Journal of Advances and Scholarly Researches in Allied Education
Vol. 15, Issue No. 11, November-2018, ISSN 2230-7540

Study on Higher-Order Kirchhoff Type Equations

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Abstract – The presence of higher order inertial collectors with heavy damping terminology is studied by Kirchhoff style equations. The Hadamard graphical norm is used to achieve inertial multiple variants in some spectral intervals for this form of equations.

Keywords: Inertial Multiple, State of Spectral Interval, Figure Van, Kirchhoff Higher-Order

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INTRODUCTION

The inertial manifold holds a significant role in the analysis of the long-term structural actions of infinite dynamical structures. It is an invariant Lipschitz finite dimensional manifold and draws in the machine space [1-3] all exposure-nential solution orbitals. It plays a key role in both complex and infinite dimensional structures of dimensions. Because its location is significant, many researchers researched the nature and attractiveness of the inertial varieties, their final-size properties, and the related problems in inertial varieties and delays. Guoguang Lin and Jingzhu Wu[4] investigated the presence at that time of low-quality, inertial multiplicators of the Bousinesq equation and the equation is very damp.

$$\begin{cases} u_{tt} - \alpha\Delta u_t - \Delta u + u^{2k+1} = f(x,y), (x,y) \in \Omega, \\ u(x,y,0) = u_0(x,y), (x,y) \in \Omega, \\ u(x,y,t) = u(x+\pi,y,t) = u(x,y+\pi,t) = 0, (x,y) \in \Omega. \end{cases}$$

The following fourth-order highly dampened time-delay wav equations analysis Zhicheng Zhang and Guoguang Lin:

$$\begin{cases} u_{tt} - \varepsilon\Delta u_t - \Delta u + \Delta^2 u = f(u_t), t > 0, \\ u_0(\Theta) = u^0(\Theta), \Theta \in [-r,0], \partial_t|_{t=0} = u. \end{cases}$$

The paper will re-rise, refine order space, merge the high order structural damping, prove to be generalised high-order high-order inertial multiplexes in some hypotheses. Kirchhoff will remain. More concerning the inertial multiple equation of Kirchhoff. The following Kirchhoff style equations discuss the initial limit value concerns in this paper:

$$\begin{cases} u_{tt} + (1 + \int_\Omega |D^m u|^p dx)^r (-\Delta)^m u + \Delta^{2m} u + \beta(-\Delta)^m u_t = f(x), \\ u(x,t) = 0, \frac{\partial^i u}{\partial v^i} = 0, i = 1, 2, \cdots, 2m-1, x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega \subset R^n. \end{cases} \tag{1}$$

In which r 0, m 0, 1, β > 0. Ω is the flat boundary boundary area ∂Ωin Rn. f(x) is a power beyond is an external normal vector, the ∆2systemic damping word is the amputation term, β(−∆)mut is a damping term for the structure. (1 +|Dmu|pdx)r(−∆)mu (1 +|Dmu|pdx)r(− leaving)mu the harsh word. And the rigid word predictions shall be made late.

Prepare

This paper describes the following spaces and icons for convenience: $f = f(x)$, $D = \nabla$, $H = L^2(\Omega)$, $H^{2m} = H^{2m}(\Omega)$, $V_1 = H^{2m}(\Omega) \times H(\Omega)$, $V_2 = H^{2m+k} \times H^k(\Omega)(k = 0, 1, 2, \cdot \cdot \cdot, 2m)$, C_0 isconstant. Respectively,(·, ·) and ‖ · ‖ V $(u, v) = ∫Ω\ u(x)v(x)dx$, (u, v) =‖u‖2.And then let's Defining the room norm and internal product V_1 and V_2: $\forall U_i \in (u_i, v_i) \in V_i$, $i = 1, 2$, we have

$$(U_1, U_2) = (D^m u_1, D^m u_2) + (v_1, v_2), \tag{2}$$

$$\|U\|^2_{V_1} = (U,U)_{V_1} = \|D^m u\|^2 + \|v\|^2, \tag{3}$$

$$\|U\|^2_{V_1} = (U,U)_{V_2} = \|D^m u\|^2 + \|v\|^2. \tag{4}$$

$g(\|D^m u\|^p_p) = (1 + \int_\Omega |D^m u|^p dx)^r$ meets the following conditions:

(a) $\|D^m u\|_{L^p} \le C\|Du\|^\alpha_{L^2}\|u\|^{1-\alpha}_{H^{2m}}$, where $\alpha = (\frac{1}{p} - \frac{n}{m} - \frac{1}{4m})/(\frac{1}{2} - \frac{1}{n} - \frac{1}{4m})$.

(b) Writing $g(\|D^m u\|^p_p) = (1 + \int_\Omega |D^m u|^p dx)^r$ by (a) we get

$$\kappa_0 \le g(\|D^m u\|^p_p) \le \kappa_1, \kappa = \begin{cases} \kappa_0, \frac{d}{dt}\|D^{m+k} u\|^p_p \ge 0, \\ \kappa_1, \frac{d}{dt}\|D^{m+k} u\|^p_p < 0. \end{cases} \text{And } \kappa_0 \ge 1, \kappa_1, C, \|u\|_{H^{2m}} \text{ is constant.}$$

Definition 2. 1 One notes that inertial multiples μ are finite-dimensional multiples that must fulfil these characteristics:

I. μ is a multiple and finite dimension of lipschitz;

II. μ, that means S(t)μ, t > 0, is a positive invariant set;

III. The exponential μ is used to attract all orbital solution.

Definition 2.2 Assume that A1: X is a supplier and F ∊ Cb(X, X) meets the following inequalities

$$\|F(U) - F(V)\|_X \leq l_F \|U - V\|_X (U, V \in X),$$

Assume that the operator A1's point range is split into 2 sections σ1 and σ2, and that σ1 is finite,

$$\Lambda_1 = sup\{Re\lambda | \lambda \in \sigma_1\}, \Lambda_2 = inf\{Re\lambda | \lambda \in \sigma_2\}. \qquad (5)$$

And

$$X_i = span\{\omega_j | j \in \sigma_i\}, i = 1, 2. \qquad (6)$$

So

$$\Lambda_2 - \Lambda_1 > 4l_F, \qquad (7)$$

We are decomposed orthogonally

$$X = X_1 \oplus X_2. \qquad (8)$$

and continuous mapping

$$P_1 : X \to X_1, P_2 : X \to X_2. \qquad (9)$$

Lemma 2.1Suppose that the eigenvalues $\mu_j^{\pm}(j \geq 1)$ are nonsubtractive, and for all *m* when *Nm*, μ^-N and $\mu^-N+1$are consecutive adjacent values.

Inertial Manifolds

Equation (1) is equal to the equation of the first order

$$U_t + \widetilde{A}U = F(U). \qquad (10)$$

where U = (u, v), v = ut ,

$$\widetilde{A} = \begin{pmatrix} 0 & -I \\ (-\Delta)^{2m} & \beta(-\Delta)^m \end{pmatrix}. \qquad (11)$$

$$F(U) = \begin{pmatrix} 0 \\ f(x) - (1 + \|D^m u\|_p^p)^r (-\Delta)^m u \end{pmatrix}. \qquad (12)$$

$$D(\widetilde{A}) = \{u \in H^{2m+k} | u \in L^2, (-\Delta)^m u \in H^{2m+k}\} \times H^m. \qquad (13)$$

The standard graph identified by the dot in X

$$(U, V)_X = ((-\Delta)^m u, (-\Delta)^m \overline{y}) + (v, \overline{z}) \qquad (14)$$

where

$U = (u, v), V = (y, z) \in X, u, y \in H^{2m+k}(\Omega); v, z \in H^{2m+k}(\Omega). \overline{y},$ Reflect y, z, conjugate respectively. Of course, in equation (11) the operator described is monotone. For U ∊ D(A~)

$$(\widetilde{A}U, U)_X = -((-\Delta)^m v, (-\Delta)^m \overline{u}) + ((-\Delta)^{2m} u + \beta(-\Delta)^m v, \overline{v}) = \beta\|D^m v\|^2 \geq 0. \qquad (15)$$

So (AU, U)X is a real non-negative.

Find the following equation with the own meaning in order to evaluate A~

$$\widetilde{A}(U) = \lambda U, U = (u, v) \in X. \qquad (16)$$

That is

$$\begin{cases} -v = \lambda u; \\ (-\Delta)^{2m} u + \beta(-\Delta)^m v = \lambda v. \end{cases} \qquad (17)$$

By swapping the first fomular equation (17) with the second equation (17), we got

$$\begin{cases} \lambda^2 u + (-\Delta)^{2m} u - \beta\lambda(-\Delta)^m u = 0; \\ u|_{\partial\Omega} = (-\Delta)^m u|_{\partial\Omega} = 0. \end{cases} \qquad (18)$$

We use the internal product in equation with the first expression (18)

$$\lambda^2 \|u\|^2 + \|(-\Delta)^m u\|^2 - \beta\lambda\|(-\Delta)^m u\|^2 = 0. \qquad (19)$$

Equation (19) is assumed to be an undefined quadratic equation of λ.

$$\lambda_k^{\pm} = \frac{\beta\delta_k \pm \sqrt{\beta^2\delta_k^2 - 4\delta_k^2}}{2}. \qquad (20)$$

Where δ_k is the eigenvalue of $(-\Delta)^m$ in $H^{2m}(\Omega)$. If $\beta^2 \geq 4$, then All A~ own values are real positive and the corresponding ownvector has the shape of $U_k^{\pm} = (u_k, -\lambda_k^{\pm} u_k)$. As regards formula (13), the following marks are rendered in order to make it easy to use the following. For all $k \geq 1$,we get

$$\|D^m u_k\| = \sqrt{\delta_k}, \|u_k\|^2 = 1, \|(-\Delta)^{2m} u_k\|^2 = \delta_k^2. \qquad (21)$$

Lemma3.1 *Remarking* $g(u) = (1 + \int_\Omega |D^m u|^p dx)^r (-\Delta)^m u,\ g : H_0^{2m+k}(\Omega) \to H_0^{2m}(\Omega)$ *is* Uniformly bound and constant internationally.

Proof. $\forall u_1, u_2 \in H_0^{2m+k}(\Omega)$

$$g(u_1) - g(u_2) = ((1 + \|D^m u_1\|_p^p)^r - (1 + \|D^m u_2\|_p^p)^r) \cdot (-\Delta)^m u_1 + (1 + \|D^m u_2\|_p^p)^r (-\Delta)^m (u_1 - u_2).$$

$$\begin{aligned} \|D^k(g(u_1 - g(u_2))\| &\le g'(\xi) \cdot \|D^{m+k} u_1 - D^{m+k} u_2\| \cdot D^{2m+k} u_1 + g(u_2)\|D^{2m+k}(u_1 - u_2)\| \\ &\le g'(\xi) \cdot \lambda_1^{\frac{m}{2}} \|D^{2m+k}(u_1 - u_2)\| \|D^{2m+k} u_1\| + g(u_2)\|D^{2m+k}(u_1 - u_2)\| \\ &\le C_0 \|D^{2m+k}(u_1 - u_2). \end{aligned} \tag{22}$$

where $\xi = \theta D^m u_1 + (1 - \theta) D^m u_2$, $\theta \in [0, 1]$. Let $l = C_0$, So l is a g(u) coefficient of lipschitz.

Theorem 3.1 *When $0 < \beta \le 2$, l is the lipschitz coefficient of $g(\|D^m u\|p)$, let $N_1 \in N$ make that $N \ge N_1$, we have*

$$\beta(\delta_{N+1} - \delta_N) \ge 8l. \tag{23}$$

The (7) spectral interval requirement is satisfied by the operator A~.

Proof. According to equations (12) and (14), writing $U = (u, v)$, $V = (u, v) \in X$, then

$$\|F(U) - F(V)\|_X = \|g(\|D^m u\|_p^p) - g(\|D^m \widetilde{u}\|_p^p)\| \le l\|u - \widetilde{u}\| \le \frac{l}{\sqrt{\beta - 2}} \|U - V\|_X. \tag{24}$$

That is $l_F \le l$. According to equation (19), the necessary and sufficient condition for $\lambda^{\pm}k$ to be $\lambda^{\pm}k$ real number that is $\beta > 2$. By assuming that $0 < \beta \le 2$, A have at most $\lambda^{\pm}k$ finite number of $2N_0$ as eigenroots, when $N_0 = 0$, $0 < \beta \le 2$, then $\Lambda_0 = max\{\lambda_k^{\pm} | k \le N_0$. When $k \ge N_0 + 1$, the eigenvalue is complex, and the real part is taken

$$Re\lambda_k^{\pm} = \frac{\beta}{2}\delta_k. \tag{25}$$

So there is $N_1 \ge N_0 + 1$ make $Re\lambda_k^{\pm} > \Lambda_0, k \ge N_1$. Let make (22) be true. Reduce the A point distribution

$$\sigma_1 = \{\lambda_k^{\pm} | k \le N\}, \sigma_2 = \{\lambda_k^{\pm} | k \ge N + 1\}. \tag{26}$$

Set the necessary subspace

$$X_1 = span\{\lambda_k^{\pm} | k \le N\}, X_2 = span\{\lambda_k^{\pm} | k \ge N + 1\}. \tag{27}$$

Inexistence k make $\delta_k^- \in \sigma_1$ and $\delta_k^+ \in \sigma_2$, which means it can't exist $U_k^- \in X_1$ and $U_k^+ \in X_2$. X1 and X2 are therefore X's ordinary subspace. (5) and (25) state that we are having

$$Re(\lambda_{N+1}^- - \lambda_N^+) = \frac{\beta}{2}(\delta_{N+1} - \delta_N). \tag{28}$$

Thus from (23) it can be understood that A follows the spectral interval.

Theorem 3.2 *When $\beta \ge 2$, l is the Lipschitz coefficient of $g(\|D^m u\|p)$ let $N_1 \in N$ be sufficiently large, so that $N \ge N_1$ and*

$$(\delta_{N+1} - \delta_N)(\frac{\beta}{2} - \frac{\sqrt{\beta^2 - 4}}{2}) \ge \frac{4l}{\sqrt{\beta - 2}} + 1 \tag{29}$$

Operator A then fulfills (7) the spectral intervals.

Proof. When $\beta > 2$, All A proper values are real positive numbers, and we know the sequences $\{\lambda^- k\}_{k \ge 1}$ and $\{\lambda^+\}_{k \ge 1}$ Four measures to explain are growing.

Step 1: Because $\lambda^{\pm}k$ is non-subtractive, according to Lemma 2.1, N is given so that $\lambda^- N$ and $\lambda^- N+1$ are adjacent values and A's own worth is split down to

$$\sigma_1 = \{\lambda_j^-, \lambda_k^+ | max\{\lambda_j^-, \lambda_k^+\} \le \lambda_N^-\}, \tag{30}$$
$$\sigma_2 = \{\lambda_j^-, \lambda_k^+ | \lambda_j^- \le \lambda_N^- \le min\{\lambda_j^-, \lambda_k^+\}\}. \tag{31}$$

Step 2: The X can be split down into

$$X_1 = span\{U_j^-, U_k^+ | \lambda_j^-, \lambda_k^+ \in \sigma_1\}, \tag{32}$$
$$X_2 = span\{U_j^-, U_k^+ | \lambda_j^-, \lambda_k^+ \in \sigma_2\}. \tag{33}$$

The purpose is to orthogenalize and meet the cross-spectral term these two subspaces. (7).

$$\Lambda_1 = \lambda_N^-, \Lambda_2 = \lambda_{N+1}^-,$$

Further decomposition $X_2 = X_C \oplus X_R$,

$$X_C = span\{U_j^- | \lambda_j^- \le \lambda_N^- \le \lambda_j^+\}, \tag{34}$$
$$X_R = span\{U_R^{\pm} | \lambda_N^- \le \lambda_k^{\pm}\}. \tag{35}$$

And assuming that $X_N = X_C \oplus X_1$. First, show the dot product of X's own values to be orthogonal for X1 and X2, such that two functions are added $\Phi : X_N \to R$ and $\Psi : X_R \to R$.

$$\Phi(U, V) = (\beta - 1)((-\Delta)^m u, (-\Delta)^m \overline{y}) + (\overline{z}, (-\Delta)^m u) + (\overline{v}, (-\Delta)^m y) + (\overline{z}, v), \tag{36}$$
$$\Psi(U, V) = \beta((-\Delta)^m u, (-\Delta)^m \overline{y}) + (\overline{z}, (-\Delta)^m u) + (\overline{v}, (-\Delta)^m y) + (\overline{z}, v). \tag{37}$$

Where $U = (u, v)$, $V = (y, z)$, y and z are conjugates of y and z, respectively. Suppose $U = (u, v) \in X_N$, then

$$\begin{aligned} \Phi(U, U) &= (\beta - 1)\|(-\Delta)^m u\|^2 + 2(\overline{v}, (-\Delta)^m u) + \|v\|^2 \\ &\ge (\beta - 1)\|(-\Delta)^m u\|^2 - 2\|v\| \cdot \|(-\Delta)^m u\| + \|v\|^2 \\ &\ge (\beta - 1)\|(-\Delta)^m u\|^2 - \|v\|^2 - \|(-\Delta)^m u\|^2 + \|v\|^2 \\ &\ge (\beta - 2)\|(-\Delta)^m u\|^2. \end{aligned} \tag{38}$$

And since $\beta > 2$, you get that $\Phi(U, U) \ge 0$ is true for all $U = (u, v) \in X_N$, Φ is positive definite. Similarly, since $U = (u, v) \in X_R$,

$$\begin{aligned} \Psi(U, U) &= \beta\|(-\Delta)^m u\|^2 + 2(\overline{v}, (-\Delta)^m u) + \|v\|^2 \\ &\ge \beta\|(-\Delta)^m u\|^2 - 2\|v\| \cdot \|(-\Delta)^m u\| + \|v\|^2 \\ &\ge \beta\|(-\Delta)^m u\|^2 - \|v\|^2 - \|(-\Delta)^m u\|^2 + \|v\|^2 \\ &\ge (\beta - 1)\|(-\Delta)^m u\|^2. \end{aligned} \tag{39}$$

And since β > 2, you get that $\Psi(U, U) \geq 0$ is true for all $U = (u, v) \in X_N$, Ψ is So hopefully optimistic. Now the inner product of X will be defined

$$\ll U, V \gg_X = \Phi(P_N U, P_N V) + \Psi(P_R U, P_R V). \tag{40}$$

Where P_N and P_R are respectively mappings of $X \to X_N$ and $X \to X_R$. Equation (40) may be revised to receive for case

$$\ll U, V \gg_X = \Phi(U, V) + \Psi(U, V). \tag{41}$$

In terms of point product XN and XC is orthogonal as regards both the X1 and X2 subspaces described in (32) and (33), that is « U^+, U^- »$_X$= 0 , For each $U^+ \in X_C$ and $U^- \in x_N$,we can deduce from Equation (35)

$$\begin{aligned} \ll U_j^+, U_j^- \gg_X &= \Phi(U_j^+, U_j^-) \\ &= (\beta-1)\|(-\Delta)^m u_j\|^2 - (\lambda_j^- + \lambda_j^+)\|D^m u_j\|^2 + \lambda_j^- \lambda_j^+ \|u_j\|^2 \\ &= (\beta-1)\delta_j^2 - (\lambda_j^- + \lambda_j^+)\delta_j + \lambda_j^- \lambda_j^+. \end{aligned} \tag{42}$$

Equation (18) notes that we

$$\lambda_j^- + \lambda_j^+ = \beta\delta_j, \lambda_j^- \lambda_j^+ = \delta_j^2, \text{ so}$$

$$\ll U_j^+, U_j^- \gg_X = \Phi(U_j^+, U_j^-) = 0. \tag{43}$$

Step 3: The lipschitz constant F next guess, where $F(U) = (0, f(x) - g(u))^T$, $g : H^{2m+k} \to H^{2m}$ and $l_F = l$. It can be shown with every equation (30) and equation (31). $U = (u, v) \in X$, we have

$$\begin{aligned} \|U\|_X^2 &= \Phi(P_1 U, P_1 U) + \Psi(P_2 U, P_2 U) \\ &\geq (\beta-2)(\|(-\Delta)^m P_1 u\|^2 + \|(-\Delta)^m P_2 u\|^2) \\ &\geq (\beta-2)\|(-\Delta)^m u\|^2. \end{aligned} \tag{44}$$

Set $U = (u, v), V = (\overline{u}, \overline{v}) \in X$, we have

$$\|F(U) - F(V)\|_X = \|g(\|D^m u\|_p^p) - g(\|D^m \overline{u}\|_p^p)\| \leq l\|u - \overline{u}\| \leq \frac{l}{\sqrt{\beta-2}}\|U - V\|_X. \tag{45}$$

CONCLUSION

$$l_F \leq \frac{l}{\sqrt{\beta-2}}. \tag{46}$$

Step 4: The formula (7) must be checked for the spectral interval condition that can be derived from: ‾Λ1 = λ−N andΛ2 = λ−N+1 above

$$\Lambda_2 - \Lambda_1 = \lambda_{N+1}^- - \lambda_N^- = \frac{1}{2}(\delta_{N+1} - \delta_N) + \frac{1}{2}(\sqrt{R(N)} - \sqrt{R(N+1)}). \tag{47}$$

Make sure that $N_1 > 0$ is such that for all $N \geq N_1, R_1(N) = 1 + \frac{2\beta}{(\beta^2-4)\delta_N^2} + \frac{1}{(\beta^2-4)\delta_N^2}$, so we can calculate

$$\sqrt{R(N)} - \sqrt{R(N+1)} + \sqrt{\beta-2}(\delta_{N+1} - \delta_N) = \sqrt{\beta-2}(\delta_{N+1} R_1(N+1) - \delta_N R_1(N)). \tag{48}$$

Well from the observations we created it is easy to understand

$$\lim_{N\to\infty} (\sqrt{R(N)} - \sqrt{R(N+1)} + \sqrt{\beta-2}(\delta_{N+1} - \delta_N)) = 0. \tag{49}$$

Then we will do this by integrating equations (46), (47), (21) and (48).

$$\Lambda_2 - \Lambda_1 > \frac{1}{2}((\delta_{N+1} - \delta_N)(\beta - \sqrt{\beta^2-4}) - 1) \geq \frac{4l}{\sqrt{\beta-2}} \geq 4l_F. \tag{50}$$

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