

Fuzzifying the Socio-Economic Status of a Person

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Abstract – This paper makes an attempt to inquire, evaluate and interpret the “socio-economic world” of a person using fuzzy logic. In the context of socio-economic, a degree of poverty is analyzed to show a status of a person. Fuzzy membership functions are applied to the attributes relating to poverty measurement with reference to issues like income, consumption, education and employment, environment, gender disparity, equality, preferential options and even health conditions. Fuzzy multi-dimensional approach is proposed to determine to what extent someone is poor.

Key words: Socio-economic status, a degree of poverty, Fuzzy Logic, Fuzzy Sets, Fuzzy Subsets, Fuzzy membership functions, poverty line fuzzy multi-dimensional, etc.

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1. INTRODUCTION

The socio-economic status of a person is a combination of a multiple factors such as Income, education, employment, health status, social capital, housing condition, social security experienced by an individual or a community etc. High or low level of socio-economic status affects the freedom of a person to achieve one's functioning in society. The socio-economic status can be altered by an individual or by the descendant of a family for better or worse depending on the capabilities available at his or her disposal.

Lofti A. Zadeh's (1965) fuzzy theory and fuzzy logic were used as a great analytical instrument to describe and evaluate ideas in socio-economic domain. So far, poor people have been categorized based on poverty line that is to say - income, consumption and expenditure.

This paper has three sections namely, the first section deals with the historical development of the concept of fuzzy logic and its needs today. Section two highlights the fuzzy sets and its applications to socio-economic problem namely, poverty and its estimation. And the section three contains a case study - a fuzzy measure of poverty in a rural context of Nalanda district, Bihar. The paper concludes with the interpretation of result of the fuzzy socio-economic status measures.

1.1 A Historical Development of the Fuzzy Logic

Fuzzy logic is a precise theory for dealing with imprecision and vagueness. It is logic of approximate reasoning. Zadeh says in his research paper that it is viewed as an attempt to formalization and mechanization of two remarkable human capabilities. (i) The capacity to converse, think and conclude in a logical climate of imprecision, ambiguity, knowledge incompleteness, conflictivity, perception of fact and probability-in essence, in an setting of imperfect facts. (ii) The capacity to undertake a broad range of physical and mental activities without weighing or estimating them.

1.2 Fore Runners to Fuzzy Logic

The development and origin of fuzzy logic lies in the ancient Greek philosophy. Philosophers have developed a concise theory of logic. The laws of reasoning are established.

Aristotelian logic

One of the laws of thought of Aristotle was (i) The *Law of the Excluded Middle* which states that every proposition must *either be True or False* which means it excludes the possibility of having a logic value other than true or false. That means there is nothing in between. This further states that a proposition can neither be affirmed nor denied i.e.

a proposition is true or its negation is true.(ii) The Extended Contradiction Rule which states that a argument cannot be right and incorrect.

Mathematical Interpretations

There is a bivalent ways of thought in the science world. Scientific claims must be valid or incorrect. The two- valued logic is also known as a classical Aristotelian logic or first- order of logic.

1. The law of the excluded middle: Every statement is accurate or wrong. It can be represented according to mathematics: $X = A \cup \text{not } A, (\text{Either } A \text{ Or } \text{not } A)$; Symbolically, $A \cup \bar{A} = X$

Either Socrates is mortal or it is not the case that Socrates is mortal That law includes Socrates' middle position, which is neither divine nor human. Therefore, either the first option (Socracy is deadly) or its negation (Socrates is not mortal) must be valid.

2. The law of Extended Contradiction: No statement can be true and false simultaneously. $X = [\text{not}(A \cap \text{not } A)]$, Symbolically, $A \cap \bar{A} = \emptyset$

Thus all classes are assumed to have sharply defined boundaries. Either an object is a member of a class or it is not a member of a class. But most classes in the real world do not have sharp boundaries. For example if we consider characteristics or properties like, tall, intelligent, beauty, happiness, tiredness, and so on, all of these characteristics lack sharp boundaries. Therefore, the classical Aristotelian logic is not designed to deal with these cases.

Parmenides and Heraclitus

These philosophers raised an objection to the two valued classical logic around 400 BC. Parmenides proposed the first revision of these laws. There were strong and immediate objections. Heraclitus also stated that facts may be valid at the same time and not real.

Plato

Plato, the famous philosopher, established the groundwork for what became considered uncertain logic [4]. He claimed that when the antithesis rumbled there was a gray region between Truth and False. Later such thinkers as Hegel, Marx and Engels, echoed the same idea in their semantic works.

Lukasiewicz: Multi-valued Logic

It was Lukasiewicz, a Polish mathematician in the year 1920, who first proposed and established a

coherent alternative to Aristotle's two-value theory. He identified three of them, which were respected logic and mathematics. The third value he proposed should be labeled 'Possible' and a numerical value should be given between 1 and 0. In 3-valued logic, everything is same as in the 2-valued logic, except for the addition of the third truth value: True, May be (intermediate) and false. These three linguistic values are represented by as follows:

$$\left\{1, \frac{1}{2}, 0\right\}, \text{True} = 1,$$

$$\text{May be or intermediate value} = \frac{1}{2}, \text{False} = 0$$

Finally he hoped to derive four- valued logic, five – valued logic and then declared that nothing could preclude the derivation of an infinite-assessed concept in theory. But at last he settled down on a four valued logic because it seemed to be most easily suitable to Aristotelian logic.

Donald Ervin Knuth

Knuth also proposed a three valued logic similar to Lukasiewicz. He from Lukasiewicz hypothesized that the dynamics of the classical two-precious or reactive –will be much more elegant than. He used the integral range $[-1, 0 + 1]$ but then this alternative failed to gain acceptable responses.

Max Black

In 1937 Black presented his concept fuzzy package, which suggested the continuum's vagueness. He said that continuum need not be continuous, but could be discrete like a dotted line. He proposed to estimate a graph by a separate line with a number given by each point.

Lotfi A. Zadeh: An Infinite –Valued Logic (A Paradigm Shift)

Fifty years ago, Lotfi A. Zadeh who had the courage and the gift to begin the grand paradigm shift. The father of fuzzy logic is named. Dissatisfied with the use of very precise mathematics to describe the imprecise real world, in 1965, the notion of an infinite –valued logic was introduced by him. In his significant work, "Fuzzy Sets", He defined the theory and extension mathematics of fuzzy set: fuzzy logic. His seminal work challenged the Aristotelian Classical logic giving a way for fuzzy logic.

Fuzzy logic or fuzzy set theory proposed "the membership function" to operate over the range of real numbers $[0, 1]$. It is a generalization of the classical logic. The underlying principle of this angry reasoning is that statistics are not essential components of human thought. The principle of fuzzy set provides a statistical ability to grasp the

complexity in human cognitive functions, including perception and reasoning.

The creation of flimsy reasoning was primarily inspired by the need for a logical structure to tackle ambiguity and lexical inaccuracy. In fuzzy logic we deal with fuzzy quantifiers, like most, few, many, and several. These fuzzy quantifiers are qualitative in nature and not quantitative.

1.2 Philosophy of Fuzzy Logic

All is a question of degree of fuzzy logic; even truth is a matter of degree. So it is proper to say that something is 'quite true' or 'it's more or less true'. Fuzzy logic offers a structure that is robust and descriptive enough for the semanticization of natural language to be a functional construct.

Fuzzy logic discusses an abstract though not exactly constructed rationale from classical predications. Fuzzy logic is capable of handling inherently or intrinsically imprecise concepts. It allows linguistic forms like slightly, quite, and very etc. Fuzzy logic is thus driven by the desire to collect and reflect real world data with ambiguity. In the most imprecise situations, smooth and classical logic will not provide sufficient account.

Zadeh challenged the very laws of thought namely (i) the law of excluded middle and (ii) the law of Contradiction. (i) The law of excluded middle: The

Union of the fuzzy set $\tilde{A}$ and its complement $\tilde{A}^c$ – does not lead to the fundamental set again. This is represented mathematically $\tilde{A} \cup \tilde{A}^c \neq X$ (ii) The law of Contradiction: The Intersection of the fuzzy set $\tilde{A}$ and its complement $\tilde{A}^c$, is not an empty set. This is mathematically expressed as $\tilde{A} \cap \tilde{A}^c \neq \emptyset$

2. CONCEPT OF FUZZY SET

A Fuzzy package includes components that differ in the sets. What Zadeh proposed was embraced first in the Far East and later implemented widely across the world. The composition of items to a collection is examined bilaterally in classical set theory: an object is either part of the set or does not. The fuzzy set theory then allows for the incremental joining of elements to a group, which is represented in the actual unity interval $[0, 1]$ by the help of a membership function. A fuzzy set, in particular, is any set that enables its members in interval $[0, 1]$ to be of different membership grades (member function). Mathematically it is defined as follows:

2.1 Definition of a Fuzzy Set:

A fuzzy $\tilde{A}$ set in X is expressed as a set of ordered pairs defined

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X, \mu_{\tilde{A}}(x): X \rightarrow [0, 1]\}$$

Mainstreaming (MF) function is X on $[0, 1]$ mapping. Within a given fuzzy package, it gives the degree of membership of an entity.

Zadeh added an item in the set or a function of an item in a set which was referred to

$$\begin{aligned} \mu_{\tilde{A}}(x) &= 1 \text{ if } x \in A \\ &= 0 \text{ if } x \notin A \\ &= (0, 1) \text{ along the boundary} \end{aligned}$$

The limit is broken in fuzzy subsets and an entity will slowly switch from association to non-appearance in Set A .

2.2 The essential characteristics of fuzzy logic

- In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
- In fuzzy logic everything is a matter of degree.
- Any logical system can be fuzzified.
- In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.
- Inference is viewed as a process of propagation of elastic constraints.

There are two main characteristics of fuzzy systems that give them better performance for specific applications.

- Fuzzy systems are suitable for uncertain or approximate reasoning, especially for the system with a mathematical model that is difficult to derive.
- Fuzzy logic allows decision making with estimated values under incomplete or uncertain information.

Four principal facets of fuzzy logic: 1. The fuzzy-set-theoretic facet, FLS; 2. The logical facet, FLI; 3. The epistemic facet, FLE; and 4. The relational facet, FLR.

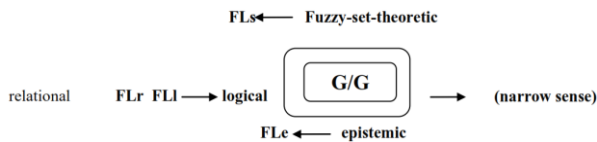


Fig: 1The core of fuzzy logic - FL is Graduation/Granulation, G/G.

The key features of the fugitive-logic are the fundamental principles of graduation and granulation which are central of FL. More importantly, all is graduated in abstract logic or, equivalently, it is lawful. This is a graduate problem. Moreover, everything is granulated or permitted in fuzzy logic, with a granule being a clump of attribute values, uniformity, consistency or usefulness. Age, for example, is granulated by defining its ideals as young, medium-aged and elderly. Linguistic variables may be viewed as granulated variables whose granular values are linguistic labels of granules. Graduation and granulation play an important role in human understanding in a qualitative way. Example as follows in the given picture below:

Continuous $\rightarrow$ quantized $\rightarrow$ granulated

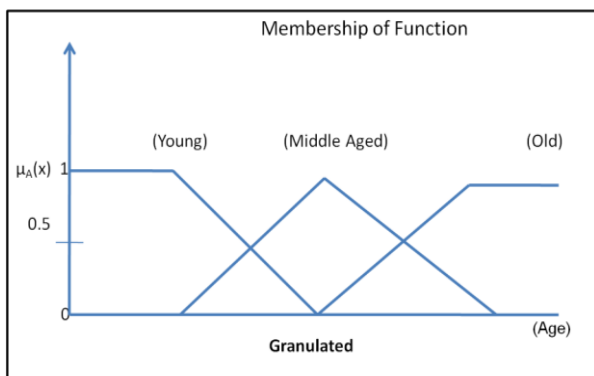


Fig: 2Granulation of Age: Young, middle aged and old are linguistic (granular) values of age.

2.3 Applications to Socio-Economic Status of a person

2.3.1 Poverty Context

A person who is poor implies poverty as lack of security, low wages, lack of employment opportunity, poor nutrition, poor access to safe drinking water, having too many children to feed, children being engaged in work to bring money to a family, poor educational opportunities, and misuse of resources etc. whereas, for a non-poor person poverty is a lack of income. According to Dr. Suresh Tendulkar, a person with an income rupees 27.00 a day in a rural area and a person with an income rupees 33.33 in an urban area are considered non-poor. Poverty line divides a set of poor into two Sets which is absurd. Since poverty line is based on the concept of Crisp Set.

Mozaffar Qizilbash describes deprivation as a ambiguous term in this sense [6]. Thus we propose to measure the *degree of poverty* incorporating multidimensional aspects of deprivation into the definition.

2.3.2 Poverty Set: A matter of degree

Poverty collection can be described according to the fluid logic principle. In fuzzy logic a statement can be possibly true to a certain degree. The degree of certainty is called the "truth value". As Zadeh claims that the theory of Fuzzy is essentially a theory of graded definitions—a theory with an elasticity of any or more.

Therefore, the poor individual or a household are assigned a degree in relation to the membership functions. A disadvantaged citizen with the lowest and 0 (not the poorest) values in a given range is allocated with a differing degree. In mathematical terms it can be represented as follows:

False: Truth value = 0, True: truth value =1, Uncertain: $0 < \text{Truth value} < 1$.

2.3.3 Poor: A Vague Predicate

Bad is a loosely term for: I it includes specific cases (an offender is not obviously poor and not obviously not poor), (ii) there are strict limits (along a presumption that a poor person does not even exist);

2.4 Review of Literature: Approaches to Poverty Assessment

Many studies have been made on poverty assessment. For the sake of brevity we highlight only a few to them.

2.4.1 Traditional Approach

The traditional approach defines the poor as all those individuals or households who fall below a critical level of minimum standard of living called the poverty line. Those individuals or families over the poverty line are deemed to be non-poor.

2.4.2 Features of the traditional approach

There are two distinct aspects of the current approach to deprivation calculation. (i) Uni-dimensional: standardized, since only one measure or one aspect of deprivation has been taken into account. And this one dimension, as income and consumption / expenditure, also is money-metrical. Revenue is used as the way to achieve basic living conditions while use shows that the essentials are currently obtained. (ii) Axis of poverty: this method splits people in two groups: poor and non-poor. The researchers or policymakers choose the poverty line, depending on the aim of their study or

purpose of the proposed programs. It could be absolute, relative or subjective or any combination of these. For example, Dr. Suresh Tendulkar, former strategy leader of India's Commission, preferred deprivation as its agenda aimed at providing disadvantaged citizens in the world the benefits of government services.

Methods: The traditional approach uses mainly three following methods to measure poverty.

- The headcount ratio, also called the poverty rate
- The poverty gap or poverty ratio or income gap and
- An index measuring the severity or intensity of poverty

The poverty line scale is the foundation of such approaches.

Shortcoming of the Traditional approach

- (i) It studies only one dimension of poverty at a time, There is a strong understanding, though, that there are several factors that lead to poverty.
- (ii) This distinctly separates the poor from the affluent. In reality, though, the boundary is breached.
- (iii) The lateral vagueness of suffering cannot be identified. To overcome the shortcomings of traditional approach, the multi-dimensional approach was developed by the Nobel laureate Dr. Amartya Sen.

2.4.3 Multidimensional Approach

Poverty is a multidimensional condition where only one element of poverty is to be observed. poverty metrics rely not on a particular variable or uniformity, but on various variables such as income or usage. The following seminal work was developed by Prof. Amartya Sen. They are summarized below:

Headcount Ratio: The proportion of a country's population below the poverty line is expressed as

$H = q/N$, where q is the percentage of people lining below the poverty line and N is the total population.

Poverty Gap: The gap between income accruing to the bottom (poor) and the top (rich) segment of the population. Wider the gap, the greater is the inequality in the distribution of income. Poverty Gap = $(Z - X_p)/Z$. where Z represents the poverty line and X_p represents the average consumer expenditure of the poor.

Severity of poverty: This method is a combination of the poverty rate, poverty gap and income inequality. It is commonly known as FGT (Foster-Greer-Thorbecke) method. It is expressed by the

$$P_{\alpha}(y, z) = \frac{1}{n} \sum_{i=1}^q \left(\frac{z - y_i}{z} \right)^{\alpha}$$

following formula:

where Z is the poverty line, y_i is the income of i^{th} household and q the number of household

where $y_i \leq z$. The poverty rate is where $\alpha=0$, the poverty ratio when $\alpha=1$ and severity of poverty is measure when $\alpha=2$. The aggregate poverty gap is simply the poverty ratio multiplied by z and n .

The works of Dr. Amartya Sen on Capabilities and Functionings played a significant role in promoting the use of multi-dimensional approach to poverty measurement. In the words of Klasen this approach finds its appropriate interpretations:

"The multidimensional approaches have relied on work by Rawls, Sen and others emphasize that the shortage of essential goods (rawls) or fundamental capability (sen) is a cause for deprivation, both of these cannot be purchased with cash because these are not given in a market system. Financial resources, they contend are just one of several means to achieve well-being and therefore, efforts should be directed at measuring well-being outcomes, rather than focus on one its imperfect proxies." This approach certainly offers a broad and more accurate picture of poverty than the traditional approach.

Capability: Dr. Amartya Sen uses a word to imply liberty of a individual in the things. Thus, deprivation means loss of ability for him or her or failure.

Freedom: The argument from Dr. Amartya Sen says that there are several options required for society in order to meet its wishes. According to him development is not meaningful without freedom to choose. Thus, we can say that if a person has no freedom of choice then he is doomed to be in poverty.

Functioning's: How do people do or may do to the items that may hold or regulate those characteristics.

Later in 1997 UNDP, as an example of the multi-dimensional measure of deprivation in terms of working loss, launched HPI (Human Deprivation Measure). The HPI applies poverty of the world to a population's living condition for the fundamental measurements of life, namely good housing, educational success and life expectancy at birth.

The Multidimensional Approach thus tackles the notion of horizontal poverty vagueness with

multifaceted dimensions as the central poverty aspect of schooling, wellness, accommodation, air, water, jobs and protection.

Methodsof multidimensional poverty measurement

The multi-dimensional approach to poverty assessments explores multiple aspects of inequality in the condition of human life and provides a definition of absolute suffering for the elderly.

Dual approaches are used to classify individuals who are multidimensional bad. They are (i) a counting methodology and (ii) dual cutoffs.

- (i) Dual cutoff method: (a) Identify all individuals deprived in any dimension within a dimension cutoff. This is the first cutoff. This limit is set for increasing dimension and defines whether a individual is deprived of that dimension. (b) Identify who is multi-dimensionally poor which expresses cross dimensional cutoff and gives deprivation in at least one third of the weighted indicators. This is the second cutoff which describes deprivation in details.
- (ii) A counting technique: all the measurements are equivalent, the second cut off indicates how many measurements a individual is lacking and is called multidimensional bad. A counting technique. The counting method is defined as this similarly weighted strategy. After this identification process, the aggregation is carried out using natural extension of the Sabina Alkire and James Foster poverty measure in multidimensional space [13], [17]. It is constructed using the formula as mentioned below.

$$MPI = M_0 = H \times A,$$

where MPI refers to Multi-dimensional poverty indicators, M_0 refers to adjusted headcount, H is the percentage of people poor and implied in the $H = q / n$ formula (where q is the amount of poor and n is the total). A is the average percentage of weighted deprivation experienced by individuals, showing the severity of the formulation of multidimensional deprivation:

$$A = \left[\sum_{i=1}^q \frac{c_i}{N} \right] \div q$$

, where A is the total amount of disadvantaged people suffering from homelessness. A is determined by adding up each person's share of the overall poverty and by splitting the sum of the needy.

Shortcoming of the multidimensional approach

Despite its elegant contributions to poverty measures it has many problems associated with this approach

including difficulties in aggregating deprivations from specific origins in various sizes or measurements. They are as follows: (i) one of the major shortcomings is that there is no consensus on what dimensions of well-being should be included in poverty analysis. (ii) There are no established criteria or methods for calculating multidimensional poverty. (iii) It has problem with regard to weights that the different dimensions contribute to overall poverty. Many aspects definitely lead to deprivation rather than others. For example, the HDI assigns equal weights to the dimensions to measure deprivation of a household or an individual. (iv) It fails to capture the vertical vagueness of poverty.

However, multidimensional approach offers more advantages than the traditional approach. The approach used in this paper falls under multidimensional approach because it includes various dimensions of poverty simultaneously. The Fuzzy approach also includes vertical and horizontal vagueness of poverty for a better conclusion or approach to the measurement of poverty problem.

2.4.4 The Fuzzy Approach

The first attempt to apply the Fuzzy concepts to Multi-dimensional poverty measures were made by Andréa Cerioli and Sergio Zani in 1990. They criticized the traditional approach as well the multi-dimensional approach and proposed a new approach: a Fuzzy approach. Our key critiques are::

1. The estimation of individual earnings is always imprecise, primarily due to the absence of correct details for respondents. A self – employed person like a tailor or a mason may not be able to indicate his/her income. It varies with a large difference from month to month. As a consequence, traditional income based indices may result in incorrect findings.

2. The abrupt distinction between poverty and non-poverty provided by Poverty Line (Rs. 5000.00) cut-off seems unrealistic. A radical transformation from severe deprivation to prosperity is closer to reality.

In order to overcome the above drawbacks many researchers suggested different approaches as to the measurement of poverty starting from 1990 to 2005.

Totally, the concept of the Fuzzy and Relatives (TFR) method to poverty reduction and social isolation was developed by Cheli and Lemmi and updated by Betti et al. (2005).

The Italian household survey results of 1994 used Chiappero-Martinetti in 2002 to support the Fuzzy Set hypothesis approach to assess the well-being of organizational and capability spatials. The study measured five areas of functionings such as health,

education, knowledge, social interaction and psychological conditions.

The implementation of this approach has been developed by a number of authors. The "time element," especially the transformation matrices method, has been focussed more on by Cheli and Betti and others (1999) and Betti et al (2005). Betti and Verma (1999, 2002, and 2004) and Verma and Betti (2002) refined their methodology to catch multi-dimensional aspects and to establish "manifest" and "latent" deprivation definitions to represent cross-section and union between different dimensions.

Totally Fuzzy (TF) Method

In 1990, Andréa Cerioli and Sergio Zani suggested the completely Fuzzy (TF) system. They said that fluffy environments require more than one level of deprivation to be used to assess a person's status, because the measurements are essentially the "degree to which individuals" in each dimension belong to the group of the disadvantaged. A poverty measure that indicates the cumulative poverty of each household according to its atmosphere is the cumulative membership feature. To assess person or household membership according i to the predictor j . They suggested to define two threshold values such as $j_{\min}$ and $j_{\max}$ such that if j for and individual is smaller than the $j_{\min}$ the person would be defined as definitely poor while if j is higher than $j_{\max}$ then the person is definitely not poor. If the individual's or household's deprivation were to fall between these two levels, the membership function will be between x_{ij} , $j_{\min}$ and $j_{\max}$. Thus, Andréa Cerioli and Sergio Zani's interpretation of membership feature is as follows:

$$\mu_j(i) = \begin{cases} 1 & \text{if } x_{ij} \leq j_{\min} \\ \frac{j_{\max} - x_{ij}}{j_{\max} - j_{\min}} & \text{if } j_{\min} < x_{ij} < j_{\max} \\ 0 & \text{if } x_{ij} \geq j_{\max} \end{cases}$$

Totally Fuzzy and Relative (TFR) Method

In 1995, Chelli and Lemmi claimed that there were two shortcomings in the completely fuzzy system. Firstly, it is unreasonable to pick two levels. Furthermore, there is little theoretical and observational support for the use of a linear model for the membership model. They argued to use a cumulative distribution function as the basis of membership function. That process was named "totally relative" since the member feature meaning is calculated entirely by the individual's relative place in

population distribution. They suggested the following membership formula:

$$\mu_j(i) = \mu_{j^{(k)}}(i) = \begin{cases} 0 & \text{if } k=1 \\ \mu_{j^{(k-1)}}(i) + \frac{F(j_i^{(k)}) - F(j_i^{(k-1)})}{1 - F(j_i^{(1)})} & \text{otherwise} \end{cases}$$

where $\mu_j(i) = 1 - F(j_i)$ or $\mu_j(i) = F(j_i)$ and k categories in them (j^k indicators k -th category of indicator j)

2.4.5 Need for Fuzzy multidimensional Approach

Fuzzy Multi-dimensional Approach (FMA) is proposed to analyze the status of a person based on socio-economic reality. This approach proceeds to link the crisp set of poverty measures with fuzzy set theory. It attempts to show where crisp set and fuzzy theories concur and differ. We add fuzzy method to the multidimensional poverty analysis because of the following reasons: (i) fuzzy method is able to handle both horizontal and vertical vagueness of poverty. (ii) It may evaluate multivariate relationships that generalize set-theoretical behaviors. (iii) Even categorical concepts often turn out to be a matter of degree. (iv) It combines set-wise thinking and continuous variables in a rigorous fashion..

2.4.6 Fuzzy Subset approach to Poverty Measurement

Consider a set E of n individuals or households and let $\underline{A}$ be a subset of E consisting of the poor, such that a fuzzy membership is given by

$\mu_{\underline{A}}(x_i)$ where $(i=1, 2, 3, \dots, n)$ denote for each individual or household in $\underline{A}$ and $\mu: \underline{A} \rightarrow [0, 1]$.

Then following critical limits in the given subset to identify the upper and lower bounds or grade or degree or membership or level of the poor.

- 1) $\mu_{\underline{A}}(x_i) = 0$ if i^{th} individual is certainly not poor;
- 2) $\mu_{\underline{A}}(x_i) = 1$ if i^{th} individual is poor;
- 3) $0 < \mu_{\underline{A}}(x_i) < 1$ if i^{th} individual exhibits a partial membership in the subset of $\underline{A}$.

Fuzzy attempts to reply: (i) How can memberships be assigned to items in a fuzzy package. (ii) How to adapt the definition of fuzzy sets to specific issues?

The first problem applies to the creation of a metric scale in order to satisfy the requirements in logical measurement schemes for membership values. This is achieved through participation.

Expression of Membership Function

A measurement of an individual's status is the value of the membership function $\mu_A(i)$. The membership function of an individual is derived by computing the degree of membership across the j^{th} functionings is given by the following formula:

$$\mu_A(x_i) = \frac{\sum_{j=1}^n x_j}{N(C_k) \times w_t}, \text{ where } w_t \in \{1, 2, \dots, n\} \quad [1]$$

The Degree of SES of $x_i = \frac{\text{Sum of the Actual Functionings Weightscore Achieved}}{\text{Total Number of Capabilities} \times \text{Functionings Weightscore}}$

where x_{ij} is the value of the membership functions for individual over the variable j^{th} or indicators. And w_t are the weights of the indicators across the each dimension set according to the indicator categories. C_k are the capabilities, $k = 1, 2, 3, \dots, n$. $N(C_k)$ is the number of capabilities chosen under consideration. $N=1, 2, 3, \dots, n$. and w_t are the functioning weight scores under considerations, where, SES refers to Socio-Economic-Status.

3. CASE STUDY

10- Individuals are selected from the primary surveyed data (Basanpur Village, Nalanda District, Bihar). They are represented by individual-1, individual-2...individual -10 respectively. 13-capabilities are taken such as land holding, housing conditions, normal clothings, food security, sanitation, means of livelihood, education, status of children, Market durables, category of debt, justification for migration, and preference for support in the results or metrics that will be used further for a validity of the fuzzy subset approach in measuring SES through poverty as determining factors for SES. These functionings or indicators are taken from National Sample Survey Organization (NSSO), Ministry of Rural Development, Government of India issued in 2011-12 for survey in Bihar, India.

3.1 Assignment of functioning scores to capability criteria: Table-1

S. No	Capabilities	0	1	2	3	4
1	Size group of operational holding of land	Nil	Less than 1 ha. of un-irrigated land (0.5 ha of irrigated land)	1 ha-2 ha of un-irrigated land (0.5 - 1.0 ha of irrigated land)	2 ha. 5 ha of un-irrigated land (1.0 - 2.5 ha of irrigated land)	More than 5 ha. of un-irrigated land (or 2.5 ha of irrigated land)
2	Type of house	Houseless	Kutcha or mud house	Semi- Pucca or semi-concrete	Pucca Concrete	Urban type of house
3	Average availability of normal wear clothing (per person in pieces)	Less than two	2 or more, but less than 4	4 or more, but less than 6	6 or more, but less than 10	10 or more
4	Food Security	Less than one square meal per day for major part of the year	Normally, one square meal per day, but less than one square meal occasionally	One square meal per day throughout the year	Two square meals per day with occasional shortage	Enough food throughout the year
5	Sanitation	Open defecation	Group latrine with irregular water supply	Group latrine with regular water supply	Clean group latrine with regular water supply and regular sweeper	Private latrine
6	Means livelihood of	Casual labour	Subsistence cultivation	Artisan	Salary	Others
7	Status of the Household Labour Force	Bonded Labour	Female and child labour	Only adult females and no child labour	Adult males only	Others
8	Literacy Status of the highest literate adult	Illiterate	Upto primary (Class V)	Complete Secondary (Passed X)	Graduate/ Professional Diploma	Post Graduate/ Professional Graduate
9	Status of children (5-14 years) any child	Not going to school, but working	Going to school and working	-----	-----	Going to School and not working
10	Ownership of consumer durables: TV, Electric fan, Kitchen Appliances like Pressure cooker, Radio	Nil	Any one	Two items only	Any three or all items	Ownership of any of the following: Computer, Telephone, Refrigerator, Colour TV, Electric Kitchen appliances, Expensive furniture, Transport vehicles

11	Type indebtedness of	For daily consumption purposes from informal sources	For production purpose from informal sources	For purpose other from informal sources	Borrowing only from institutional Agencies	No indebtedness and possess assets
12	Reason for migration from household region	Casual work	Seasonal employment	Other forms of livelihood	Non-migration	Other purposes
13	Preference of Assistance	Wage employment/TPDS (Targeted Public Distribution System)	Self-Employment	Training and Skill Up gradation	Housing	Loan/Subsidy more than Rs. One lakh or No assistance needed

The Total Score for an individual across all the 13- (thirteen Capabilities) will vary between 0 and 52.

($N(C_k) \times W_t = 13 \times 4 = 52$, i.e 13 Capabilities are multiplied by the maximum Functionings score).

For Individual -1, actual functioning score is

$$\sum_{j=1}^n x_j$$

calculated as
 $= 3+2+1+2+0+1+3+3+4+3+0+2+2 = 26$.

Similarly, the calculation is carried out for other individual cases and given in the table below:

Functioning's score achieved values: Table-2

Individuals	Total Functioning Score from the Survey	Functioning's score achieved
Individual -1	3+2+1+2+0+1+3+3+4+3+0+2+2	26
Individual -2	2+4+3+4+4+3+3+4+4+4+3+3+3	44
Individual -3	2+3+1+2+1+3+3+3+1+2+1+2+2	26
Individual -4	1+2+1+3+0+1+3+2+1+2+2+2+3	23
Individual -5	4+1+4+4+0+1+3+2+1+2+2+3+1	28
Individual -6	1+2+2+2+1+1+4+3+4+3+3+4+2	32
Individual -7	1+1+3+4+0+1+4+4+0+1+4+3+3	29
Individual -8	2+2+3+4+0+1+4+4+4+0+4+3+3	34
Individual -9	0+0+2+3+2+3+4+2+2+0+4+3+1	26
Individual -10	2+2+1+3+0+0+1+1+4+1+0+2+3	20

Fuzzification: Membership Function Calculations

Using the Formulae [1], The following Membership Values is calculated. For example:

$$\mu_{SES}(x_{ij=0.1,13}) = \mu_{SES}(x_{ij}) = \frac{\sum_{j=1}^n x_j}{N(C_k) \times w_i} = \frac{3+2+1+2+0+1+3+3+4+3+0+2+2}{13 \times 4} = \frac{26}{52} = 0.5$$

Therefore, from equation [1], we get the membership value for the individual-1 is 0.5 as the degree of poverty status.

In a similar fashion, all the calculations are done. By using the equations [1] we get the following membership values are given in the table below.

Fuzzy membership value table: Table-3

Number of Individuals	Membership Function Values
1	0.5
2	0.84
3	0.5
4	0.44
5	0.53
6	0.61
7	0.55
8	0.65
9	0.5
10	0.38

Socio-Economic Status has satisfaction score which follows fuzzy linguistic values as follows:

$$\mu_{SES}(x_{ij}) = \{ \text{very low, low, Average, High, Very High} \}$$

$$\mu_{SES}(x_{i=1,2,...,10}) = \{(1,0.5), (2,0.84), (3,0.5), (4,0.44), (5,0.53), (6,0.61), (7,0.55), (8,0.65), (9,0.5), (10,0.38)\}$$

Fuzzy Socio-Economic Status values (FSSES): Table-4

Number of Individuals, $\mu_{SES}(x_{i=1,2,...,10})$	Membership Function Values (Scores), $\{\mu_{SES}(x_{ij})\}$	Fuzzy linguistic Status Scores, $\{\mu_{SES}(x_{ij})\}$
1	0.5	Average
2	0.84	Extremely High
3	0.5	Average
4	0.44	Low
5	0.53	Average
6	0.61	High
7	0.55	High
8	0.65	Very high
9	0.5	Average
10	0.38	Very low

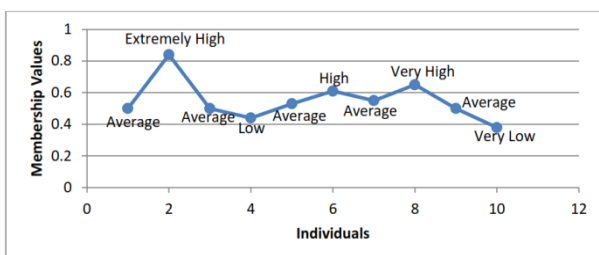


Fig: 3. Graphical representations of FSSES

4. CONCLUSIONS

RESULTS AND INTERPRETATIONS:

Using fuzzy subset membership function it is found that individual-10, has very low socio-economic status, Individual-4 has low status, and individual-1, individual-3, individual-5 and individual-9 have average socio-economic status, individual-6 and individual-7 have High socio-economic status and individual-2 and individual-8 have very high Socio-economic status.

Hence, any policy of eradication could be made according to their socio-economic status. SES through fuzzy poverty assessment basically tries to assess the level of an individual or a household to decide if it is the target group for the government assistance.

Using FMA approach, it is concluded that fuzzification by the Membership Function Approach (FMA) is able to handle vagueness, impreciseness and complexity, strengthening the connection between fuzzy subset theory and empirical poverty data analysis and socio-economic of a person. Hence, the title of this paper, "Fuzzifying Socio-Economic Status of a Person" is justified.

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