Journal of Advances and Scholarly Researches in Allied Education
Vol. 15, Issue No. 11, November-2018, ISSN 2230-7540

An Analysis upon the Geometrical Coordination Theory of Two Dimensions

Sikander*

Assistant Professor of Mathematics, A.I.J.H.M. College Rohtak, Haryana, India

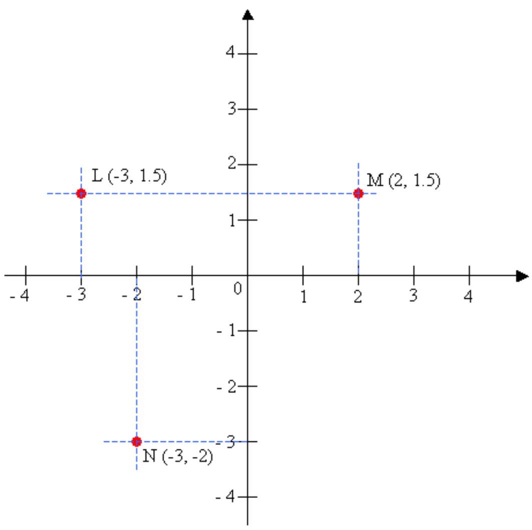
Abstract – Coordinate Geometry is viewed as one of the most intriguing ideas of arithmetic. Coordinate Geometry (or the expository geometry) depicts the connection among geometry and algebra through diagrams including curves and lines. It gives geometric angles in Algebra and empowers to take care of geometric issues. It is a piece of geometry where the situation of points on the plane is portrayed utilizing an arranged pair of numbers. Here, the ideas of coordinate geometry are clarified alongside its formulas and their determinations.

---------------------------------x---------------------------------

INTRODUCTION

The coordinate plane is an essential idea for coordinate geometry. It portrays a two-dimensional plane as far as two opposite tomahawks: x and y. The x-pivot demonstrates the flat bearing while the y-hub shows the vertical heading of the plane. In the coordinate plane, points are demonstrated by their situations along the x and y-tomahawks.

For example: In the coordinate plane underneath, point L is spoken to by the coordinates (– 3, 1.5) on the grounds that it is positioned on – 3 along the x-pivot and on 1.5 along the y-hub. Likewise, you can make sense of why the points M = (2, 1.5) and N = (– 3, – 2).



Slopes-

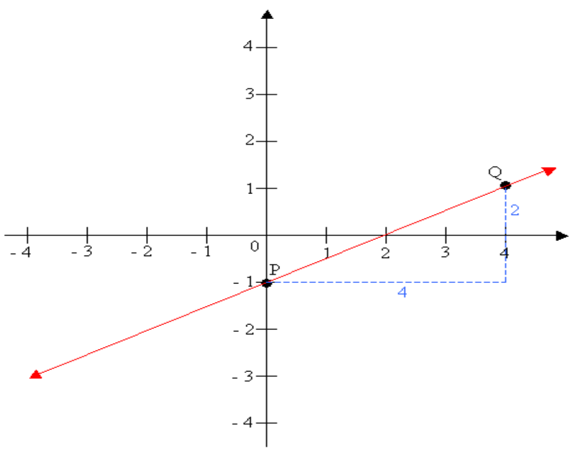
On the coordinate plane, the slant of a line is called the slope. Slope is the ratio of the change in the y-value over the change in the x-value.

Given any two points on a line, you can calculate the slope of the line by using this formula:

$$slope = \frac{change\ in\ y\ value}{change\ in\ x\ value}$$

For example: Given two points, P = (0, –1) and Q = (4,1), on the line we can calculate the slope of the line.

$$slope = \frac{change\ in\ y\ value}{change\ in\ x\ value} = \frac{1-(-1)}{4-0} = \frac{2}{4} = \frac{1}{2}$$



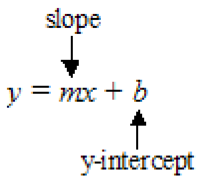
Y-intercept -

The y-intercept is where the line intercepts (meets) the y-axis.

For example: In the above diagram, the line intercepts the y-axis at (0,–1). Its y-intercept is equals to –1.

Equation Of A Line -

In coordinate geometry, the equation of a line can be written in the form, $y = mx + b$, where m is the slope and b is the y-intercept. (see a mnemonic for this formula)



For example: The equation of the line in the above diagram is:

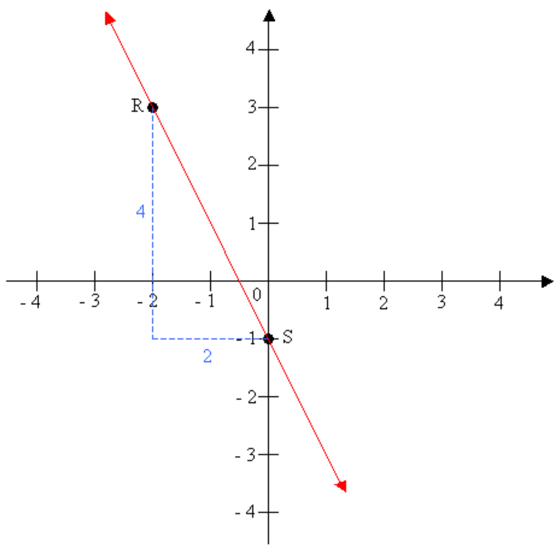
$$y = \frac{1}{2}x - 1$$

Negative Slope -

Let's look at a line that has a negative slope.

For example: Consider the two points, R(–2, 3) and S(0, –1) on the line. What would be the slope of the line?

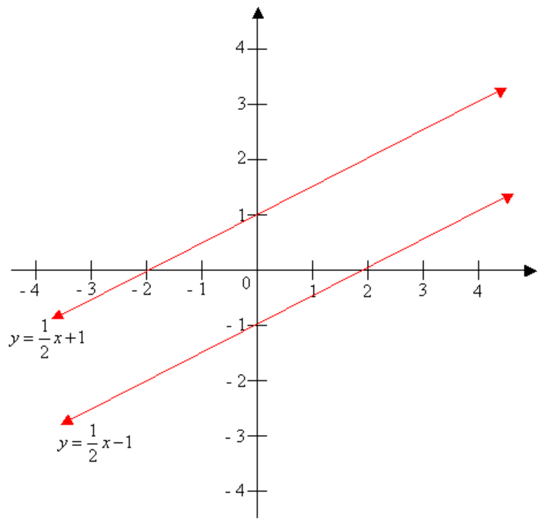
$$slope = \frac{change\ in\ y\ value}{change\ in\ x\ value} = \frac{-1-3}{0-(-2)} = \frac{-4}{2} = -\frac{2}{1}$$



The y-intercept of the line is –1. The slope is –2. The equation of the line is: $y = -2x - 1$

Slopes of Parallel Lines-

In coordnate geometry, two lines are parallel if their slopes (m) are equal.



For example: The line

$$y = \frac{1}{2}x - 1$$

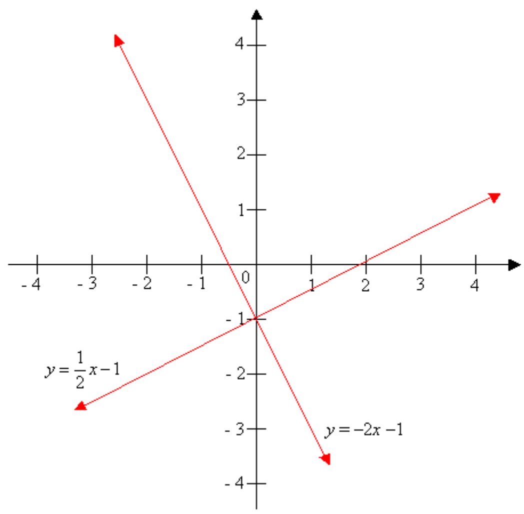
is parallel to the line

$$y = \frac{1}{2}x + 1.$$

Their slopes are both the same.

Slopes Of Perpendicular Lines-

In the coordinate plane, two lines are perpendicular if the product of their slopes (m) is –1.



For example: The line

$$y = \frac{1}{2}x - 1$$

is perpendicular to the line $y = -2x - 1$. The product of the two slopes is

$$\frac{1}{2}\times(-2)=-1$$

Midpoint Formula-

Some coordinate geometry questions may expect you to discover the midpoint of line sections in the coordinate plane. To discover a point that is somewhere between two given points, get the normal of the x-values and the normal of the y-values.

The midpoint between the two points (x_1,y_1) and (x_2,y_2) is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

For example: The midpoint of the points A(1,4) and B(5,6) is

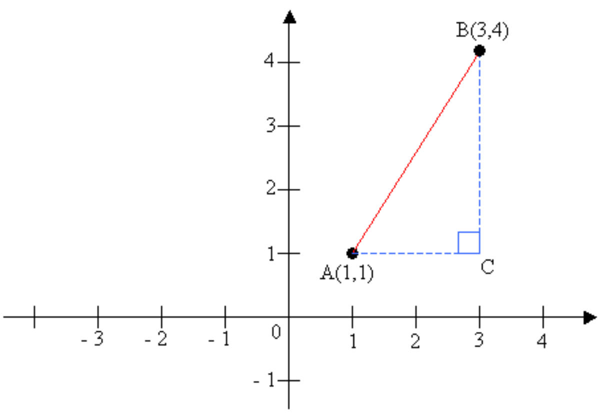
$$\left(\frac{1+5}{2},\frac{4+6}{2}\right)=\left(\frac{6}{2},\frac{10}{2}\right)=(3,5)$$

Distance Formula-

In the coordinate plane, you can use the Pythagorean Theorem to find the distance between any two points.

The distance between the two points (x_1,y_1) and (x_2,y_2) is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



For example: To find the distance between A(1,1) and B(3,4), we form a right angled triangle with $\overline{AB}$ as the hypotenuse. The length of $\overline{AC}$ = 3 – 1 = 2. The length of $\overline{BC}$ = 4 – 1 = 3. Applying Pythagorean Theorem:

$$\overline{AB}^2=2^2+3^2$$

$$\overline{AB}^2=13$$

$$\overline{AB}=\sqrt{13}$$

System of geometry in which points, lines, shapes, and surfaces are spoken to by algebraic articulations. In plane (two-dimensional) coordinate geometry, the plane is generally characterized by two tomahawks at right edges to one another, the flat x-pivot and the vertical y-hub, meeting at O, the starting point. A point on the plane can be spoken to by a couple of Cartesian coordinates, which characterize its situation as far as its separation along the x-pivot and along the y-hub from O. These separations are, individually, the x and y coordinates of the point.

Lines are spoken to as equations; for example, y = 2x + 1 gives a straight line, and y = 3x2 + 2x gives a parabola (a curve). The diagrams of shifting equations can be drawn by plotting the coordinates of points that fulfill their equations, and signing up the points. One of the benefits of coordinate geometry is that geometrical solutions can be acquired without illustration however by controlling algebraic articulations. For example, the coordinates of the point of crossing point of two straight lines can be dictated by finding the extraordinary estimations of x and y that fulfill both of the equations for the lines, that is, by understanding them as a couple of synchronous equations. The curves considered in straightforward coordinate geometry are the conic areas (circle, oval, parabola, and hyperbola), every one of which has a trademark equation.

COORDINATE SYSTEMS IN THE PLANE

When we talk about "the point with coordinates (x,y)" or "the curve with equation f(x,y)", we will dependably have as a top priority cartesian coordinates (Section 1.2). In the event that a formula includes another kind of coordinates, this will be expressed unequivocally.

Substitutions and Transformations-

Formulas for changes in coordinate systems can prompt disarray on the grounds that (for example) moving the coordinate tomahawks up has indistinguishable impact on equations from moving items down while the tomahawks remain fixed. (To peruse the following section, you can move your eyes down or slide the page up.)

To dodge disarray, we will cautiously recognize transformations of the plane and substitutions, as clarified underneath. Comparative contemplations will apply to transformations and substitutions in three dimensions (see Section 9.1).

Substitutions-A substitution**, or** change of coordinates**, relates the coordinates of a point in one coordinate system to those of *the same point in a different coordinate system*. Usually one coordinate system has the superscript ′ and the other does not, and we write**

$$\begin{cases} x = F_x(x',y'), \\ y = F_y(x',y') \end{cases} \qquad \text{or} \qquad (x,y) = F(x',y'). \qquad (1)$$

This means: given the equation of an object in the unprimed coordinate system, one obtains the equation of the *same* object in the primed coordinate system by substituting $F_x(x',y')$ for x and $F_y(x',y')$ for y in the equation. For instance, suppose the primed coordinate system is obtained from the unprimed system by moving the axes up a distance d. Then $x=x'$ and $y=y'+d$. The circle with equation $x^2+y^2=1$ in the unprimed system has equation $x'^2+(y'+d)^2=1$ in the primed system. Thus transforming an implicit equation in (x,y) into one in (x',y') is immediate.

The point $P=(a,b)$ in the unprimed system, with equation $x=a$, $y=b$, has equation $F_x(x',y')=a$, $F_y(x',y')=b$ in the new system. To get the primed coordinates explicitly one must solve for x' and y' (in the example just given we have $x'=a$, $y'+d=b$, which yields $x'=a$, $y'=b-d$). Therefore if possible we give the **inverse equations**

$$\begin{cases} x' = G_{x'}(x,y), \\ y' = G_{y'}(x,y) \end{cases} \qquad \text{or} \qquad (x',y') = G(x,y),$$

which are equivalent to (1) if $G(F(x,y))=(x,y)$ and $F(G(x,y))=(x,y)$. Then to go from the unprimed to the unprimed system one merely plugs the known values of x and y into these equations. This is also the best strategy when dealing with a curve expressed parametrically, that is: $x=x(t)$, $y=y(t)$.

Transformations-A transformation **associates to each point (*x*,*y*) *a different point in the same coordinate system*; we denote this by**

$$(x,y) \mapsto F(x,y), \qquad (2)$$

where F is a map from the plane to itself (a two-component function of two variables). For example, translating down by a distance d is accomplished by $(x,y) \mapsto (x,\ y-d)$ (Section 2). Thus the action of the transformation on a point whose coordinates are known (or on a curve expressed parametrically) can be immediately computed.

If, on the other hand, we have an object (say a curve) defined *implicitly* by the equation $C(x,y)=0$, finding the equation of the transformed object requires using the *inverse transformation*

$$(x,y) \mapsto G(x,y)$$

defined by $G(F(x,y))=(x,y)$ and $F(G(x,y))=(x,y)$. The equation of the transformed object is $C(G(x,y))=0$. For instance, if C is the circle with equation $x^2+y^2=1$ and we are translating down by a distance d, the inverse transformation is

$$(x,y) \mapsto (x,\ y+d)$$

(translating up), and the equation of the translated circle is $x^2+(y+d)^2=1$. Compare the example following (1) .

Using transformations to perform changes of coordinates-Usually, we will not give formulas of the form (1) for changes between two coordinate systems of the same type, because they can be immediately derived from the corresponding formulas (2) for transformations, which are given here (Section 2). We give two examples for clarity.

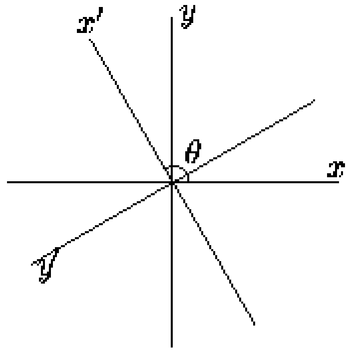


Figure 1: Change of coordinates by a rotation.

Let the two cartesian coordinate systems (x,y) and (x',y') be related as follows (Figure 1): they have the same origin, and the positive x'-axis is obtained from the positive x-axis by a (counterclockwise) rotation through an angle θ. If a point has coordinates (x,y) in the unprimed system, its coordinates (x',y') in the primed system are the same as the coordinates in the unprimed system of a point that undergoes the *inverse rotation*, that is, a rotation by an angle $\alpha=-\theta$. According to (2.1.1) , this transformation acts as follows:

$$(x,y) \mapsto \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}(x,y) = (x\cos\theta + y\sin\theta,\ -x\sin\theta + y\cos\theta). \qquad (3)$$

Therefore the right-hand side of (3) is (x',y'), and the desired substitution is

$$x' = x\cos\theta + y\sin\theta, \quad y' = -x\sin\theta + y\cos\theta.$$

Switching the roles of the primed and unprimed systems we get the equivalent substitution

$$x' = x\cos\theta + -y\sin\theta, \quad y' = x\sin\theta + y\cos\theta$$

(since the *x*-axis is obtained from the *x'*-axis by a rotation through an angle -θ).

Similarly, let the two cartesian coordinate systems (*x*,*y*) and (*x'*,*y'*) differ by a translation: *x* is parallel to *x'* and *y* to *y'*, and the origin of the second system coincides with the point (x_0,y_0) of the first system. The coordinates (*x*,*y*) and (*x'*,*y'*) of a point are related by

$$x = x' + x_0, \qquad x' = x - x_0,$$
$$y = y' + y_0, \qquad y' = y - y_0. \qquad (4)$$

Cartesian Coordinates in the Plane-

In **cartesian coordinates** (or **rectangular coordinates**), the ``address'' of a point *P* is given by two real numbers indicating the positions of the perpendicular projections from the point to two fixed, perpendicular, graduated lines, called the **axes**. If one coordinate is denoted *x* and the other *y*, the axes are called the ***x*-axis** and the ***y*-axis**, and we write *P*=(*x*,*y*). Usually the *x*-axis is drawn horizontal, with *x* increasing to the right, and the *y*-axis is drawn vertical, with *y* increasing going up. The point *x*=0, *y*=0 is the **origin**, where the axes intersect. See Figure 1.

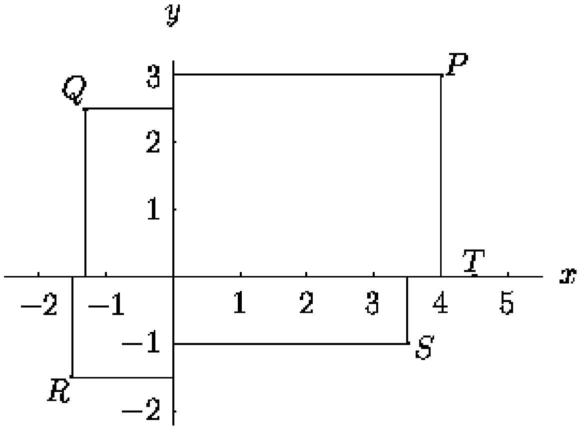


Figure 1: In cartesian coordinates, *P*=(4,3), *Q*=(-1.3,2.5), *R*=(-1.5,-1.5), *S*=(3.5,-1), and *T*=(4.5,0). The axes divide the plane into four **quadrants**: *P* is in the first quadrant, *Q* in the second, *R* in the third, and *S* in the fourth. *T* is on the positive *x*-axis.

Polar Coordinates in the Plane-

In **polar coordinates** a point *P* is also characterized by two numbers: the distance $r \geq 0$ to a fixed **pole** or **origin** *O*, and the angle θ the ray *OP* makes with a fixed ray originating at *O*, which is generally drawn pointing to the right (this is called the **initial ray**). The angle θ is only defined up to a multiple of 360° or 2π. In addition, it is sometimes convenient to relax the condition $r \geq 0$ and allow *r* to be a signed distance, so (*r*,θ) and (-*r*, θ+180°) represent the same point (Figure 1).

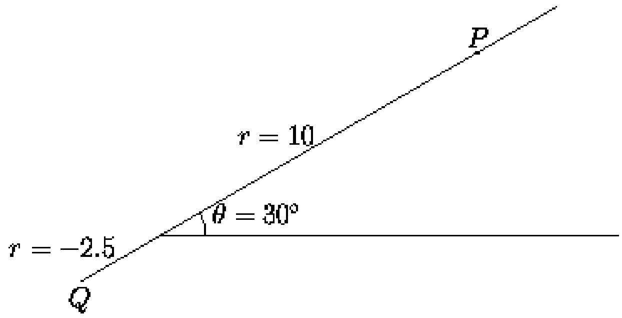


Figure 1: Among the possible sets of polar coordinates for *P* are: (10, 30°), (10, 390°) and (10, -330°). Among the sets of polar coordinates for *Q* are: (2.5, 210°) and (-2.5, 30°).

Relations between Cartesian and Polar Coordinates-Consider a system of polar coordinates and a system of cartesian coordinates with the same origin. Assume the initial ray of the polar coordinate system coincides with the positive *x*-axis, and that the ray θ=90° coincides with the positive *y*-axis. Then the polar coordinates (*r*,θ) and the cartesian coordinates (*x*,*y*) of the same point are related as follows:

$$\begin{cases} x = r\cos\theta, \\ y = r\sin\theta, \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2}, \\ \theta = \arctan\dfrac{y}{x}, \end{cases} \qquad \begin{cases} \sin\theta = \dfrac{y}{\sqrt{x^2+y^2}}, \\ \cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}. \end{cases}$$

Homogeneous Coordinates in the Plane-

A triple of real numbers (*x*:*y*:*t*), with $t \neq 0$, is a set of **homogeneous coordinates** for the point *P* with cartesian coordinates (*x*/*t*, *y*/*t*). Thus the same point has many sets of homogeneous coordinates: (*x*:*y*:*t*) and (*x'*:*y'*:*t'*) represent the same point if and only if there is some real number α such that *x'*=α*x*, *y'*=α*y*, *t'*=α*t*. If *P* has cartesian coordinates (x_0,y_0), one set of homogeneous coordinates for *P* is (x_0,y_0,1).

When we think about indistinguishable triple of numbers from the cartesian coordinates of a point in three-dimensional space (Section 9.1), we compose it (x,y,t) rather than (x:y:t). The association between the point in space with cartesian coordinates (x,y,t) and the point in the plane with homogeneous coordinates (x:y:t) winds up obvious when we consider the plane t=1 in space, with cartesian coordinates given by the initial two coordinates x, y of room (Figure 1). The point (x,y,t) in space can be associated with the starting point by a line L that converges the plane t=1 in the point with cartesian coordinates (x/t, y/t), or homogeneous coordinates (x:y:t).

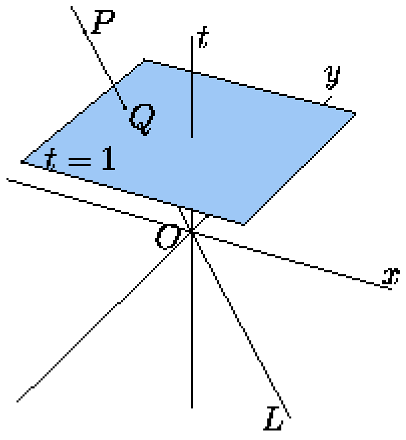


Figure 1: The point P with spatial coordinates (x,y,t) tasks to the point Q with spatial coordinates (x/t, y/t, 1). The plane cartesian coordinates of Q are (x/t, y/t), and (x:y:t) is one lot of homogeneous coordinates for Q. Any point on hold L (aside from the beginning O) would likewise extend to P'.

Projective coordinates are helpful for a few reasons, one the most significant being that they enable one to bring together all symmetries of the plane (just as different transformations) under a solitary umbrella. Every one of these transformations can be viewed as linear maps in the space of triples (x:y:t), thus can be communicated as far as matrix augmentations. (See Section 2.2).

In the event that we think about triples (x:y:t) with the end goal that at any rate one of x, y, t is nonzero, we can name the points in the plane as well as points "at limitlessness". In this way (x:y:t) speaks to the point at unendingness toward the line with slope y/x.

Oblique Coordinates in the Plane-

The accompanying generalization of cartesian coordinates is now and then valuable. Think about two tomahawks (graduated lines), meeting at the birthplace yet not really perperdicularly. Give the edge between them a chance to be . In this system of diagonal coordinates, a point P is given by two genuine numbers showing the places of the projections from the point to every pivot, toward the different hub. See Figure 1. The primary pivot (x-hub) is commonly drawn on a level plane. The case =90° yields a cartesian coordinate system.

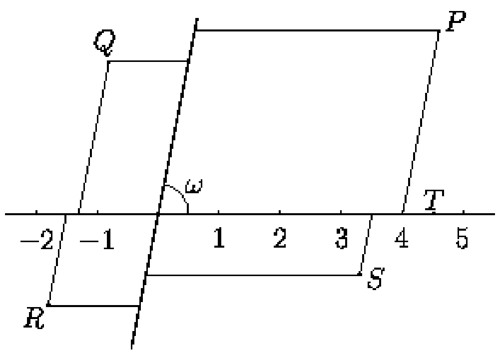


Figure 1: In oblique coordinates, *P*=(4,3), *Q*=(-1.3,2.5), *R*=(-1.5,-1.5), *S*=(3.5,-1), and *T*=(4.5,0). Compare Figure 1.2.1.

OTHER TRANSFORMATIONS OF THE PLANE

Similarities-

A transformation of the plane that jelly shapes is known as a comparability. Each closeness of the plane is gotten by creating a corresponding scaling transformation (otherwise called a homothety) with an isometry. A relative scaling transformation focused at the birthplace has the structure

$$(x, y) \mapsto (ax, ay),$$

where $a \neq 0$ is the scaling factor (a real number). The corresponding matrix in **homogeneous coordinates** is

$$H_a = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In **polar coordinates**, the transformation is

$$(r, \theta) \mapsto (ar, \theta).$$

Affine Transformations-

A transformation that jelly lines and parallelism (maps parallel lines to parallel lines) is a relative transformation. There are two significant specific instances of such transformations:

A no proportional scaling transformation focused at the starting point has the structure

$$(x, y) \mapsto (ax, by),$$

where $a, b \neq 0$ are the scaling factors (real numbers). The corresponding matrix in **homogeneous coordinates** is

$$H_{a,b} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

A **shear** preserving horizontal lines has the form

$$(x, y) \mapsto (x + ry, y),$$

where *r* is the shearing factor (see Figure 1). The corresponding matrix in **homogeneous coordinates** is

$$S_r = \begin{bmatrix} 1 & r & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

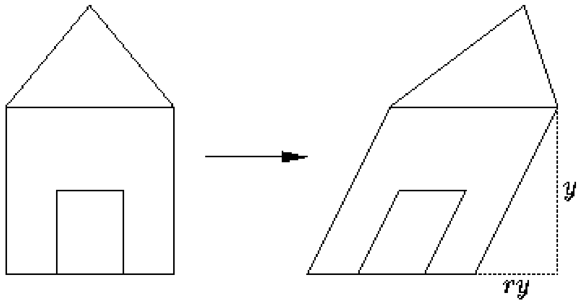


Figure 1: A shear with factor r=½.

Every affine transformation is obtained by composing a scaling transformation with an isometry, or a shear with a homothety and an isometry.

Projective Transformations-

A transformation that maps lines to lines (yet does not really save parallelism) is a projective transformation. Any plane projective transformation can be communicated by an invertible 3×3 matrix in homogeneous coordinates; on the other hand, any invertible 3×3 matrix characterizes a projective transformation of the plane. Projective transformations (if not relative) are not characterized on the majority of the plane, however just on the supplement of a line (the missing line is "mapped to infinity").

A typical example of a projective transformation is given by a point of view transformation . Carefully this gives a transformation starting with one plane then onto the next, yet in the event that we recognize the two planes by (for example) fixing a cartesian system in every, we get a projective transformation from the plane to itself.

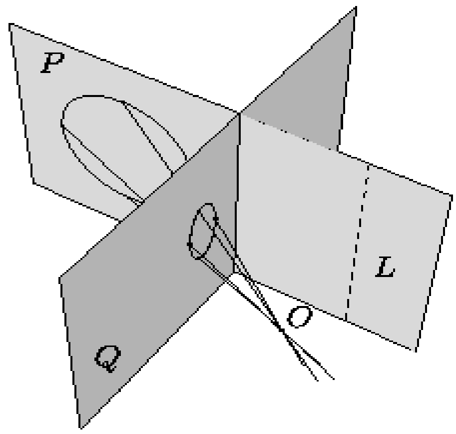


Figure 1: A point of view transformation with focus O, mapping the plane P to the plane Q. The transformation isn't characterized at stake L, where P crosses the plane parallel to Q and going throught O.

LINES

The (cartesian) equation of a straight line is linear in the coordinates x and y, that is, of the structure ax+by+c=0.

The slope of this line is - a/b, the convergence with the x-pivot (or x-catch) is x=-c/an, and the crossing point with the y-hub (or y-capture) is y=-c/b. In the event that a=0 the line is parallel to the x-pivot, and if b=0 the line is parallel to the y-hub.

(In a slanted coordinate system everything in the former passage stays valid, with the exception of the estimation of the slope.)

When $a^2+b^2=1$ and $c \leq 0$ in the equation *ax*+*by*+*c*=0, the equation is said to be in **normal form**. In this case *c* is the **distance of the line to the origin**, and

$$\omega = \arcsin a = \arccos b$$

is the angle that the perpendicular dropped to the line from the origin makes with the positive *x*-axis (Figure 1).

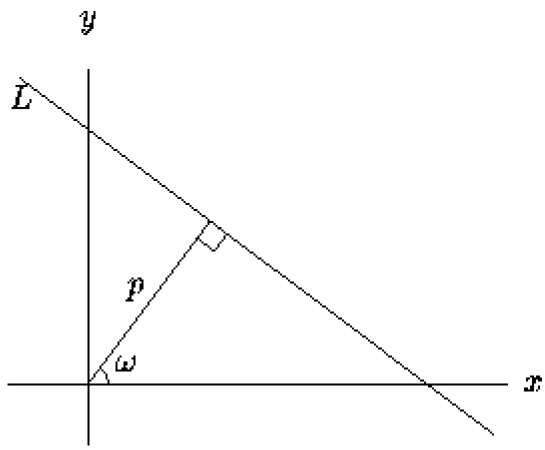


Figure 1: The normal form of *L* is *x* cos ω+ *y* sin ω=*p*.

To reduce an arbitrary equation $ax+by+c=0$ to normal form, divide by $\pm\sqrt{a^2+b^2}$, where the sign of the radical is chosen opposite the sign of *c* when $c \neq 0$ and the same as the sign of *b* when $c=0$.

Lines with prescribed properties-

Line of slope *m* intersecting the *x*-axis at $x=x_0$:

$$y=m(x-x_0)$$

Line of slope *m* intersecting the *y*-axis at $y=y_0$:

$$y=mx+y_0$$

Line intersecting the *x*-axis at $x=x_0$ and the *y*-axis at $y=y_0$:

$$x/x_0+y/y_0=1$$

(This formula remains true in **oblique coordinates**.)

Line of slope *m* going though (x_0,y_0):

$$y-y_0=m(x-x_0)$$

Line going through points (x_0,y_0) and (x_1,y_1) :

$$\frac{y-y_1}{x-x_1}=\frac{y_0-y_1}{x_0-x_1} \qquad or \qquad \begin{vmatrix} x & y & 1 \\ x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}=0$$

(These formulas remain true in **oblique coordinates**.)

Slope of line going through points (x_0,y_0) and (x_1,y_1):

$$(y_1-y_0)/(x_1-x_0)$$

Line going through points with **polar coordinates** (r_0,θ_0) and (r_1,θ_1) :

$$r(r_0 \sin(\theta-\theta_0)-r_1 \sin(\theta-\theta_1))= r_0r_1 \sin(\theta_1-\theta_0).$$

Distances-

The **distance** between two points in the plane is the **length of the line segment** joining the two points. If the points have **cartesian coordinates** (x_0,y_0) and (x_1,y_1), this distance is

$$\sqrt{(x_1-x_0)^2+(y_1-y_0)^2}.$$

If the points have **polar coordinates** (r_0,θ_0) and (r_1,θ_1), this distance is

$$\sqrt{r_0^2+r_1^2-2r_0r_1\cos(\theta_0-\theta_1)}.$$

If the points have **oblique coordinates** (x_0,y_0) and (x_1,y_1), this distance is

$$\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+2(x_1-x_0)(y_1-y_0)\cos\omega},$$

where ω is the angle between the axes (Figure1.5.1).

The point *k*% of the way from $P_0=(x_0,y_0)$ to $P_1=(x_1,y_1)$ is

$$\left(\frac{kx+(100-k)x}{100},\frac{ky+(100-k)y}{100}\right).$$

(The same formula works also in oblique coordinates.) This point divides the segment P_0P_1 in the ratio *k*:(100-*k*). As a particular case, the **midpoint** of P_0P_1 is given by

$$(\tfrac{1}{2}(x_1+x_2),\ \tfrac{1}{2}(y_1+y_2)).$$

The **distance** from the point (x_0,y_0) to the line $ax+by+c=0$ is

$$\frac{ax_0+by_0+c}{\sqrt{a^2+b^2}}$$

Angles-

The **angle** between two lines $a_0x+b_0y+c_0=0$ and $a_1x+b_1y+c_1=0$ is

$$\arctan\frac{b_1}{a_1}-\arctan\frac{b_0}{a_0}=\arctan\frac{a_0b_1-a_1b_0}{a_0a_1+b_0b_1}.$$

In particular, the two lines are **parallel** when $a_0b_1=a_1b_0$, and **perpendicular** when $a_0a_1=-b_0b_1$.

The **angle** between two lines of slopes m_0 and m_1 is

$$\arctan((m_1-m_0)/(1+m_0m_1)).$$

In particular, the two lines are **parallel** when $m_0=m_1$ and **perpendicular** when $m_0m_1=-1$.

Concurrence and Collinearity-

Three lines $a_0x+b_0y+c_0=0$, $a_1x+b_1y+c_1=0$, and $a_2x+b_2y+c_2=0$ are **concurrent** if and only if

$$\begin{vmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

(This remains true in **oblique coordinates**.)

Three points (r_0, θ_0), (r_1, θ_1) and (r_2, θ_2) are **collinear** if and only if

$$\begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

(This remains true in **oblique coordinates**.)

Three points with polar coordinates (r_0, θ_0), (r_1, θ_1) and (r_2, θ_2) are collinear if and only if

$$r_1r_2 \sin(\theta_2-\theta_1)+ r_0r_1 \sin(\theta_1-\theta_0)+ r_2r_0 \sin(\theta_0-\theta_2)=0.$$

REFERENCES

1. Boris A. Rosenfeld and Adolf P. Youschkevitch (2006). "Geometry", in Roshdi Rashed, ed., Encyclopedia of the History of Arabic Science, Vol. 2, pp. 447-494 [470], Routledge, London and New York:

2. Boyer (2001). "Euclid of Alexandria". pp. 104. "The Elements was not, as is sometimes thought, a compendium of all geometric knowledge; it was instead an introductory textbook covering all elementary mathematics-"

3. E. Calabi (2007) *in* Algebraic geometry and topology: a symposium in honor of S. Lefschetz (Princeton University Press, 2007);

4. Kline (2002) "Mathematical thought from ancient to modern times", Oxford University Press, p. 1032. Kant did not reject the logical (analytic a priori) possibility of non-Euclidean geometry, see Jeremy Gray, "Ideas of Space Euclidean, Non-Euclidean, and Relativistic", Oxford, 1989; p. 85. Some have implied that, in light of this, Kant had in fact predicted the development of non-Euclidean geometry, cf. Leonard Nelson, "Philosophy and Axiomatics," Socratic Method and Critical Philosophy, Dover, 1995; p.164.

5. P. Griffiths and J. Harris (2008). Principles of algebraic geometry (Wiley, 2008)

6. R. Rashed (2004). The development of Arabic mathematics: between arithmetic and algebra, London

7. R. Rashed (2004). The development of Arabic mathematics: between arithmetic and algebra, London.

Corresponding Author

Sikander*

Assistant Professor of Mathematics, A.I.J.H.M. College Rohtak, Haryana, India

sikanderbeniwal2212@gmail.com