Study of Nonlinear Functional Differential Equations in Branch Algebras

Annu Yadav[1]* Dr. Ashwani Kumar[2]

[1] Research Scholar, Sunrise University, Alwar, Rajasthan

[2] Associate Professor, Sunrise University, Alwar, Rajasthan

Abstract – Presently the theory of nonlinear functional differential equation is a very active area of mathematical research and applications of theory are of fundamental importance in the formulation and analysis of various classes of operator's equation which arise in the physical, biological, engineering and technological sciences. Usually the mathematical models or the equations that have been used to describe a particular phenomenon or process of the universe contain a certain parameter, which has some specific physical interpretation, but whose value is not known. If any such phenomena involving so mentioned parameter and which satisfy certain probabilistic or measure theoretic behavior with respect to the parameter, then we say it is a random phenomenon.

Key Words – Nonlinear Functional, Differential Equation, Branch Algebras

---------------------------------x---------------------------------

INTRODUCTION

It should be remembered that the methods used for disrupting the dynamic structures defined by non linear differential and integral equations are beneficial for the nonlinear study. Any differential equations that describe a complex system do not have a solution, so it may be beneficial to interrupt these issues. The disturbed differential equations are classified into various forms. The hybrid differential equation (i.e. quadratic disruptions of a nonlinear differential equation) is a main form of disturbance. See for more facts and sources. Recently, much attention has been given to this issue. We note that the hybrid theory of fixed points is used to establish the theory of life in hybrid equations. We refer the reader to for more information. Hybrid differential equation of the first order:[1]

$$\begin{cases} \frac{d}{dt}\left[\frac{\theta(t)}{u(t,\theta(t))}\right] = v(t,\theta(t)), & \text{a.e. } t \in J, \\ \theta(t_0) = \theta_0 \in \mathbb{R}, \end{cases}$$

Where

$$J = [0,T),\ u \in C(J \times \mathbb{R}, \mathbb{R} \setminus \{0\})$$

And

$$v \in C(J \times \mathbb{R}, \mathbb{R})$$

They became the first writers to explain the nature of hybrid differential equations and the implications of similarity, as well as numerous fundamental inequalities. Indeed, with the principle of inequality, Dhage and Lakshmikantham proved the presence of severe solutions and a comparable consequence. In order to effectively model many physical phenoms, the theme of a fractional calculus dealing with derivatives and integrals of arBitrary orders was used. Obviously in the different fields of science and engineering, such as liquid flux, signalling and image analysis, fractal theory, control theory, electromagnetic theory, the fitting of experimental results, optics, potential theory, biochemical, chemical, propagation and viscoelasticity, multiple applications have been detected in this area. FHDEs have been added for some recent developments on the topic. The FHDE with R-L gap operators took the initiative[2]

$$\begin{cases} D^q\left[\frac{\theta(t)}{u(t,\theta(t))}\right] = v(t,\theta(t)), & \text{a.e. } t \in J, 0 < q < 1, \\ \theta(t_0) = \theta_0 \in \mathbb{R}, \end{cases}$$

Where

$$J = [0,T),\ u \in C(J \times \mathbb{R}, \mathbb{R} \setminus \{0\})$$

And

$$v \in C(J \times \mathbb{R}, \mathbb{R})$$

In addition to that, they proved the existence theorem for FHDEs with the mixed Lipschitz and Carathéodory conditions. Initial problems with values in hybrid fractional integro-differential equations were explored by Sitho et coll.:

$$\begin{cases} D^q[\frac{\theta(t)-\sum_{i=1}^{m} I^{\beta_i} w_i(t,\theta(t))}{u(t,\theta(t))}] = v(t,\theta(t)), \quad \text{a.e. } t \in J, \\ \theta(0) = 0 \in \mathbb{R}, \end{cases}$$

Where D^q denotes the R-L fractional derivative of order $q, 0 < \alpha \leq 1, I^{\phi}$ is the R-L fractional integral of order $\phi > 0, \phi \in \{\beta_1, \beta_2, \ldots, \beta_m\}, J = [0,T), u \in C(J \times \mathbb{R}, \mathbb{R} \setminus \{0\}),$ and $v, w \in C(J \times \mathbb{R}, \mathbb{R})$ with $w(0,0) = 0$.

Coupled systems analysis inside fractional differential equations is concerned, largely because such studies exist in a variety of recent problems and a two-point issue of the boundary value corresponding to a combined structure of fractional differential equations has been analysed. The basic solutions, namely the pairing of the nonlinear fractional reaction-diffusion equations. Furthermore, following problems of Dirichlet boundary importance of hybrid couplings:[3]

$$\begin{cases} {}^cD^{\delta}[\frac{x(t)}{f_1(t,x(t),y(t))}] = h_1(t,x(t),y(t)), \quad t \in J, 1 < \delta \leq 2, \\ D^{\gamma}[\frac{y(t)}{f_2(t,x(t),y(t))}] = h_2(t,x(t),y(t)), \quad t \in J, 1 < \gamma \leq 2, \\ x(0) = x(1) = 0, \qquad y(0) = y(1) = 0, \end{cases}$$

Where

$$f_i \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R} \setminus \{0\})$$

And

$$h_i \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R}), i = 1,2$$

Here, We are aiming at strategies for the fractional hybrid differential equations method

$$\begin{cases} D^p[\frac{\theta(t)-w(t,\theta(t))}{u(t,\theta(t))}] = v(t,\vartheta(t)), \quad t \in J, \\ D^p[\frac{\vartheta(t)-w(t,\vartheta(t))}{u(t,\vartheta(t))}] = v(t,\theta(t)), \quad t \in J, 0 < p < 1, \\ \theta(0) = 0, \qquad \vartheta(0) = 0. \end{cases}$$

First and foremost, we demonstrate a combined fixed point theorem that generalizes a fixed point Dhage theorem in Banach algebras.

DIFFERENTIAL EQUATION

A differential equation in mathematics is an equation connected to one or more functions. In implementations, functions are usually physical, derivatives reflect shifts in rates and the difference equation describes a relation between the two. This is common; thus, in many disciplines, including chemistry , physics, economics and biology, differential equations play a prominent place.[4]

The analysis of differential equations includes, above all, studying their solutions, and the properties of their solutions (the set of functions which satisfy each equation). Explicit formulas can solve only the simplest differential equations; but, without calculating them precisely, certain properties of solutions of a given differential equation can be calculated.

Many solutions may be approximated numerically using computers where a closed form expression for the solutions is not available. The theory of dynamic systems relies on qualitative study of the systems of differential equations, although several methods for numerical determination of solutions have been established with some accuracy.[5]

TYPES OF DIFFERENTIAL EQUATION

There can be separated into many groups with various equations. This groups of differential equations may help to guide the choice of a solution, in addition to explaining the properties of the equation itself. The common variations between ordinary / partial, linear / non-lineal, and homogenous / heterogeneous are included in the equation. There are also more properties and subclasseries that can be very helpful in particular situations. This list is not very comprehensive.[6]

Ordinary differential equations

An ordinary differential equation is a balance comprising an unknown component of a real or complex variable x, the derivatives of that variable, and some functions of x. Generally, the uncertain feature is defined by an x-dependent variable. Thus x is also referred to as the independent equation predictor. In comparison to the term partial differential equation there can be more than one independent variable, the term "common" is added.

Linear differential equations are differential equal to the unknown function and its derivatives, which are linear. Their philosophy is well known and their solutions in integrals may in certain situations be articulated.[7]

The bulk of ODEs in physics are linear. Many special functions may therefore be described as solutions for linear differential equations.

In general, since a closed-form expression cannot express the solutions of a differential equation, computational methods are widely used to solve differential equations in a machine.

Partial differential equations

The differential equation, which includes undefined multivariate functions and its partial derivatives, is part of a differential equation. PDEs are used to formulate problems involving many variables' functions and are either solved by means of closed solutions or are used to construct a related computer model. (This is counter to standard differential equations that deal with the functions and derivatives of a single variable.)

PDEs may be used in defining a large spectrum of natural processes, including acoustic, heat, electrostatic, electrodynamics, fluid flow, elasticity or quantum mechanics. The PDEs may also formally formalize certain apparently separate physical processes. As normal differential equations mostly model single-dimensional, multi-dimensional structures are often modeled by part-differential equations. Stochastic partial equations generalize randomness modeling through partial equations.[8]

Non-linear differential equations

A differential equation, which is not a linear equation with an undefined and derivative function, shall be a nonlinear differential equation (that is, linearity or nonlinearity are not seen in the function statements here). Exactly few approaches are possible to address nonlinear differential equations; usually established methods are based on the equation with specific symmetries. Nonlinear differential equations are typical of instability and can be very complex over longer periods of time. Also the fundamental issues concerning existence, singularity and extendibility of solutions for nonlinear differential equations and the accuracy of initial and limit value concerns for nonlinear PDEs have been severe problems and their resolution is considered a huge advance in mathematical theory in specific cases. If, therefore, a properly constructed representation of a meaningful physical mechanism is the difference equation therefore one expects a solution.

Linear differential equations also occur as nonlinear equations approximations. Such approximations are applicable only in minimal terms. For eg, the harmonic oscillator equation is a nonlinear pendulum equation approximation valid for tiny oscillations in amplitudes.[9]

Examples

In the first series of instances, u is an unfamiliar function of x, and c and − are known constants. Both ordinary and partial differential equations contain two large classifications, the distinction between linear and nonlinear differential equations and the differential and heterogeneous.

Standard differential equation Heterogeneous linear constant coefficient:

$$\frac{du}{dx} = cu + x^2.$$

Homogenous linear equation of the second order:

$$\frac{d^2u}{dx^2} - x\frac{du}{dx} + u = 0$$

Homogenous linear coefficient of second order ordinary difference equation defining the harmonic oscillator:

$$\frac{d^2u}{dx^2} + \omega^2 u = 0$$

Heterogeneous nonlinear differential equation of the first order:

$$\frac{du}{dx} = u^2 + 4$$

Second-order nonlinear equation defining the motion of a longitudinal pendulum L (due to sine function):

$$L\frac{d^2u}{dx^2} + g\sin u = 0$$

The uncertain function u relies on two variables x and t or x and y in the next collection of instances.

The homogeneous partial differential linear equation of first order:

$$\frac{\partial u}{\partial t} + t\frac{\partial u}{\partial x} = 0$$

Laplace theorem, a homogenous linear coefficient of second order, partial differential theorem of the Elliptic form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Homogeneous non-linear non-differential equation of third order:

$$\frac{\partial u}{\partial t} = 6u\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial x^3}$$

BANACH ALGEBRA

The Banach algebra, named after Stefan Banach, is, in mathematics, and particularly functional analysis, an associative algebra A above the actual or complex numbers, at the same time also a Banach space, a normal space that is comprehensive of the normally induced metric. The norm must be complied with.[9]

$$\forall x, y \in A : \|x\, y\| \leq \|x\|\, \|y\|$$

This guarantees a constant multiplication method.

Banach algebra is considered a unital algebra if it has the identity part of a multiplication of which the norm 1. Any Banach algebra A (whether it has an identity element or not) can be embedded isometrically into a unital Banach algebra A_e so as to form a closed ideal of A_e. Often one assumes a priori that the algebra under consideration is unital: for one can develop much of the theory by considering A_e and then applying the outcome in the original algebra. This isn't necessarily the case, though. In Banach algebra, for example, one cannot describe without an identity all the trigonometric functions.

True Banach algebras can vary greatly from the theory of complex Banach algebras. The continuum of an element in a nontrivial Banach algebra, for example, will never be vacant although it may be vacant for some elements of a genuine Banach algebra.

Banach algebras can be described over p-adic numbers fields as well. This is used in p-adic analysis.

DIFFERENTIAL AND INTEGRAL EQUATIONS IN BANACH ALGEBRAS

Dhage and Regan introduced and created the analysis for nonlinear differential equations in 2001.

Let $J=[0,T]$ be a closed and bounded interval in $\mathbb{R}$. Then the general form of the nonlinear ordinary differential equations in Banach algebra is[10]

$$L\left(\frac{x(t)}{f(t,x(t))}\right) = g\, x(t) \quad a.e. \quad t \in J \tag{1.1}$$

Subject to either some initial condition

$$x \in \mathcal{J} \tag{1.2}$$

Or some boundary condition

$$x \in \mathcal{B} \tag{1.3}$$

Where, the function

$$f : J \times \mathbb{R} \longrightarrow \mathbb{R} - \{0\}$$

Is continuous,

$$L : C^n(J, \mathbb{R}) \longrightarrow L^1(J, \mathbb{R})$$

Is an n^{th} order linear differential operator defined by

$$L = a_0 \frac{d^n}{dt^n} + a_1 \frac{d^{n-1}}{dt^{n-1}} + \ldots.. + a_n$$

For some continuous functions

$$a_i : J \rightarrow \mathbb{R}$$

And

$$N : C\,(J, \mathbb{R}) \longrightarrow L^1(J, \mathbb{R})$$

is a Nemetsky operator defined by

$$Nx = g\left(t, \frac{dx}{dt}, \ldots\ldots, \frac{d^{n-1}x}{dt^{n-1}}\right)$$

With

$$g : J \times \mathbb{R}^{n-1} \longrightarrow \mathbb{R}$$

The problem (1.1)-(1.2) is referred to as the original nonlinear differential equations problem and the problem (1.1)-(1.3) is named the nonlinear equations problem in banach algebras in nth order limit values.[6]

The first ordinary non-linear differential equation problem of initial values,

$$\left.\begin{array}{l} \dfrac{d}{dt}\left[\dfrac{x(t)}{f(t,x(t))}\right] = g(t,x(t)) \quad \text{a.e.} \quad t \in J \\ x(0) = x_0 \in \square \end{array}\right\} \tag{1.4}$$

Studied for life outcomes under acceptable conditions in Dhage and Regan. Likewise, the topic of standard non-linear differential first-order boundary value problem,

$$\left.\begin{array}{l} \dfrac{d}{dt}\left[\dfrac{x(t)}{f(t,x(t))}\right] = g(t,x(t)) \quad \text{a.e.} \quad t \in J \\ x(0) = x(T) \end{array}\right\} \tag{1.5}$$

Is in Dhage and addressed. A. via the Carathion principle with nonlinear differential equations and monotone principle. In addition, initial value problems of nonlinear differential equations in the ordinary second order

$$\left.\begin{array}{l}\dfrac{d^2}{dt^2}\left[\dfrac{x(t)}{f(t,x(t))}\right]=g(t,x(t)) \quad \text{a.e.} \quad t\in J \\ x(0)=x_0, \qquad x'(0)=x_1\end{array}\right\} \tag{1.6}$$

In Dhage it is also discussed whether drastic solution occurs and operates under some conditions of caratheodonicity and monotonicity. During this thesis several nonlinear differential equations in Banach algebras with original and boundary conditions are addressed. They expand and generalize previously established findings for the differential equations (1.4), (1.5) and (1.6).[5]

REFERENCES

[1] T. T. Soong (1973). Random Differential Equations in Sciences and Engineering. Academic Press, New York.

[2] V. Lakshmikantham, S. G. Deo, Ordinary differential equation. Tata McGraw Hill publishing company limited, New Delhi.

[3] G. F. Simmons and S. G. Krantz, Differential equation theory and technique and practice. Tata McGraw Hill publishing company limited, New Delhi.

[4] M. Joshi, R. K. Bose, Nonlinear functional anal ysis.

[5] G. Adomian (1967). Theory of random systems. Trans. 4" Prague Conf. Information Theory Statist. Decision Functions and Random Processes (1965), pp. 205-222.

[6] L. Arnold (1974). Stochastic Differential Equations: Theory and Application. Wiley, New York.

[7] P. B. Bailey, L. F. Shampine and P. E. Walkman (1968). Nonlinear Two Point Boundary Value Problems. Academic Press, New York and London.

[8] S. R. Bernfield and V. Lakshinikantham (1980). An Introduction to Nonlinear Boundary Value Problems. Academic Press, New York.

[9] A. T. Bharucha (1972). Reid, Random Integral Equations. Academic Press, New York.

[10] J. R. Blum (1965). Theory of Random Functions. Garden and Breach, New York.

Corresponding Author

Annu Yadav*

Research Scholar, Sunrise University, Alwar, Rajasthan