x, T^{m(j)+1}x) + d(T^{n(j)}x, T^{n(j)+1}x) + d(T^{m(j)+1}x, T^{n(j)+1}x) \leqslant d(T^{m(j)}x, T^{m(j)+1}x) + d(T^{n(j)}x, T^{n(j)+1}x) + \varphi(d(T^{m(j)}x, T^{n(j)}x)) + \psi(d(T^{m(j)+1}x, T^{m(j)}x)) + \chi(d(T^{n(j)+1}x, T^{n(j)}x))$$

that is

$$d(T^{m(j)}x, T^{n(j)}x) - \varphi(d(T^{m(j)}x, T^{n(j)}x)) \leqslant d(T^{m(j)}x, T^{m(j)+1}x) + d(T^{n(j)}x, T^{n(j)+1}x) + \psi(d(T^{m(j)+1}x, T^{m(j)}x)) + \chi(d(T^{n(j)+1}x, T^{n(j)}x))$$

and, letting $j \to +\infty$ $\varepsilon - \lim_j \varphi(d(T^{m(j)}x, T^{n(j)}x)) \leqslant 0.$

Hence, by (1.3) and (1.5) it follows that $\varepsilon = 0$, a contradiction. Since X is a complete metric space, the proof is complete.

2. In this segment we should demonstrate two fixed point theorems: the first for non-continuous, the second for continuous mappings.

Theorem 1. Let $X, T, \varphi, \psi, \chi$ be as in the Lemma.

Furthermore, we suppose that $\chi(r) < r$ if $r > 0.$

Then T has a unique fixed point.

Proof. Uniqueness is obvious by virtue of hypotheses (I1), (I3) and (I4). So let us show existence. From the Lemma there is a $z \in X$ such that $T^n x \to z,$ as $n \to +\infty$, for each $x \in X.$ Since

$$d(z, Tz) \leqslant d(z, T^n x) + d(T^n x, T^{n+1}x) + d(T^{n+1}x, Tz) \leqslant d(z, T^n x) + d(T^n x, T^{n+1}x) + \varphi(d(T^n x, z)) + \chi(d(Tz, z)) + \psi(d(T^n x, T^{n+1}x))$$

we have

$$d(z, Tx) - \chi(d(z, Tz)) \leqslant 2d(z, T^n x) + d(T^n x, T^{n+1}x) + \psi(d(T^n x, T^{n+1}x))$$

and, letting $n \to +\infty,$ we have $0 \leqslant d(z, Tz) - \chi(d(z, Tz)) \leqslant 0;$ thus, $z = Tz$ follows.

Theorem 2. Let $X, T, \varphi, \psi, \chi$ be as in the Lemma. If, in addition, T is continuous, then it has a unique fixed point.

Proof. Uniqueness follows as in Theorem 1.4. Let $z \in X$ be such that $T^n x \to z,$ as $n \to +\infty$ From continuity of T we obtain $\lim_n T^{n+1}x = Tz$ and, since T is asymptotically regular, we have $z = Tz.$

2-METRIC SPACE

The idea of generalized n-metric space was presented long back, in the year 1928, by Menger. From that point forward, a few mathematicians have created on Menger's thought. These works have been seen in various written works, among which, crafted by Blumenthal and Pauc are powerful in understanding the issue bitterly. In the year 1938, Vulich presented an idea of higher dimensional standard in linear spaces. Be that as it may, the territory has not been additionally considered by specialists for a long time. In the year 1958, Froda chipped away at the territory of p-metric. Be that as it may, another improvement in the field started in the year 1962. In this year, Gähler distributed initial one of the arrangement of papers entitled ―2-metric spaces and their topological structures‖. Gähler has rethought Menger's examination by taking n = 2. We realize that the idea of a metric is to be viewed as a generalization of the thought of the separation between two points. Then again, the thought of 2-metric space can be considered as a generalization of the idea of territory. The territory in the Euclidean plane is extraordinarily dictated by given three points in the plane. Be that as it may, for a fixed component among triplets, a metric gives a 2-metric. On the off chance that any of the triplets corresponds with any point, at that point the points are collinear and we can acquire a metric. Additionally it can be effectively watched that a 2-metric gives a pseudo metric.

Definition : Let X be a non-empty> set. A real valued function d on $X \times X \times X \to R^+$ *is said to be a 2-metric on A' if (i) given distinct elements* x, y *of* X, *there exists an element* z *of* X *such that* $d(x,y,z) \neq 0$.

(ii) $d(x,y,z) = 0$ when at least two of x, y, z are equal,

(iii) $d(x,y,z) = d(x,z,y) = d(y,z,x)$ for all x, y, z in X, and

(iv) $d(x,y,z) \leq d(x,y,w) + d(x,w,z) + d(w,y,z)$ for all x, y, z, w in X.

When d is a 2-metric on X, then the ordered pair (X, d) is called a 2-metric space.

Gahler has additionally demonstrated that in spite of the fact that d is a continuous function of any of its contentions, it require not be continuous in two contentions. On the off chance that it is continuous in two contentions then it is continuous in every one of the three contentions. In a nutshell, we can state that a 2-metric d which is continuous in the greater part of its contentions will be called continuous.

We now give a few cases of 2-metric space which have been recognized from the study of writings.

COMMON FIXED POINT THEOREM IN e-£-FUZZY METRIC SPACE-

In 1965, Zacleh (1965) presented fuzzy set, which is additionally generalized to intuitionistic fuzzy set by Atauassov (1986) and $\mathcal{L}$- fuzzy set by Goguen (1967), Thereafter a few creators characterized fuzzy metric spaces in various ways. Subsequently a few fuzzy fixed point theorems are likewise settled in their new settings.

We first review the preliminaries required for ensuing results.

Definition 1. Let $\theta : [0, \infty) \times [0, \infty) \to [0, \infty)$ be a continuous map with respect to each variable. Then θ is called a 5-action if and only if it satisfies the following conditions

(i) $\theta(0, 0) = 0$ and $\theta(t, s) = \theta(s, t)$ for all $t, s \geq 0,$

(ii) $\theta(s, t) < \theta(u, v)$ if $s < u$ and $t \leq v$ or $s \leq u$ and $t < v,$

(iii) for each $r \in \mathrm{Im}(\theta) - \{0\}$ and for each $s \in (0, r]$, there exists $t \in (0, r]$ such that $\theta(t, s) = r,$ where $\mathrm{Im}(\theta) = \{\theta(s, t) : s \geq 0, t \geq 0\},$

(iv) $\theta(s, 0) \leq s,$ for all $s > 0.$

Definition 2. Let X be a nonempty set. A mapping $d_\theta : X \times X \to [0, \infty)$ is called a θ- metric on Ar with respect to B - action, if d_θ satisfies the following

(i) $d_\theta(x, y) = 0$ if $x = y,$

(ii) $d_\theta(x, y) = d_\theta(y, x),$ for all $x, y \in X,$

(iii) $d_\theta(x, y) \leq \theta(d_\theta(x, z), d_\theta(z, y)),$ for all $x, y, z \in X.$

Then (X, d_θ) is called a θ- metric space.

Remark 1. If $\theta(s, t) = s + t,$ then the θ- metric space becomes a metric space.

Remark 2. If $\theta(s, t) = b(s + t), b \geq 1,$ then the θ- metric space is the 6-metric space.

Now we define $\theta - \mathcal{L}$ - fuzzy metric space following the definition given by George and Veeramani,

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Definition 2.3. The 3-triplet $(X, \mathcal{M}, \mathcal{T})$ is said to be an $\theta-\mathcal{L}$- fuzzy metric space, if A' is an arbitrary (non-empty) set. $\mathcal{T}$ is a continuous t- norm 011 $\mathcal{L}$ and $\mathcal{M}$ is an $\mathcal{L}$-fuzzy set 011 $X^2 \times (0, \infty)$ with respect to B~ action. $\theta \in \mathcal{M}$ satisfying the following conditions for every x, y, z in X and t, s in $(0, \infty)$

(i) $\mathcal{M}(x, y, t) >_L 0_{\mathcal{L}}$,

(ii) $\mathcal{M}(x, y, t) = 1_{\mathcal{L}}$ for all $t > 0$, iff $x = y$,

(iii) $\mathcal{M}(x, y, t) = \mathcal{M}(y, x, t)$

(iv) $\mathcal{T}(\mathcal{M}(x, y, t), \mathcal{M}(y, z, s)) \leq_L \mathcal{M}(x, z, \theta(t, s))$,

(v) $\mathcal{M}(x, y, .) : (0, \infty) \to L$ is continuous and $\lim_{t\to\infty} \mathcal{M}(x, y, t) = 1_{\mathcal{L}}$.

In this case $\mathcal{M}$ is called a $\theta-\mathcal{L}$-fuzzy metric space. If $\mathcal{M} = \mathcal{M}_{M,N}$ is an intuitionistic fuzzy set, then the 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be a θ- intuitionistic fuzzy metric space.

CONCLUSION

The assessment is focused on a couple of new sorts of fixed point theorems in some spaces, for instance, cone metric spaces and fuzzy metric spaces together with their applications. In this examination, we have shown the most basic fixed point theorems in the informative examination of issues in the associated science: Banach's and Schauder's fixed point theorem. An early presentation for fixed point theorems for nonexpansive mappings is the proof of a type of the fixed point theorem of Browder, G¨ohde, Kirk. As we will see, fixed point hypothesis for nonexpansive mappings advantage generally of geometric properties of Banach spaces.

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