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The Development of Topology Based Open Sets, Closed Sets, and Limit Points: An Analytical Study

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***Abstract** – General topology has its basic establishments in real and complex investigation, which made critical vocations of the interrelated thoughts of open set, of closed set, and of a limit motivation behind a set. This article inspects how those three ideas developed and advanced during the late nineteenth and mid twentieth hundreds of years, because of Weierstrass, Cantor, and Lebesgue. Specific consideration is paid to the various types of the Bolzano–Weierstrass Theorem found in the last's unpublished talks. A fruitless early, unpublished introduction of open sets by Dedekind is inspected, just as how Peano and Jordan nearly presented that idea.*

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INTRODUCTION

The most punctual thought was that of the limit purpose of a set, due to Weierstrass be that as it may, scattered by Cantor, while that of a closed set (because of Cantor) emerged fairly later. The possibility of an open set (except for Dedekind's concise, unpublished work about it) came most recent of all. The exceptionally moderate dispersion of the idea of open set is astounding in perspective on its significance now.

In the wake of examining how these thoughts created in analysis (where they were just observed as tools of analysis and not part of a different subject of topology), we think about the advancement of the idea of topological space. Such spaces are presently generally dependent on the idea of an open set. There was a time of development during which it was not clear what idea ought to be taken as crude and what aphorisms ought to be accepted for such a space. This advancement of the thought of a theoretical topological space started in 1904 with the introduction, by the French investigator Maurice Fréchet [1904], of his idea of a L-space, which took the limit of an endless grouping as the crude thought.

For a couple of decades there was a challenge among ideas significantly more general than the one which in the end got prevailing and which is as yet predominant today under the name of "topological space." In analysis, the idea of a metric space was vital for quite a few years, especially during the 1920s as Banach spaces (total normed vector spaces, where the standard gives the metric), until some other time by the more general idea of topological vector space.

AN ABORTIVE APPROACH TO OPEN SETS

Cantor alluded uniquely to a point "interior to" an interval or, a couple of years after the fact, to "interior points" of a continuous point-set. All things considered, Cantor's 1879 definition of interior guide was close toward that set forward by Giuseppe Peano in his book Geometric Applications of the Infinitesimal Calculus. Peano considered a point-set An (in a space of one, two, or three measurements) and described a point p to be inside to An if there is a positive number r to such a degree, that all of those focuses whose good ways from p isn't as much as r have a spot with A. In his next two definitions, Peano went past what Cantor had done and expressed that a point p was said to be exterior to An if p is interior to the supplement of A. At long last, p was said to be a boundary purpose of An if p was neither interior nor exterior to A. Peano realized that on the off chance that A contains a few yet not every one of the points of space, at that point An essentially has a boundary point, which could conceivably have a place with A.

Peano's thoughts could without much of a stretch have prompted the idea of open set at the time, however in certainty didn't. He characterized the boundary of a set as the assortment of all its boundary points. At that point, had he wished to do as such, he could have characterized a set to be open on the off chance that it was indistinguishable from the set of all its interior points.

Ironicly thoughts very like Peano's and Jordan's had been created numerous years sooner by Dedekind in an unpublished original copy, which was first distributed in Dedekind's gathered works in 1931, on account of Emmy Noether.

LIMIT POINTS AND CONNECTEDNESS

The possibility of connectedness of a point-set was engaged with that of a continuum in the mid-nineteenth century, at once before any idea of closed or open set had been proposed. As Wilder commented, Bolzano's after death Paradoxes of the Infinite (1851) declared that "a continuum is available when, and just when, we have a total of basic elements (moments or points or substances) so organized that every individual from the total has, at every person and adequately little distance from itself, in any event one other individual from the total for a neighbor". Cantor strenuously questioned Bolzano's definition of a continuum since, under this definition, a set comprising of a few separated continua would be a continuum. It might be, in any case, that Bolzano's definition of continuum drove Cantor to plan what he called "connectedness," i.e., a set M is connected if for each positive ε and each an and b in M there is some limited n and a few points p1,p2, . . . , pn with the end goal that the distances ap1,p1p2, . . . , pnb are for the most part not exactly ε.

Cantor's definition of connectedness imparted a few detriments to Bolzano's definition of continuum, albeit obviously nobody pointed them out at the time. For example, Cantor's definition made the set of every discerning number connected, and similarly the set of every unreasonable number; in Bolzano's terms, these two sets would be continua. (Afterward, topologists would view both of these sets as examples of "completely disconnected" sets.)

A very unique definition of connectedness was proposed by Jordan in his 1892 article on distinct integrals. There, limiting himself to closed and bounded sets, he characterized a set E in to be connected ("d'un seul occupant") if and just if E can't be partitioned into two closed and "separated" sets. Jordan promptly proceeded to demonstrate that a closed and bounded set is connected in his sense if and just on the off chance that it is connected in Cantor's sense. He rehashed his discourse of such ideas (limit point, separated sets, closed set, connected set) in his Cours d'analyse. Plainly what we currently see as topological ideas were seen by Jordan as parts of analysis and as tools to be utilized in analysis, as opposed to as a different and unmistakable field of arithmetic.

SERIOUS ERRORS ABOUT CLOSED SETS

A number of genuine errors about closed sets were made when the new century rolled over, errors that were associated with the later idea of compactness. The first of these errors was expected to Hurwitz. The incongruity is that Schoenflies even refered to the proper page of Jordan, however didn't see that by forgetting about "unbounded," he had made the case bogus. This error was connected with Schoenflies' overemphasis on the significance of closed sets: "The most significant pointsets from the hypothetical perspective are the closed and the ideal sets. These are the ones most habitually experienced in analysis and geometry".

A progression of related errors pursued. Schoenflies contended that the one–one continuous picture of an ideal set P is great. (In any case, this can come up short if P is unbounded.) His indicated evidence depended on the case that a vast set of points must have a limit point. (This is an erroneous type of the Bolzano–Weierstrass Theorem and comes up short, for example, if P is the set of characteristic numbers.) He made a comparative however more grounded case about closed sets, yet this is similarly as bogus as the case about flawless sets. At that point he contended that if a function is continuous at each purpose of an ideal set P , then the function is consistently continuous on P. Be that as it may, this fizzles if P is taken to be all nonnegative points on the real line and the function is f (x) = x2. Last, he affirmed that if Pn is closed and Pn+1 is a nonempty subset of Pn for every single positive whole number n, at that point the convergence of all the Pn is nonempty. In the event that he had required that P1 be bounded, at that point his declaration would have been valid, yet else it is effectively discredited. (Let Pn be the set of every single real number at the very least n.)

HAUSDORFF AND GENERAL TOPOLOGY

The possibility of an open set in a conceptual space (instead of n-dimensional Euclidean space, where the thought was expected to Baire and Lebesgue) was started by Felix Hausdorff with regards to his topological spaces. Nonetheless, what Hausdorff called a topological space is a more specific thought than what is currently all around called a topological space. What he utilized as a crude thought seems to be "neighborhood of a point." To keep away from vagueness, we will call his spaces "neighborhood spaces." Hausdorff characterized a local space to be a set E, whose individuals were designated "points," together with an assortment of subsets of E. These subsets were called neighborhoods and were dependent upon four aphorisms:

(A) Every point x has a place with at any rate one neighborhood of x, and each area of x contains x.

(B) If U and V are neighborhoods of x, at that point there is some local W of x with the end goal that $W \subseteq U \cap V$.

(C) If a point y has a place with a local U of x, at that point there is some local V of y to such an extent that V is a subset of U.

(D) If x and y are unmistakable points, at that point there is a local U of x and a local V of y to such an extent that U and V are disjoint.

Following giving his sayings for a topological space, Hausdorff characterized what he implied by an "interior point" of a subset An of a topological space. Specifically, x is an interior purpose of An if some area of x is a subset of A. What's more, x was said to be a boundary purpose of An if x has a place with A yet isn't an interior purpose of A. At that point a set A was characterized to be an open set ("Gebiet") if the entirety of its points are interior points. At last, he indicated that the association of any family (countable or uncountable) of open sets is open and that the crossing point of limitedly many open sets is open. (By indicating that the association of any uncountable group of open sets is open, he went past what Lebesgue had done in 1905 with open sets.)

It ought to be seen that Hausdorff's neighborhoods don't really compare to neighborhoods when, similar to the case today, the idea of open set is taken as crude for topological spaces. Specifically, for a local space E containing in any event two points, the entire space E need not be an area of any point, for in the event that the main neighborhood of any point x is simply {x}, at that point every one of Hausdorff's adages are fulfilled, in spite of the fact that the entire space X isn't an open set. Be that as it may, if the cutting edge idea of topological space with "open set" is taken as the crude thought, at that point an area of x is characterized to be any set V to such an extent that x has a place with some open subset of V . Consequently the entire space is then open and is an area of every one of its points, in inconsistency to our example of a Hausdorff neighborhood space. (What for Hausdorff was the set of neighborhoods of a space is today called an "area base" for the space.)

In the wake of characterizing open sets regarding neighborhoods, Hausdorff went to collection points and closed sets. He characterized p to be a collection point ("Häufungspunkt") of a set B if each area of p contained unendingly numerous points of B. (This was in finished concurrence with Cantor's idea of limit point.) A set was characterized to be closed on the off chance that it contained all its amassing points. At that point Hausdorff demonstrated that the crossing point of any group of closed sets was closed and that the association of any limited number of closed sets was closed.

Hausdorff received from Fréchet the name "reduced" (which, above, we called "Fréchet-smaller") for any set every one of whose unending subsets has an amassing point. What Hausdorff called the "Borel Theorem" was the suggestion that if a closed and Fréchet-smaller set M is a subset of the association of an unbounded succession S of open sets, at that point M is as of now a subset of the association of some limited subset of S.

Hausdorff's second adage of countability (i.e., the set of all neighborhoods is countable) had different ramifications for the open and closed sets. One of these outcomes was what is presently known as the countable chain condition: any set of disjoint open sets is countable. Another was that the set of every single open set has a similar cardinality as the set of every single closed set, to be specific that of the set of every single real number. Besides, this saying inferred a more keen type of the Borel Theorem, with S having any interminable cardinality instead of fundamentally being countable.

TOPOLOGICAL SPACES

In the past sections, we examined the convergence of successions, the progression of functions, and the compactness of sets. We communicated these properties regarding a metric or standard. A few sorts of convergence, for example, the pointwise convergence of real-esteemed functions characterized on an interval, can't be communicated as far as a metric on a function space. Topological spaces give a general system to the investigation of convergence, coherence, and compactness. The principal structure on a topological space isn't a distance function, however an assortment of open sets; thinking straightforwardly as far as open sets regularly prompts more noteworthy clarity as well as more prominent generality.

Definition 1 A topology on a nonempty set X is an assortment of subsets of X, called open sets, to such an extent that:

(a) the empty set $\emptyset$ and the set *X* are open;

(b) the union of an arbitrary collection of open sets is open;

(c) the intersection of a finite number of open sets is open.

(d) A subset An of X is a closed set if and just if its supplement, $A^c = X \setminus A$, is open.

(e) All the more officially, an assortment $\mathcal{T}$ of subsets of X is a topology on X if:

(f) (a) $\emptyset, X \in \mathcal{T}$;

(g) (b) on the off chance that $G_\alpha \in \mathcal{T}$ for $\alpha \in A$, at that point $\bigcup_{\alpha \in A} G_\alpha \in \mathcal{T}$;

(h) (c) on the off chance that $G_i \in \mathcal{T}$ for $i = 1, 2 \dots, n$, at that point $\bigcap_{i=1}^{n} G_i \in \mathcal{T}$.

(i) We call the pair $(X, \mathcal{T})$ a topological space; in the event that $\mathcal{T}$ is obvious from the unique circumstance, at that point we regularly allude to X as a topological space.

(j) Example 1 Let X be a nonempty set. The assortment, $\{\emptyset, X\}$ comprising of the vacant set and the entire set, is a topology on X, called the minor topology or rash topology. The power set $\mathcal{P}(X)$ of X, comprising of all subsets of X, is a topology on X, called the discrete topology.

(k) Example 2 Let (X, d) be a metric space. At that point the set of every single open set characterized in Definition 1 is a topology on X, called the metric topology. For example, a subset G of $\mathbb{R}$ is open as for the standard, metric topology on $\mathbb{R}$ if and if for each $x \in G$ there is an open interval I with the end goal that $x \in I$ and $I \subset G$.

Example 3 Let $(X, \mathcal{T})$ be a topological space and *Y* a subset of *X.* Then

S = {$H \subset Y \mid H = G \cap Y$ for some $G \in \mathcal{T}$}

is a topology on *Y.* The open sets in *Y* are the intersections of open sets in *X* with *Y.* This topology is called the *induced* or *relative topology* of *Y* in *X*, and *(Y,S)* is called a topological subspace of $(X, \mathcal{T})$. For instance, the interval [0,1/2) is an open subset of [0,1] with respect to the induced metric topology of [0,1] in R, since $[0, 1/2) = (-1/2, 1/2) \cap [0, 1]$.

A set $V \subset X$ is a *neighborhood* of a point $x \in X$ if there exists an open set $G \subset V$ with $x \in G$. We do not require that *V* itself is open. A topology $\mathcal{T}$ on *X* is called *Hausdorff* if every pair of distinct points $x, y \in X$ has a pair of nonintersecting neighborhoods, meaning that there are neighborhoods V_x of *x* and V_y of *y* such that $V_x \cap V_y = \emptyset$ (see Figure 4.1). When the topology is clear, we often refer to *X* as a Hausdorff space. Almost all the topological spaces encountered in analysis are Hausdorff. For example, all metric topologies are Hausdorff. On the other hand, if *X* has at least two elements, then the trivial topology on *X* is not Hausdorff.

We can express the notions of convergence, continuity, and compactness in terms of open sets. Let *X* and *Y* be a topological spaces.

Definition 2 A sequence (x_n) in *X converges* to a limit $x \in X$ if for every neighborhood *V* of x, there is a number *N* such that $x_n \in V$ for all $n \geq N$.

This definition says that the sequence eventually lies entirely in every neighborhood of *x.*

Definition 3 A function $f : X \to Y$ is *continuous* at $x \in X$ if for each neighborhood *W* of *f(x)* there exists a neighborhood *V* of *x* such that $f(V) \subset W$. We say that f is continuous on *X* if it is continuous at every $x \in X$.

Theorem 1 Let $(X, \mathcal{T})$ and *(Y,S)* be two topological spaces and $f : X \to Y$.

Then f is continuous on *X* if and only if $f^{-1}(G) \in \mathcal{T}$ for every $G \in \mathcal{S}$.

Thus, a continuous function is characterized by the property that the inverse image of an open set is open. We leave the proof to Exercise 4.4.

Definition 4 A function $f : X \to Y$ between topological spaces *X* and V is a *homeomorphism* if it is a one-to-one, onto map and both f and f^{-1} are continuous. Two topological spaces *X* and *Y* are *homeomorphic* if there is a homeomorphism $f : X \to Y$.

Homeomorphic spaces are indistinguishable as topological spaces. For example, if $f : X \to Y$ is a homeomorphism, then *G* is open in *X* if and only if *f(G)* is open in *Y,* and a sequence (x_n) converges to *x* in *X* if and only if the sequence $(f(x_n))$ converges to *f(x)* in *Y.*

A one-to-one, onto map f always has an inverse f^{-1}, but f^{-1} need not be continuous even if f is.

Example 4 We define $f : [0, 2\pi) \to \mathbb{T}$ by $f(\theta) = e^{i\theta}$, where $[0, 2\pi) \subset \mathbb{R}$ with the topology induced by the usual topology on $\mathbb{R}$, and $\mathbb{T} \subset \mathbb{C}$ is the unit circle with the topology induced by the usual topology on C. Then, as illustrated in Figure 4.2, f is continuous but f^{-1} is not.

Definition 5 A subset if of a topological space *X* is *compact* if every open cover of *K* contains a finite subcover.

It follows from the definition that a subset *K* of *X* is compact in the topology on *X* if and only if *K* is compact as a subset of itself with respect to the relative topology of *K* in *X.* This contrasts with the fact that a set $G \subset Y$ may be relatively open in *Y,* yet not be open in *X.* For this reason, while we define the notion of relatively open, we do not define the notion of relatively compact.

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BASES OF OPEN SETS

The collection of all open sets in a topological space is often huge and unwieldy. The topological properties of metric spaces can be expressed entirely in terms of open balls, which form a rather small subset of the open sets. In this section we introduce subsets of a topological space that play a similar role to open balls in a metric space.

Definition 1 A subset *B* of a topology $\mathcal{T}$ is a *base* for $\mathcal{T}$ if for every $G \in \mathcal{T}$ there is a collection of sets $B_\alpha \in \mathcal{B}$ such that $G = \bigcup_\alpha B_\alpha$. A collection $\mathcal{N}$ of neighborhoods of a point $x \in X$ is called a *neighborhood base* for *x* if for each neighborhood *V* of *x* there is a neighborhood $W \in \mathcal{N}$ such that $W \subset V$. A topological space *X* is *first countable* if every $x \in X$ has a countable neighborhood base, and *second countable* if *X* has a countable base.

Example 1 The collection of all open intervals *(a, b)* with $a, b \in \mathbb{R}$ is a base for the standard topology on $\mathbb{R}$. The collection of all open intervals $(a, b) \subset \mathbb{R}$ with rational endpoints $a, b \in \mathbb{Q}$ is a countable base for the standard topology on $\mathbb{R}$. Thus, the standard topology is second countable.

Example 2 Let *X* be a metric space and *A* a dense subspace of *X.* The set of open balls $B_{1/n}(x)$, with $n \geq 1$ and $x \in A$ is a base for the metric topology on *X.*

A metric space is first countable, and a separable metric space is second countable.

Example 3 If *X* is topological space with the discrete topology, then the collection of open sets $\mathcal{B} = \{\{x\} \mid x \in X\}$ is a base. The discrete topology is first countable, and if *X* is countable, then it is second countable.

It is often useful to define a topology in terms of a base.

Theorem 1 A collection of open sets $\mathcal{B} \subset \mathcal{T}$ is a base for the topology $\mathcal{T}$ on a set *X* if and only if $\mathcal{B}$ contains a neighborhood base for *x* for every $x \in X$.

Proof. Suppose $\mathcal{B}$ is a base for $\mathcal{T}$. If *N* is a neighborhood of $x \in X$, then there is an open set $G \in \mathcal{T}$ such that $x \in G \subset N$. Since $\mathcal{B}$ is a base, there are sets $B_\alpha \in \mathcal{B}$ such that $\bigcup_\alpha B_\alpha = G$. Therefore, there is an α such that $x \in B_\alpha$ and $B_\alpha \subset N$. It follows that $\mathcal{B}$ contains a neighborhood base for *x.*

Conversely, if a collection of open sets $\mathcal{B}$ contains a neighborhood base for every $x \in X$, then for every open set $G \in \mathcal{T}$ and every $x \in G$ there exists $B_x \in \mathcal{B}$ such that $x \in B_x \subset G$. Therefore, $\bigcup_x B_x = G$, so $\mathcal{B}$ is a base for the topology.

Example 4 Suppose that *X* is the space of all real-valued functions on the interval [a, *b].* We may identify a function $f : [a, b] \to \mathbb{R}$ with a point $\prod_{x\in[a,b]} f(x)$ in $\mathbb{R}^{[a,b]}$, so $X = \mathbb{R}^{[a,b]}$ is the [a, 6]-fold Cartesian product of $\mathbb{R}$. Let $x = \{x_1, \ldots, x_n\}$, where $x_i \in [a, b]$, and $y = \{y_1, \ldots, y_n\}$, where $y_i \in \mathbb{R}$, be finite subsets of [a, 6] and $\mathbb{R}$, respectively. For $\epsilon > 0$, we define a subset $B_{x,y,\epsilon}$ of *X* by

$$B_{x,y,\epsilon} = \{f \in X \mid |f(x_i) - y_i| < \epsilon \text{ for } i = 1, \ldots, n\}. \quad (1)$$

The topology of pointwise convergence is the smallest topology on *X* that contains the sets $B_{x,y,\epsilon}$ for all finite sets $x \subset [a, b]$, $y \subset \mathbb{R}$, and $\epsilon > 0$. We have $f_n \to f$ with respect to this topology if and only if $f_n(x) \to f(x)$ for every $x \in [a, b]$. If $f \in X$ and $y_i = f(x_i)$, then the sets $B_{x,y,\epsilon}$ form a neighborhood base for $f \in X$. This topology is not first countable.

The set $B_{x,y,\epsilon}$ in (1) is called a *cylinder set.* It has a rectangular base

$$(y_1 - \epsilon, y_1 + \epsilon) \times (y_2 - \epsilon, y_2 + \epsilon) \times \ldots \times (y_n - \epsilon, y_n + \epsilon)$$

in the $x_1, x_2, \ldots, x_n$ coordinates, and is unrestricted in the other coordinate directions. More picturesquely, $B_{x,y,\epsilon}$ is sometimes called a "slalom set," because it consists of all functions whose graphs pass through the "slalom gates" at x_i with radius ϵ and center y_i.

A base for the topology of pointwise convergence is given by all finite intersections of sets of the form $B_{x,y,\epsilon}$. In fact, it is sufficient to take the sets of the the form

$$\{f \in X \mid |f(x_i) - y_i| < \epsilon_i \text{ for } i = 1, \ldots, n\} \quad (2)$$

where $n \in \mathbb{N}$, $\{x_1, \ldots, x_n\} \subset [a, b]$, $\{y_1, \ldots, y_n\} \subset \mathbb{R}$, and $\epsilon_i > 0$. The sets of functions in (2) with intervals of variable width $\epsilon_i > 0$ generate the same topology as the sets with intervals of a fixed width because $B_{x,y,\epsilon}$ with $\epsilon = \min \epsilon_i > 0$ is contained inside the set in (2).

We say that a topological space $(X, \mathcal{T})$ is *metrizable* if there is a metric on *X* whose metric topology is $\mathcal{T}$. For a metrizable space, we can give sequential characterizations of compact sets, closed sets (Proposition 1.41), and continuous functions. These sequential characterizations may not apply in a nonmetrizable topological space.

There is, however, a generalization of sequences, called *nets*, that can be used to express all the above properties in an analogous way. We will not make use of nets in this book.

For example, the closure $\overline{A}$ of a subset A of a topological space *X* is the smallest closed set that contains *A.* If *X* is metrizable, then $\overline{A}$ is the set of limits of convergent sequences whose terms are in *A*, but if *X* is a not metrizable, then this procedure may fail. We call the setof limit points of sequences in *A* the *sequential closure* of *A* and denote it by $\overline{A}^S$. The sequential closure is a subset of the closure, but it may be a strict subset, as illustrated by the following example.

Example 5 Consider the space of all functions $f : [0,1] \to \mathbb{R}$ with the topology of pointwise convergence. For each $m, n \geq 1$, we let

$$f_{m,n}(x) = [\cos(m!\pi x)]^{2n}.$$

We define functions f_m and / by the pointwise limits,

$$f_m(x) = \lim_{n\to\infty} f_{m,n}(x) = \begin{cases} 1 & \text{if } x = k/m!,\ k = 0,\ldots,m!, \\ 0 & \text{otherwise,} \end{cases}$$

$$f(x) = \lim_{m\to\infty} f_m(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q}\cap[0,1], \\ 0 & \text{otherwise.} \end{cases}$$

Let $A = \{f_{m,n} \mid m, n \geq 1\}$. Then these limits show that

$$f_m \in \overline{A}^S, \qquad f \in \overline{\overline{A}^S}^S.$$

It is possible to show that the pointwise limit of a sequence of continuous functions on [0,1] is continuous on a dense subset of [0,1]. Since f is nowhere continuous in [0,1], it is not the pointwise limitof any subsequence of the continuous functions $f_{m,n}$. Therefore, $f \in \overline{A}$ but $f \notin \overline{A}^S$. This example shows that the topology of pointwise convergence on the real-valued functions on [0,1] is not metrizable.

A linear space with a topology defined on it, which need not be derived from a norm or metric, such that the operations of vector addition and scalar multiplication are continuous is called a *topological linear space,* or a *topological vector space.* The space of real-valued functions on a set with the topology of pointwise convergence is an example of a topological linear space. Topological linear spaces, such as the Schwartz space, also arise in connection with distribution theory.

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