

A Study on the Usage of Number Theory for the Formulation of Mathematical Problems

K. Gunasekar^{1*} Dr. Ashwini Kumar Nagpal²

¹ Research Scholar (Part Time) OPJS University, Churu, Rajasthan

² Research Guide, Department of Mathematics, OPJS University, Churu, Rajasthan

Abstract – The main focus of this thesis is to study the different properties of Pythagorean triples, to find triples with the sum of any two coordinates is some fixed power, finite sum of polynomial expressions, triangular numbers, pseudoprimes and some primality tests we discussed some basics and known results in Number Theory which are essential for our research work.

Keywords: Number Theory, Pythagorean Triple

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INTRODUCTION

Srinivasa Ramanujan has recorded many beautiful continuous fraction extensions in his notebooks. Chapter 12 of Ramanujan's second notebook is entirely devoted to continuous fractions. Other continuous fractions can be found in other chapters, in particular in his second notebook, Chapter 16[94]. The unorganised portions of the second and third notebooks of Ramanujan contain about 60 additional results for the continued fractions. This part of Ramanujan's work has been addressed and developed by several authors, including G. E. Andrews, M. D. Carlitz, R. Carlitz Hirschhorn L. Carlitz Gordon W. A. Allalam and M. Scoville B. E. H. Ismail and K. G. Ramanathan R. Y. Denis S. Bhargava and Adiga Bhargava from Chandrashekar, Adiga, and D. D. Somashekara Adiga and Somashekara A. Verma, Denis, K. It's Rao Srinivasa and N. The most famous and only continuous fraction shown in Ramanujan's published papers is the continuous fraction of Rogers-Ramanujan and is given by Rogers-Ramanujan.

Srinivasa Ramanujan has recorded a few results in his second scratch pad [94, pp. 257-262] in the hypotheses of the elliptical capacity of the elective bases relating to the traditional hypothesis. K. Venkatachaliengar inspected a portion of the sections in the 'lost' note pad of Ramanujan[96] dedicated to elective hypotheses. Ramanujan had a rare appreciation of number n . The vast majority of the hypotheses, in particular the isolated conditions and speculations of the elliptical capacity to the elective bases created by Ramanujan, were devoted to the representational arrangement for $1/r$. So as to set up some wonderful equations for $1/r$ expressed by Ramanujan J. M. Borwein and P. B. Borwein has

built up "comparing speculation." Ramanujan has recorded 23 delightful P-Q estimated time of arrival work characters, including the remainder of the estimated time of arrival capacity in Chapter 25 of his subsequent scratch pad. A large proportion of these P-Q personalities have been shown by Bemdt and Zhang to use the different measured conditions of Ramanujan having a place with the hypothesis of mark 2 (traditional hypothesis). In order to build up the rest of the P-Q characters, Bemdt used the hypothesis of a particular structure. As of late, Bhargava, Adiga and Mahadeva Naika have acquired another class of P-Q personalities on the basis of certain special conditions having a place with the 'elective hypothesis of mark 4.' In the same way, Baruah[80] has demonstrated five special conditions of Ramanujan. These P-Q personalities are incredibly helpful in calculating class invariants and estimating the proportions of theta-capacity. Ramanujan has recorded a few estimates of $p(q)$ in his scratch pad[94]. Bemdt and Chan[21] demonstrated each of these qualities and some new estimates $p(q)$ not guaranteed by Ramanujan on the use of Ramanujan's measured conditions and class-invariants.

In this postulation, we are zeroing in the investigation of some known and new outcomes in the Elementary Number Theory, which examines the arrangement of positive numbers that are additionally called common numbers.

From ancient times, mathematicians divided the regular numbers into different types of numbers, some of which are as follows[40]:

Even numbers: 2; 4; 6; 8;

Odd numbers: 1; 3; 5; 7; 9;

Prime numbers: 2; 3; 5; 7;

Composite Numbers: 4; 6; 8; 9;

Square numbers: 4; 9; 16; 25;

Triangular numbers: 1; 3; 6; 10

Perfect numbers: 6; 28; 496;

Fibonacci numbers: 1; 1; 2; 3; 5; 8;

There are some standard questions in the theory of numbers like:

Sum of Squares.

Que. Can the total of two squares be a square ?

The response to this investigation is an rnative given by Pythagoras (569-470 B.C.) to the western world. Pythagorean hypothesis [44] states that there is a triangle [65] with sides x ; y and z , which satisfies the condition $x^2 + y^2 = z^2$ which is known as Pythagorean condition. The triple x ; y and z of positive integer numbers that satisfy the Pythagorean condition is called Pythagorean triples.

For example: $3^2 + 4^2 = 5^2$; $5^2 + 12^2 = 13^2$ etc.

The Pythagorean were the rst to give a method of determining in nitely many pyth-gorean triples [chapter 2, theorem (2.1.2)].

About 300 B.C. The presence of Euclid's Elements and the assortment of 13 books has changed over arithmetic from numerology to deductive science. In Book X, he gave a technique to get all the Pythagorean triplets without confirmation. Provides a link between the crude Pythagorean triples and the Mean Value The-orem. Likewise, he talked about the Pythagorean triples whose triangles have a typical hypotenuse, inradius, edge, zone, legsum. Kak and Kothapalli examine the use of crude Pythagorean triplets in cryptography. Roy and Sonia[60] have a di-rect strategy to create all potential crude and non-crude triples for some random number and have developed a method for delivering Pythagorean quadruples and tuples.

The Pythagorean Triple Metric Sequences are used to close the connexion. In addition, the Pythagorean Quadraple has been disused.

Trios of positive enters.

Que. Que. Can we and the trios of positive integer numbers have the ultimate goal that the integer of any two coordinates is some xed power?

We explained this investigation in the subtleties of Chapter 3. We've found out that such triples are in a very large number that Arvind has discovered four unmistakable positive numbers with the ultimate goal that any two of them is a square.

Number Shapes.

The numbers that are organised as some numerical gures, such as tri-edge, square, and so on, are known as gurate numbers.

The numbers that look like a triangle are called three-sided num-bers.

For example: 1; 3; 6; 10; are three-sided numbers.

The numbers that look like squares are called square numbers.

Examples are: 1; 4; 9; 16; 25; square numbers.

The numbers that are arranged in the shape of a pentagon are called pentagonal num-bers.

Examples are: 1; 5; 12; 22; 35; pentagonal numbers.

Likewise, the numbers that are arranged in the shape of Polygon are called polygonal numbers.

A polygonal number denoted by $P_d(n)$ and is a number of the form

$$P_d(n) = 1 + [1 + (d - 2)] + [1 + 2(d - 2)] + \dots + [1 + (n - 1)(d - 2)]$$

$P_d(n)$ is a d -gonal number of order n [n th d -gonal number].

Que: Is there a link between the three-sided number and the other polygonal number-bers? In addition, the response to this investigation is YES.

Each polygonal number can be written about three-sided numbers So the tri-precise number assumes a significant function in the investigation of the number hypothesis.

The connexion between the Triangular Numbers and the Prime Numbers is described in subtleties. Likewise, it provides clarification of the writing of positive whole number as far as Tri-precise numbers are concerned.

From ancient times, three-sided numbers draw attention to individuals all over the world. The intrigue is not limited to mathematicians and scientists. Further some fascinating properties of three-sided numbers can be found in.

The nitude of the primes.

Que. Are there infinitely many prime numbers?

Que. Are there numerous primes compatible with 1 modulo 4 and congruent with 3 modulo 4?

The appropriate response to these enquiries is also affirmative. Indeed, these investigations are the specific instances of the notable Dirichlet hypothesis. The iniquity of the primes is known from the time of Euclid, while the verification of the iniquity of the primes concerning the arithmetic movement is given by Dirichlet after the broken evidence given by Legendre.

Dirichlet Theorem: If $a; d \in \mathbb{N}$ and $\gcd(a; d) = 1$, then the infinite sequence $a + d; a + 2d; a + 3d; \dots$ contains quite a lot of primes.

For proof of theorem, Dirichlet used a complex analysis, so that in the history of Number Theory Dirichlet's original proof is considered to be non-elementary and analytic.

From Dirichlet theorem, one can easily conclude that,

1. There are infinitely many prime numbers.
2. There are infinitely many primes of the form $4n + 1$ and also of the form $4n + 3$, etc.

Preliminaries

Definition 1.2.1 (first number). A positive integer $p > 1$ with no proper positive factor other than 1 is called a prime number.

An integer $n > 1$ that is not prime is called a composite number. The set of primes is countably infinite.

Theorem 1.2.1 (Fundamental Theorem of Arithmetic). Every integer $n > 1$ can be written as a product of a prime uniquely, with the prime factors in the product in a non-decreasing order [9].

For integers $a; b$ with $b \neq 0$, we say that a is a factor (or divisor) of b if $b = ka$ for some $k \in \mathbb{Z}$, and in this case we write $a|b$. For integer $n > 1$ and integers $a; b$; we say that a is congruent to b modulo n , denoted by $a \equiv b \pmod{n}$ if n is divided by $b - a$. This is the equivalence relationship of the set of integers \mathbb{Z} [6].

Result 1.2.1. [34] If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then

1. $a + k \equiv b + k \pmod{n}$, $ak \equiv bk \pmod{n}$ for any $k \in \mathbb{Z}$;
2. $a \equiv c \pmod{n}$ and $b \equiv d \pmod{n}$ implies $a + b \equiv c + d \pmod{n}$;
3. $ac \equiv bd \pmod{n}$;
4. $a^k \equiv b^k \pmod{n}$, $ak \equiv bk \pmod{nk}$ for any $k \in \mathbb{Z}$

5. If $k \in \mathbb{Z}$ is a common divisor of a and b with $\gcd(k; n) = 1$, then $k^a \equiv k^b \pmod{n}$

6. If $a \equiv b \pmod{n_i}$, $1 \leq i \leq k$, then $a \equiv b \pmod{N}$ where $N = \text{lcm}(n_1; n_2; \dots; n_k)$. In particular $a \equiv b \pmod{n_1 n_2}$ if $\gcd(n_1; n_2) = 1$, that is $n_1; n_2$ are relatively prime,

$a \equiv b \pmod{n_1 n_2 \dots n_k}$ if $n_1; n_2; \dots; n_k$ are pairwise relatively prime.

(vii) If p is a prime and $a^2 \equiv b^2 \pmod{p}$, then $a \equiv b \pmod{p}$ or $a \equiv -b \pmod{p}$:

Result 1.2.2. For $a; b \in \mathbb{Z}$ and integer $n > 1$;

let $a +_n b$ = the least nonnegative remainder when $a + b$ is divided by n ; $a -_n b$ = the least nonnegative remainder when ab is divided by n ;

Then for $n > 1$; $U(n) = \{k \in \mathbb{Z} : k < n \text{ and } \gcd(k; n) = 1\}$ is an abelian group under $+_n$ with identity 1 and its order is $\phi(n)$; where ϕ is called Euler's phi function.

For any $a \in \mathbb{Z}$; with $\gcd(a; n) = 1$ we have $r \in U(n)$ such that $a \equiv r \pmod{n}$: Then we define $o(a)$ as $o(a) = o(r) = k$ is the least positive integer such that $r^k \equiv 1 \pmod{n}$

(i.e. $a^k \equiv r^k \equiv 1 \pmod{n}$). Clearly $a^{o(a)} \equiv 1 \pmod{n}$ and $o(a)|\phi(n)$: Note that $o(a)$ is

multiplicative and $(p^k)^{\phi(p^k)} \equiv p^{\phi(p^k)} \pmod{p^k}$ for a prime p and $k \in \mathbb{N}$:

A positive integer n is prime iff $\phi(n) = n - 1$:

For a prime p , a is its own inverse in group $U(p)$ if $a \equiv 1 \pmod{p}$ or $a \equiv -1 \pmod{p}$:

Theorem 1.2.2 (Wilson's Theorem). [47] An integer $n > 1$ is prime iff $(n - 1)! \equiv -1 \pmod{n}$:

Theorem 1.2.3 (Fermat's Little Theorem (FLT)). [47] If p is a prime, then $a^p \equiv a \pmod{p}$ for all integers a : If $\gcd(a; p) = 1$, then $a^{p-1} \equiv 1 \pmod{p}$. Converse of FLT is not true for example, $2^{341} \equiv 2 \pmod{341}$ and $341 = 11 \cdot 31$ which is not prime.

Theorem 1.2.4 (Euler's Theorem). [40] If integer $n > 1$ and $\gcd(a; n) = 1$, then $a^{\phi(n)} \equiv 1 \pmod{n}$:

Remark 1.2.1 (Contrapositive of FLT (Primality test)). For integer $n > 1$; if there exist $a \in \mathbb{Z}$ with $a^n \not\equiv a \pmod{n}$, then n is a composite number.

Example 1.2.1. As $2^{63} \equiv 2^{60} \cdot 2^3 \equiv (2^6)^{10} \cdot 8 \equiv (64)^{10} \cdot 8 \equiv 1^{10} \cdot 8 \equiv 8 \pmod{63}$ i.e. $2^{63} \not\equiv 2 \pmod{63}$, so 63 is not prime.

OR As $\gcd(2; 63) = 1$ and $2^{62} \equiv 4 \pmod{63}$; i.e. $2^{62} \not\equiv 6 \pmod{63}$, i.e. $(63) \nmid 6 \cdot 62$; so 63 is not a prime.

Theorem 1.2.5 (Chinese Remainder Theorem). [39] Let $n_1; n_2; \dots; n_r$ be positive integers that are pairwise relatively prime and their product be N . The system of r simultaneous linear congruences

$$x \equiv a_1 \pmod{n_1}$$

$$x \equiv a_2 \pmod{n_2}$$

.....

$$x \equiv a_r \pmod{n_r}$$

has a unique solution (mod N):

$$x \equiv N_1x_1a_1 + N_2a_2x_2 + \dots + N_r a_r x_r \pmod{N}$$

$$\text{where } N_i x_i \equiv 1 \pmod{n_i} \text{ and } N_i = \frac{N}{n_i} \quad (1 \leq i \leq r).$$

Definition 1.2.2 (Legendre Symbol and Jacobi Symbol). [39] Let p be an odd prime, a $\in \mathbb{Z}$ and $p \nmid a$: If $x^2 \equiv a \pmod{p}$ has solution, then we say that a is a quadratic residue (mod p), otherwise a quadratic nonresidue (mod p).

There are $\frac{1}{2}(p-1)$ quadratic residues and equally many quadratic nonresidues (mod p). Legendre symbol $\left(\frac{a}{p}\right)$ is defined as :

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{if } a \text{ is quadratic residue (mod } p) \\ -1 & \text{if } a \text{ is quadratic nonresidue (mod } p) \end{cases}$$

Properties:[39]] Let $a, b \in \mathbb{Z}$ such that $p \nmid ab$ (p an odd prime). Then

$$(i) \ a \equiv b \pmod{p} \Rightarrow \left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$$

$$(ii) \ \left(\frac{ab}{p}\right) = \left(\frac{a}{p}\right) \left(\frac{b}{p}\right)$$

$$(iii) \ \left(\frac{a^2}{p}\right) = 1$$

$$(iv) \ \left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}}$$

$$(v) \ \left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}} = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod{8} \\ -1 & \text{if } p \equiv \pm 3 \pmod{8} \end{cases}$$

Let $m = p_1 p_2 \dots p_r$ where the p_i are odd primes, not necessarily distinct. Let $\gcd(a, m) = 1$. Let $\left(\frac{a}{p_i}\right)$ denote the Legendre symbol for each $i, 1 \leq i \leq r$. Then $\left(\frac{a}{m}\right) = \left(\frac{a}{p_1}\right) \left(\frac{a}{p_2}\right) \dots \left(\frac{a}{p_r}\right)$. Here left side $\left(\frac{a}{m}\right)$ is Jacobi symbol. Properties of Jacobi symbols are similar to Legendre symbol.

1.2.1 Taxicab Number

There is an illustrious story in the history of mathematics about the great Indian mathematician Srinivasan Ramanujan. In Cambridge, while working with Hardy (1913), Ramanujan was ill and admitted to Putney Hospital. Hardy came to visit him in a taxi, the number of which was very dull for him in 1729. Actually, Ramanujan found that number to be very interesting. He said that 1729 was the smallest number which could be expressed in two different ways as a sum of two positive cubes. As a result, the numbers which are the taxicab numbers are defined as those for which solutions are found in the positive integers of the equation, $m = a^3 + b^3 = c^3 + d^3$ where $fa; bg = 6 fc; dg$.

The n th taxicab number is a positive integer which can be expressed as a sum of two cubes of positive integers in n different ways. The smallest number of the taxi is $T_a(n)$.

The concept of the second taxi number was first mentioned in 1657 by Bernard Frenicle de Bessy and was made famous in the early 20th century by the story of Srinivasa Ramanujan and G. H. Hardy, man. G. In 1938. Uh, H. Hardy and E. M. Wright has shown that such numbers exist for all positive integers, and that their proof is easily converted into a programme to generate such numbers. However, the proof does not state at all whether the numbers thus generated are the smallest positive and therefore cannot be used for the actual value of $T_a(n)$. Following six taxicab (smallest in size) are known .

$$\begin{aligned} T_a(1) &= 2 = 1^3 + 1^3 \\ T_a(2) &= 1729 = 1^3 + 12^3 = 9^3 + 10^3 \\ T_a(3) &= 87539319 = 167^3 + 436^3 = 228^3 + 423^3 = 255^3 + 414^3 \\ T_a(4) &= 6963472309248 = 2421^3 + 19083^3 = 5436^3 + 18948^3 = 10200^3 + 18072^3 = 13322^3 + 16630^3 \\ T_a(5) &= 48988659276962496 = 38787^3 + 365757^3 = 107839^3 + 362753^3 = 205292^3 + 342952^3 = 221424^3 + 336588^3 = 231518^3 + 331954^3 \\ T_a(6) &= 24153319581254312065344 = 582162^3 + 28906206^3 = 3064173^3 + 28894803^3 = 8519281^3 + 28657487^3 = 16218068^3 + 27093208^3 = 17492496^3 + 26590452^3 = 18289922^3 + 26224366^3 \end{aligned}$$

$T_a(2)$ is also known as the Hardy-Ramanujan number. The subsequent taxicab numbers were found with the help of supercomputers. John Leech obtained $T_a(3)$ in 1957. $T_a(4)$ found in 1991 [59]. J. A. Dardis found $T_a(5)$ in 1994 and it was confirmed by David W. Wilson [19] in 1999. In 2003 Calude S.etal found $T_a(6)$ [10]. $T_a(6)$ was announced by Uwe Hollerbach on March 9, 2008. In [14] upper bound for taxicab and cabtaxi numbers are given.

Cubefree number means a positive integer that is not divisible by any p^3 where p is a prime. If a cubefree taxicab number T is written as $T = x^3 + y^3$, then x and y are relatively prime. Among the taxicab numbers $T_a(n); 1 \leq n \leq 6$; only $T_a(1)$ and $T_a(2)$ are

cubefree taxicab numbers. The smallest cubefree taxicab number with three representations was discovered by Paul Vojta in 1981 while he was a graduate student. It is

$$15170835645 = 517^3 + 2468^3 = 709^3 + 2456^3 = 1733^3 + 2152^3 = 3^2 \times 5 \times 7 \times 31 \times 37 \times 199 \times 211.$$

The smallest cubefree taxicab number with four representations was discovered by Stuart Gascoigne and independently by Duncon Moore in 2003. It is $1801049058342701083 = 92227^3 + 1216500^3 = 136635^3 + 1216102^3 = 341995^3 + 1207602^3 = 600259^3 + 1165884^3$.

Positive integers, three representations, not cube free

$$1207602^3 = 600259^3 + 1165884^3$$

Positive integers, three representations, not cube free

$$87530319 = 436^3 + 167^3 = 423^3 + 228^3 = 41^3 + 255^3 = 3^2 \times 7 \times 31 \times 67 \times 223$$

$$1148834232 = 1044^3 + 222^3 = 929^3 + 718^3 = 846^3 + 816^3 = 2^4 \times 3^3 \times 7 \times 13 \times 211 \times 277.$$

1.3 Structure of the Thesis

1.3.1 Chapter wise Overview

Chapter 2. Pythagorean Triples

Section 2 examinations the different properties of crude Pythagorean triples with its applications to different zones of Mathematics. We likewise demonstrated some new properties of Pythagorean triples. We have talked about certain nuts and bolts of compatible numbers. We have additionally talked about

- 1) Existence of Pythagorean triples as far as A. P. what's more, nonexistence in G. P.
- 2) Nonexistence of Pythagorean triples regarding Harmonic Progressions
- 3) Irrationality of some genuine numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and so on with the assistance of various properties of Pythagorean triples. For uses of Pythagorean triples one may utilize and properties of Pythagorean triples.

For utilizations of Pythagorean triples one may utilize and.

we have found different kinds of triples of positive numbers with the end goal that the total of any two directions is an ideal squares, 3D shapes, fourth powers, fth powers and so forth. We have examined the accompanying outcomes in subtleties in this part. For the instance of impeccable squares use

R1: For assurance of unmistakable $a; b; c \in \mathbb{N}$ with $n \geq 2$, to such an extent that $a + b; a + c; b + c$ are n th intensity of positive numbers.

R2: Determination of a trio $(a; b; c)$ of particular positive numbers with the end goal that $ja^2; ja^2; ja^2; ja^2$ are immaculate squares.

These sorts of triples are emerges from Pythagorean triples which we talked about in section 2.

R3: Determination of a trio $(a; b; c)$ of particular positive whole numbers with the end goal that $a+b; a+ c; b + c$ are flawless squares.

R4: Triplets with the assistance of a known trio.

R5: Triplets from four tuples with a similar property.

With the assistance of the conditions

$$a = \frac{1}{2}(r^n + s^n - t^n)$$

$$b = \frac{1}{2}(r^n - s^n + t^n)$$

$$c = \frac{1}{2}(-r^n + s^n + t^n)$$

we have chosen $s; t$ in terms of r and found the triplets with sum of any two coordinates is cube, fourth power of positive integers. We have generalised this result for given any positive integer n .

R6: Four tuples of distinct positive integers such that sum of any two of its coordinates is cube of a positive integer.

Also with the help of Taxicab numbers we have found triplets with sum of any two coordinates as cube of a positive integer.

There are many different techniques for finding the value of $\sum_{r=1}^n g(r)$ where $g(r)$ is a polynomial in r [30]. We've developed a very simple technique to find the value of $\sum_{r=1}^n g(r)$. Knowing the result for higher degrees, we have achieved results for lower degrees through a process like differentiation. These results are also obtained through Newton's forward difference interpolation formula and through the use of difference operator operation. In this chapter, we have developed various techniques for the study of the finite sum of polynomial expressions, which are:

Integral Technique which a) determines P_n for $p = 1, 2, 3, \dots$ and

Degrees by a process like differentiation. These results are also obtained by Newton's forward difference interpolation formula and by using difference operator operation. In this chapter we have

developed different techniques for the study of finite sum of polynomial expressions which are :

- 1) Integral Technique which a) determines $\sum_{k=1}^n k^p$ for $p = 1; 2; 3;$ and b) Determines $Pg(n)$ when $g(n)$ is a polynomial.
- 2) Differentiation Techniques to determine $\sum_{k=1}^n Pn^k$, $p \geq 0$.
- 3) Forward Difference Techniques With the help of these we have proved the following formulae

$$\begin{aligned} \text{a) } 1 + 2 + 3 + \dots + n &= \frac{n(n+1)}{2} \\ \text{b) } 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(n+1)(2n+1)}{6} \\ \text{c) } 1^3 + 2^3 + 3^3 + \dots + n^3 &= \left[\frac{n(n+1)}{2} \right]^2 \\ \text{d) } 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \\ \text{e) } 1^5 + 2^5 + 3^5 + \dots + n^5 &= \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12} \\ \text{f) } 1^6 + 2^6 + 3^6 + \dots + n^6 &= \frac{n^7}{7} + \frac{n^6}{2} + \frac{n^5}{2} - \frac{n^3}{6} + \frac{n}{42} \text{ etc.} \end{aligned}$$

Triangular numbers are gurate numbers as they are represented by some xed geo-metric patterns.

PROBLEM STATEMENT:

“A study on the usage of number theory for the formulation of mathematical problems”

Problem 1. For what integer n is $\frac{2n-1}{n+7}$ an integer?

Proof. If $2n-1$ $n+7$ is an integer, then

$$n + 7 | 2n - 1.$$

Utilizing Theorem 1.4, we have that

$$n + 7 | 2(n + 7).$$

Using a comparable speculation again, this time with derivation we have

$$n + 7 | 2(n + 7) - (2n - 1)$$

which in the wake of rearranging implies

$$n + 7 | 15.$$

All the components of 15 are $-15, -5, -3, -1, 1, 3, 5, 15$ and these are for the most part the qualities that $n + 7$ can take, so the arrangements are:

$$n = -22, -12, -10, -8, -6, -4, -2, 8.$$

Notice, that negative arrangements are obviously permitted if the inquiry doesn't determine something else!

OBJECTIVES OF THE STUDY

1. To study the Pythagorean triples delineates interest in joining number theory and geometry.
2. To study Arithmetic of algebraic objects is indeed very close to that of natural numbers.

CONCLUSION

The proposed strategies examine the added substance and the multiplicative properties of the number-growth capability. Number-crunching capacity includes a true or complex valued capacity $a(n)$ to determine the normal number set that expresses some math property of n . Added substance and multiplication capacity is the most significant amount of juggling capacity. Added substance work is the math set $f(n)$ of + ve number n , when a, b is co-prime, the output capacity is the capacity option. This capacity $f(n)$ is considered to be a completely added substance if $f(a+b) = f(a) + f(b)$ + ve numbers and b are not co-prime. Completely added substance is to be used in this case by simple use with absolutely multiplicative functions. At the point where f is a completely added substance, work $f(1) = 0$. The audit of ongoing writing on the added substance and the multiplication capacity is discussed. The properties of the two capacities are examined in detail. Adjusted binomial ring elements for added substance and multiplication capacity are proposed. The ring structure over the mixture of the added substance and the multiplicative capacity can be performed with the modified binomial capacity. The Ring is a set of R , formed with double parallel tasks of expansion and duplication, correspondingly spoken to expansion and deduction. Fundamentally, R is also an abelian set, with zero as the character. Clearly, in the ring, it is plausible to extend, deduct and increase without isolating the set, anyway in the slant field, division can also be proceeded. Further, Isomorphism and Ring Homomorphisms are inferred in this hypothesis.

Added capacity of the substance in aggregate primes is the set of multiplication capacity squares. The fundamental hypothesis of math is that every $n > 1$ can be connoted as a result of unmistakable prime elements. Added substance Prime number Theory was concerned about the delineation of whole numbers as aggregates of primes or of closely related whole numbers. The premiums are described as multiplicative properties, while the issue includes the properties of the added substance. Each + ve number can only be specified as a different prime number. In this strategy, prime number hypothesis, added substance issue including prime, binomial infringement point law for added substance prime markers, prime component hypothesis for added substance arrangement and finally additive works are described. In addition, the

modified ergodic hypotheses of the added substance and the multiplication capacity are further investigated. This hypothesis manages dynamic strategies with an invariant measure and related issues. It is used for different segments of arithmetic which, in large part, incorporates the development of periodicity properties for unrivalled type techniques. Another critical part of the hypothetical ergodic idea is the question of the metric application of strategies. A fantastic part of this hypothesis and its use of stochastic techniques is used by different ideas of entropy for dynamic strategies.

In addition, the portrayal of ergodicity has broken down. At that point, the meanings of the ergodic hypothesis are discussed. In addition, the ergodic hypothesis, including algorithmically arbitrary grouping, is depicted. In the long run, the modified ergodic hypotheses are determined. The grouping of the added substance and the multiplication capacity, characterised at an arbitrary stage, is inspected. In an irregular stage, R_n is the symmetrical gathering and enveloping of all possible capacities that bijectively map the set of first n numbers $\{1, 2, \dots, n\}$ into itself. What could be compared to Kolmogorov-Rogozin imbalance is used to decide on the centralization of totally added substance work characterised by irregular stages. For the multiplicative capacity, the Voronoi summability is used to dissect the value of the multiplicative capacity grouping on irregular change.

REFERENCES

1. C. Adiga (1992). *On the representations of an integer as a sum of two or four triangular numbers*, Nihonkai Mathematical Journal, vol. 3, No.2, pp. 125-131.
2. C. Adiga, B. C. Berndt, S. Bhargava and G. N. Watson (1985). *Chapter 16 of Ramanujan's second notebook: Theta-functions and q-series*, Mem. Amer. Math. Soc, No. 315, 53, Amer. Math. Soc, Providence, 1985.
3. C. Adiga, R.Y. Denis and K.R. Vasuki (2001). *On some continued fraction expansions for the ratios of $2W_2$* , Special Functions: Selected Articles, P. K. Baneiji (Ed.), Scientific Publishers (India), pp. 1-16.
4. C. Adiga, M. S. Mahadeva Naika and Ramya Rao (2002). *Integral representations and some explicit evaluations of a continued fraction of Ramanujan*, JP Jour. Algebra, Number Theory & Appl. 2(1), pp. 5-20.
5. C. Adiga, M. S. Mahadeva Naika and K. Shivashankara (2002). *On some P-Q eta function identities of Ramanujan*, Indian J. Math., vol. 44, No. 3, pp. 253-267.
6. C. Adiga and D. D. Somashekara (1998). *On some Rogers-Ramanujan type continued fraction identities*, Mathematical Balkanica, New Series, vol. 12N0S. 1-2, pp. 37-45.
7. C. Adiga, Taekyun Kim, M. S. Mahadeva Naika and H. S. Madhusudhan, *On Ramanujan's cubic continued fraction and explicit evaluations of theta-functions*, Indian J. Pure and Applied Mathematics, (to appear).
8. C. Adiga, K. R. Vasuki and M. S. Mahadeva Naika (2001). *On some new identities involving integrals of theta-functions*, Advanced Studies in Contemporary Mathematics, 3, No. 2, pp. 1-11.
9. C. Adiga, K. R. Vasuki and M. S. Mahadeva Naika (2002). *Some new explicit evaluations of Ramanujan's cubic continued fraction*, New Zealand Journal of Mathematics, 31, pp. 1-6.
10. W. A. Al-Salam and M. E. H. Ismail (1983). *Orthogonal polynomials associated with the Rogers-Ramanujan continued fraction*, Pacific J. Math., 104, pp. 269-283.

Corresponding Author

K. Gunasekar*

Research Scholar (Part Time) OPJS University,
Churu, Rajasthan