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A Two Dissimilar Unit Parallel System with Administrative Delay in Repair

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ABSTRACT:-

This paper deals with a two non-identical unit parallel system model assuming the concept of administrative delay for getting the repairman available with the system. All the failure time distribution is taken exponentially. Various measures of system effectiveness have been obtained by making use of Poission processes and supplementary variable technique.

INTRODUCTION

Various authors including (Gopalan and D'Souza, 1975), (Goel and Gupta, 1983), (Gupta and Singh, 1985), have analysed the two-unit cold standby system models assuming that the repair facility is instantaneously available so that the repair is started immediately upon the failure of a unit provided that he/she is not busy in repairing the other unit. In this paper when an operative unit fails, the repair is not available immediately due to some administrative action. In this paper investigate a two non-identical unit parallel system model assuming the concept of administrative delay for getting the repairman available with the system. Each unit has two modes Normal (N) and total failure (F). Using the supplementary variable technique the following reliability characteristics of interest have been obtained.

Reliability of the system and mean time to system failure (MTSF).

(i) Point wise availability of the system in terms of its Laplace Transform.

(ii) Expected up time of the system in the interval $(0, t)$.

(iii) Actual steady-state probabilities for the system being in various states.

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(iv) Expected busy period of the repairman in the interval $(0, t)$.

(v) Expected profit incurred by the system in $(0, t)$ and in steady state.

The behaviour of MTSF and profit function have also been studies through graphs in respect of some parameters in a particular case.

Notations

α_1 : Constant failure rate of unit-1.

α_2 : Constant failure rate of unit-2.

η_1 : Constant rate of arrival of the repairman at the system when one unit is failed.

η_2 : Constant rate of arrival of the repairman at the system when both units are failed.

$\mu_i(x), g_i(x)$: General rate of repair and p.d.f. of repair time if unit-i. *i* = 1, 2

$\mu(x), g(x)$: General rate of simultaneous repair and corresponding p.d.f. of repair time of both the units.

$P_j(t)$: Probability that the system is in state s_j at epoch *t*. *j* = 0, 1, 2,…7

$Q_k(x, t)\,dx$: Probability that the system is in state s_k at epoch *t* and has sojourned in this state for during between *x* and $x + dx$.

* : Laplace transformation.

O : Operative

W : Waiting for repair

r : Under repair

Model

Possible states of the system and transition into the states for this model are shown in fig. 1. There are s_0, s_1, s_2, s_3, s_5 are up states. States s_4, s_6, s_7 are down states.

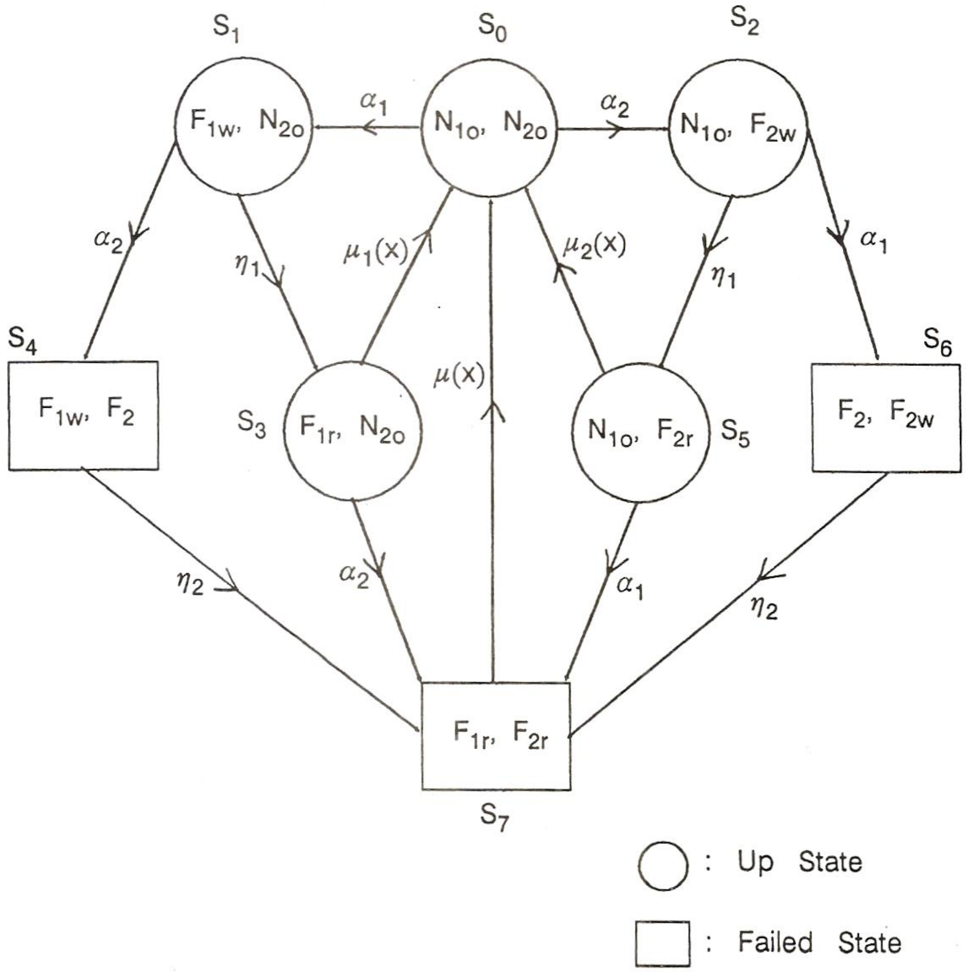

Fig. 1: State Transition Diagram

Simple probabilistic considerations give the following equations, the integral are all from 0 to ∞.

$$P_0(t+\Delta t) = P_0(t)\left[1-(\alpha_1+\alpha_2)\Delta t\right] + \int Q_3(x_3 t)\mu(x)\,dx\,\Delta t$$

$$+\int Q_5(x,t)\,\mu_2(x)\,dx\,\Delta t + \int Q_7(x,t)\,\mu(x)\,dx\,\Delta t + O(\Delta t)$$

$$P_1(t+\Delta t) = P_1(t)\left[1-(\alpha_2+\eta_1)\Delta t\right] + P_0(t)\alpha_1\Delta t + O(\Delta t)$$

$$P_2(t+\Delta t) = P_2(t)\left[1-(\alpha_1+\eta_1)\Delta t\right] + P_0(t)\alpha_2\Delta t + O(\Delta t)$$

$$Q_3(x+\Delta t, t+\Delta t) = Q_3(x,t)\left[1-\{\alpha_2+\mu_1(x)\}\Delta t\right] + O(\Delta t)$$

$$P_4(t+\Delta t) = P_4(t)\left[1-\eta_2\,\Delta t\right] + P_1(t)\alpha_2\Delta t + O(\Delta t)$$

$$Q_5(x+\Delta t, t+\Delta t) = Q_5(x,t)\left[1-\{\alpha_1+\mu_2(x)\}\Delta t\right] + O(\Delta t)$$

$$P_6(t+\Delta t) = P_6(t)\left[1-\eta_2\,\Delta t\right] + P_2(t)\alpha_2\Delta t + O(\Delta t)$$

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$$Q_7(x+\Delta t, t+\Delta t) = Q_7(x,t)\left[1-\mu(x)\right] + O(\Delta t)$$

Thus, we have

$$P_1^*(s) = \frac{\alpha_1}{s+\alpha_2+\eta_1} P_0^*(s) = A_1(s)\, P_0^*(s)$$

$$P_2^*(s) = \frac{\alpha_2}{s+\alpha_1+\eta_1} P_0^*(s) = A_2(s)\, P_0^*(s)$$

$$P_3^*(s) = \frac{\eta_1\alpha_1}{s+\alpha_2+\eta_1}\left\{\frac{1-g_1^*(s+\alpha_2)}{(s+\alpha_2)}\right\} P_0^*(s) = A_3(s) P_0^*(s)$$

$$P_4^*(s) = \frac{\alpha_1\alpha_2}{(s+\eta_2)(s+\alpha_2+\eta_1)} P_0^*(s) = A_4(s) P_0^*(s)$$

$$P_5^*(s) = \frac{\eta_1\alpha_2}{(s+\alpha_1+\eta_1)}\left\{\frac{1-g_2^*(s+\alpha_1)}{(s+\alpha_1)}\right\} P_0^*(s) = A_5(s) P_0^*(s)$$

$$P_6^*(s) = \frac{\alpha_1\alpha_2}{(s+\eta_2)(s+\alpha_1+\eta_1)} P_0^*(s) = A_6(s) P_0^*(s)$$

$$P_7^*(s) = \left[\eta_2\{A_4(s)+A_6(s)\}+\alpha_1 A_5(s)+\alpha_2 A_3(s)\right]\left\{\frac{1-g^*(s)}{s}\right\} P_0^*(s) = A_7(s) P_0^*(s)$$

$$P_0^*(s) = \left[s+\alpha_1+\alpha_2-\frac{\eta_1\alpha_1 g_1^*(s+\alpha_2)}{s+\alpha_2+\eta_1}-\frac{\eta_1\alpha_2 g_2^*(s+\alpha_1)}{s+\alpha_1+\eta_1}\right.$$

$$\left.-\eta_2\{A_4(s)+A_6(s)\}\, g^*(s) - \{\alpha_1 A_5(s)+\alpha_2 A_3(s)\}\, g^*(s)\right]^{-1}$$

The probability that the system will be in state s_0 in the long run is given by

$$P_0 = \lim_{t\to\infty} P_0 = \lim_{s\to 0} s\, P_0^*(s)$$

$$A_1 = A_1(0) \quad A_2 = A_2(0) \quad A_3 = A_3(0) \quad A_4 = A_4(0)$$

$$A_5 = A_5(0) \quad A_6 = A_6(0) \quad A_7 = A_7(0)$$

Availability Analysis:

$$A(\infty) = \lim_{s\to 0} s\, A^*(s)$$

$$= (1+A_1+A_2+A_3+A_5)P_0$$

Reliability and MTSF

$$R^*(s) = LT[R(T)]$$

$$\eta_2 = \mu(x) = 0$$

$$R^*(s) = [P_0^*(s) + P_1^*(s) + P_2^*(s) + P_3^*(s) + P_5^*(s)] \Big/ \eta_2 = \mu(x) = 0$$

$$MTSF = \int R(t)\, dt = \lim_{s \to 0} R^*(s)$$

$$= [1 + A_1 + A_2 + A_3 + A_5] \times \left[\alpha_1 + \alpha_2 - \frac{\eta_1 \alpha_1 g_1^*(\alpha_2)}{\alpha_2 + \eta_1} = \frac{\eta_2 \alpha_2 g_2^*(\alpha_1)}{\alpha_1 + \eta_1}\right]^{-1}$$

Busy period analysis of the repairman during $(0, t)$.

Busy period of repairman in repairing a unit-1 during $(0, t)$ is

$$\mu_b^1(t) = \int_0^t P_3(u)\, du$$

$$\mu_b^{1*}(s) = A_3(s) P_0^*(s) / s$$

Busy period of repairman in repairing a unit-2 during $(0, t)$ is

$$\mu_b^2(t) = \int_0^t P_5(u)\, du$$

$$\mu_b^*(s) = A_5(s) P_0^*(s) / s$$

Busy period of repairman in repairing two units during $(0, t)$ is

$$\mu_b^3(t) = \int_0^t P_7(u)\, du$$

$$\mu_b^*(s) = P_7^*(s) / s = A_n(s)\, P_0^*(s) / s$$

Profit Analysis in $(0, t)$

$P(t)$ = Total revenue in $(0, t)$ – Expected cost of repair in $(0, t)$

$$= K_0 \mu_{up}(t) - K_1 \mu_b^1(t) - K_2 \mu_b^2(t) - K_3 \mu_b^3(t)$$

where

$$\mu_{up}(t) = \int_0^t A(u)du$$

K_0 = revenue per-unit up-time

K_1 = per-unit of time repair cost of failed unit-1

K_2 = per-unit of time repair cost of failed unit-2

K_3 = per-unit of time repair cost of both the failed unit-1 and unit-2.

In steady state

$$P = \lim_{t\to\infty} \frac{P(t)}{t}$$

$$= [(1 + A_2 + A_2 + A_3 + A_5)\, K_0 - K_1 A_4 - K_2 A_6 - K_3 A_7\,]\, P_0$$

Particular Case

All the repair time distribution are exponential.

$$g_1(x) = \mu_1\, e^{-\mu_1 x} \quad g_2(x) = \mu_2\, e^{-\mu_2 x} \quad g(x) = \mu\, e^{-\mu x}$$

$$m = \frac{1}{\mu} \quad m_{\alpha_1} = \frac{\mu_2}{(\alpha_1 + \mu_2)^2} \quad m_{\alpha_2} = \frac{\mu_1}{(\alpha_2 + \mu_1)^2}$$

$$g_1^*(\alpha_2) = \frac{\mu_1}{\alpha_2 + \mu_1} \quad g_2^*(\alpha_1) = \frac{\mu_2}{\alpha_1 + \mu_2}$$

$$A_3 = \frac{\eta_1 \alpha_1}{(\alpha_2 + \eta_1)(\alpha_1 + \mu_2)}$$

$$A_5 = \frac{\eta_1 \alpha_2}{(\alpha_1 + \eta_1)(\alpha_1 + \mu_2)}$$

$$A_3 = \frac{-\eta_1 \alpha_1 (2\alpha_2 + \mu_1 + \eta_1)}{(\alpha_2 + \eta_1)^2\, (\alpha_2 + \mu_1)^2}$$

$$A_5 = \frac{-\eta_1 \alpha_2 (2\alpha_1 + \mu_2 + \eta_1)}{(\alpha_1 + \eta_1)^2\, (\alpha_1 + \mu_2)^2}$$

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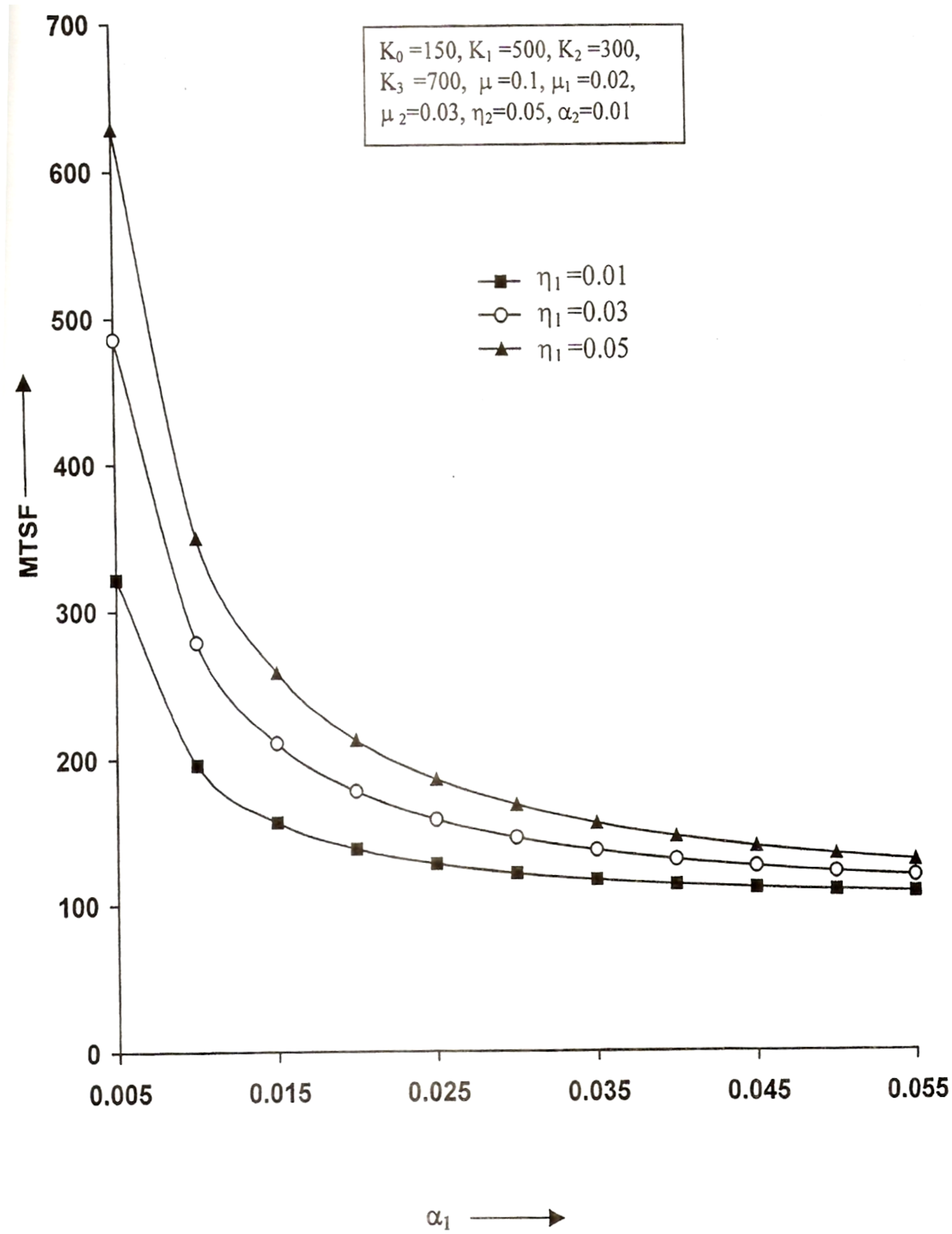

Fig. 2: Relation between MTSF and Failure rate

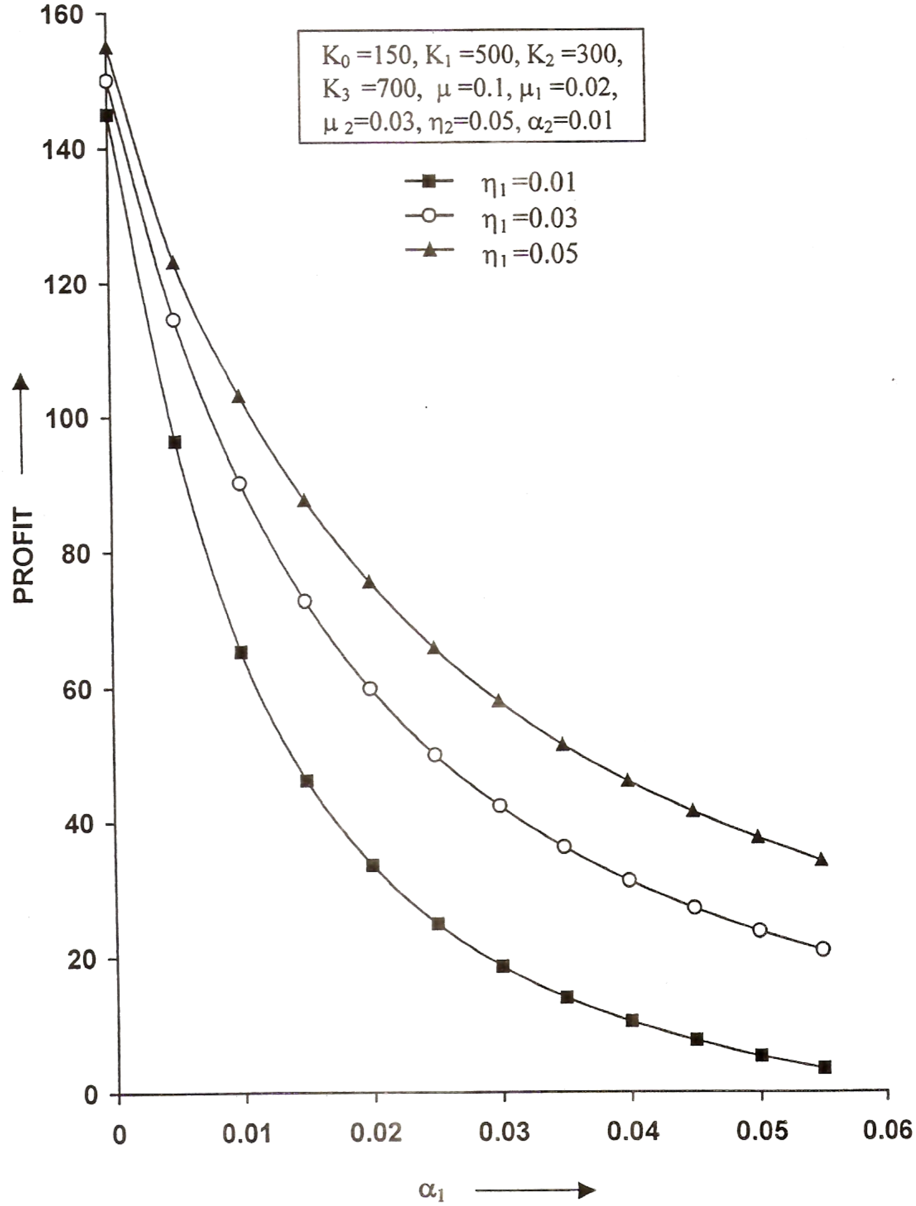

Fig. 3

CONCLUSION

Relation between failure and profit from fig. 2 MTSF decreases when failure rate increases. In fig. 3 if failure rate increases then profit decreases.

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