Some Important Theorem on Balanced Fuzzy Set

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Abstract: In this paper we have obtained some important result on balanced fuzzy set and their properties.

Key words: Fuzzy set, convex fuzzy set and balanced fuzzy set.

INTRODUCTION – 1.1

Let X is a non-empty set called universal set. Then, by a fuzzy set on X is meant a function A:X $\rightarrow$(0,1), 'A' is called a membership function, A(x) is called the membership grade of X and we write

A = {(x, A(x)) : x $\in$ X}

A fuzzy set A in a linear space E is set to be convex if for every $\lambda \in [0,1]$

$\lambda A+(1-\lambda)A \subset A$

A fuzzy set A in a vector space E is said to be balanced if $\lambda A \subset A$ for every scalar λ with $|\lambda| \leq 1$

THEOREM – 1.2

Let A and B are balanced fuzzy set in a vector space E over K then, A+B is also a balanced fuzzy set in a vector space E.

PROOF

Let us assume that A and B are balanced fuzzy set in a vector space E over the field K, then we have $\lambda A \subset A$ for every scalar λ with $|\lambda| \leq 1$ and also $\lambda B \subset B$ for all scalar λ with $|\lambda| \leq 1$. Now we have

$\lambda(A+B) = \lambda A + \lambda B$

$\subset A+B$

This shows that $\lambda(A+B) \subset A+B$.

Hence (A+B) is a balanced fuzzy set in E.

THEOREM – 1.3

Let us consider $\{A_i\}_{i \in I}$ is a family of balanced fuzzy set in a vector space E, then $A = \bigcap_{i \in I} A_i$ is a balanced fuzzy set in E.

PROOF

Let $\{A_i\}_{i \in I}$ be a family of balanced fuzzy set in a vector space E. Then, we have

$\lambda A_i \subset A_i$ for every scalar λ with $|\lambda| \leq 1$

That is $A_i(\lambda x) \geq A_i(x)$ for every scalar $|\lambda| \leq 1$ ….(i)

Again let $A = \bigcap_{i \in I} A_i$

thus $A(y) = \inf_{i \in I} A_i(y)$ for every $y \in E$

Hence $A(\lambda x) = \inf_{i \in I} A_i(\lambda x)$

now from (i) we have

$A(\lambda x) \geq \inf_{i \in I} A_i(x)$

$= A(x)$ for all scalar λ with $|\lambda| \leq 1$ and every $x \in E$

Hence $A = \bigcap A_i$ is a balanced fuzzy set in E.

REFERENCES:

(1) Ganesh, M.(2006) Introduction Of Fuzzy Sets And Fuzzy Logic, P.H.I. New Delhi.

(2) Katsaras, A.K. And Liu, D.B. (1977), "Fuzzy Vector Spaces And Fuzzy Topological Vector Spaces", J.Math. Anal. Appl., 58, Pp. 135-146.

(3) Nanda, S. Fuzzy Fields And Fuzzy Spaces, Fuzzy Sets And System 19(1986) 89-94.