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**A STUDY ON THE RECONSTRUCTION OF  
BAYESIAN DIFFUSES OPTICAL TOMOGRAPHY**

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# A Study on the Reconstruction of Bayesian Diffuses Optical Tomography

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**Abstract** – Following the assembly of a triple-modality SPECT-CT-OT small animal imaging system provided intrinsically co-registered projection data of all three submodalities and under the assumption and investigation of dual-labeled probes consisting of both Huorophores and radionuclides, a novel multi-modal reconstruction strategy is presented in this paper aimed at improving fluorescence mediated tomography (FMT). The following reconstruction procedure is proposed: Firstly, standard X-ray CT image reconstruction is performed employing the FDK algorithm. Secondly, standard SPECT image reconstruction is performed using OSEM. Thirdly, from the reconstructed CT volume data the surface boundary of the imaged object is extracted for finite element definition. Finally, the reconstructed SPECT data is used as a priori information within a Bayesian reconstruction framework for optical (FMT) reconstruction. We provide results of this multi-modal approach using phantom experimental data and illustrate that this strategy does suppress artifacts and facilitates quantitative analysis for optical imaging studies.

We present a combined classification and reconstruction algorithm for diffuse optical tomography (DOT). DOT is a nonlinear ill-posed inverse problem. Therefore, some regularization is needed. We present a mixture of Gaussians prior, which regularizes the DOT reconstruction step. During each iteration, the parameters of a mixture model are estimated. These associate each reconstructed pixel with one of several classes based on the current estimate of the optical parameters. This classification is exploited to form a new prior distribution to regularize the reconstruction step and update the optical parameters.

## INTRODUCTION

Diffuse Optical Tomography (DOT) is a non-invasive imaging modality that makes use of the light in the Near- Infrared (NIR) spectrum . The inverse problem in DOT involves reconstruction of spatially varying absorption and scattering properties as well as fluorophore lifetime and yield in tissues from boundary measurements. These fundamental quantities can be utilized to obtain tissue oxy- and deoxyhemoglobin concentrations, blood oxygen saturation. water, fat. and contrast agent uptake in tissue. The unique physiological and biochemical information offered by DOT is very valuable for practical applications such as breast cancer diagnosis. Cognitive activity monitoring. Brain tumor and hemorrhage detection with a growing list of applications in fluorescence tomographic imaging.

Recently several research groups reported development of hybrid imaging systems combining optical methods with high resolution anatomical imaging techniques. These include a concurrent X-ray tom synthesis-DOT system at Massachusetts General Hospital. MRI-DOT/DOS (Diffuse Optical

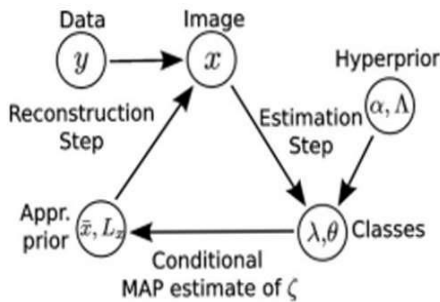
Spectroscopy) systems at University of Pennsylvania, University of California at Irvine and Dartmouth College. These multi-modality developments arc all motivated by the fact that DOT offers unique functional information (such as tissue oxy- and deoxy- hemoglobin concentrations) while high resolution anatomical imaging modalities provide complementary information for disease diagnosis and understanding with superior localization and spatial resolution. A number of studies have shown that use of a *priori* high resolution anatomical images leads to improved diffuse optical image reconstruction as well as improved spatial resolution.

In all the studies referenced above, the performance of the DOT image reconstruction relies on the assumption that the correlation between the anatomical and optical images is high. However, there may be regions in the optical image that do not have any anatomical counterparts. As a result the assumption of strong optical-anatomy correlation may cause undesirable, erroneous bias in optical image reconstruction. The hierarchical Bayesian framework affords such flexibility in designing prior image and

noise models to address the optical-anatomy correlation.

## OPTICAL TOMOGRAPHY

Diffuse optical tomography is a new medical imaging modality with potential applications in functional imaging of the brain and in breast cancer detection. This method seeks to recover optical parameters of blood and tissue from boundary measurements of light transmission in the visible and near-infrared range. The reconstructed images of the spatial distribution of tissue parameters can be related directly to physiologically important properties such as blood and tissue oxygenation state. Instrumentation for optical



tomography is portable and relatively inexpensive, and can provide a viable alternative to currently available systems such as functional magnetic resonance imaging.

Data acquisition systems consist of a light source such as an infrared laser, illuminating the body surface at different source locations in succession. The light which has propagated through the tissue is then measured at multiple detector locations on the surface. Biological tissue is strongly scattering at the wavelengths used in optical tomography, which generally makes the recovery of tissue parameters from the boundary data a highly nonlinear problem.

The experimental systems in use today utilize either ultra-short input pulses (time-domain systems) or continuous intensity-modulated input (frequency domain systems). In the former case, measurements consist of the temporal dispersion of the transmitted pulse, measured at a resolution in the order of picoseconds. In the latter case, the measurements consist of the complex intensity of the transmitted photon density wave, most commonly measured in terms of the phase shift and modulation amplitude. The frequency domain version of the problem is considered in this paper.

## FORWARD MODEL OF OPTICAL TOMOGRAPHY

In order to characterize a forward model, we need to model the photon propagation in the scattering medium. In this work, we focus on the reconstruction of absorption coefficients, hence we keep scattering

coefficient and consequently the diffusion coefficient constant throughout our calculations. As a result, the use of the following diffusion equation given in frequency domain is sufficient to define the forward model:

$$-\frac{i\omega}{c}\phi(r) - D\nabla^2\phi(r) + \mu_a(r)\phi(r) = A\delta(r_s)$$

(1)

$\phi(r)$  represents the spatially varying total field due to the point source  $A\delta(r_s)$  located at  $r = r_s$ .  $\omega$  denotes the frequency,  $c$  is the speed of light and  $i = \sqrt{-1}$ .  $D$  is the spatially invariant diffusion coefficient and  $\mu_a(r)$  stands for the spatially varying absorption coefficient.

The nonlinear relationship that is observed between  $\mu_a(r)$  and  $\phi(r)$  in equation (1) increases the computational burden in the reconstruction of the absorption coefficient distribution. In order to overcome this difficulty, we use a linear model, which is suitable for a medium with absorption coefficient values less than 0.5, such as human breast.

Let  $\phi(r) = f(\mu_a(r))$  be the nonlinear function relating the spatial distribution of absorption coefficient  $\mu_a(r)$  to the total field  $\phi(r)$ . A linear relationship between  $\phi(r)$  and  $\mu_a(r)$  can be obtained using the perturbation theory<sup>21-23</sup> by expressing the spatially varying absorption coefficient as  $\mu_a(r) = \mu_{a0} + \delta\mu_a(r)$ , where  $\mu_{a0}$  is the spatially invariant background absorption coefficient and  $\delta\mu_a(r)$  is the spatially varying component.

We have applied Rytov approach using a first order approximation to express the phase of the total field  $\phi(r, r_s)$  as:

$$\Phi(r, r_s) = \Phi_0(r, r_s) + \Phi_{sc}(r, r_s) \quad (2)$$

Where the phase term  $\Phi_0(r, r_s)$  is due to the constant background absorption coefficient  $\mu_{a0}$  and the term  $\Phi_{sc}(r, r_s)$  is due to only the differential absorption  $\delta\mu_a(r)$  of the medium.

In the case of DOT, multiple source-detector pairs with multiple frequencies are used. The medium of interest is discretized into  $N$  pixels and the forward problem is expressed in terms of a system of linear equations that relates the diffuse Rytov phase

$\Phi_{sc}(r)$  to the spatially varying component  $\delta\mu_a(r)$  of the absorption coefficient distribution  $\mu_a(r)$ :

$$\begin{bmatrix} \Phi_{sc}^{f_1}(r_{d_1}, r_{s_1}) \\ \vdots \\ \Phi_{sc}^{f_k}(r_{d_m}, r_{s_m}) \\ \Phi_{sc}^{f_p}(r_{d_1}, r_{s_1}) \\ \vdots \\ \Phi_{sc}^{f_p}(r_{d_m}, r_{s_m}) \end{bmatrix} = \begin{bmatrix} W_{11}^{f_1} & \cdots & W_{1N}^{f_1} \\ \vdots & \ddots & \vdots \\ W_{m1}^{f_1} & \cdots & W_{mN}^{f_1} \\ W_{11}^{f_p} & \cdots & W_{1N}^{f_p} \\ \vdots & \ddots & \vdots \\ W_{m1}^{f_p} & \cdots & W_{mN}^{f_p} \end{bmatrix} \times \begin{bmatrix} \delta\mu_a(r_1) \\ \vdots \\ \delta\mu_a(r_k) \\ \delta\mu_a(r_{k+1}) \\ \vdots \\ \delta\mu_a(r_N) \end{bmatrix} \quad (3)$$

where  $\Phi_{sc}(r_{d_i}, r_{s_i})$  is the real part of the diffuse perturbative Rytov phase for the  $i^{th}$  source-detector pair:  $W_{ij}^{f_k}$  is the real value of the weight for the  $j^{th}$  pixel and the  $i^{th}$  source-detector pair at frequency  $f_k$  and  $\delta\mu_a(r_j)$  is the differential absorption coefficient of the  $j^{th}$  pixel. The relation above can be expressed equivalently as follows:

$$y = W \times x \quad (4)$$

Where  $y$  is the measurement vector holding the perturbative Rytov phase  $\Phi_{sc}(r, r_s)$  for each source-detector pair.  $W$  Denotes the linear forward model (weight matrix) which relates the differential absorption coefficient distribution  $x = [\delta\mu_a(r_1) \quad \delta\mu_a(r_2) \quad \cdots \quad \delta\mu_a(r_N)]^T$  to the measurement vector  $y$ .

### OPTICAL TOMOGRAPHY AS A NONLINEAR III-POSED INVERSE PROBLEM

The general task of **image reconstruction** is to compute an image from the measured data. Imaging can be regarded as a parameter estimation problem or so called **inverse problem**.

#### The forward problem and inverse problem

A more rigorous treatment of the mathematics will be given in the next chapter. Here we shall be satisfied with a rather general treatment.

Suppose the data is described by a general vector  $y$  and that the image can be described by a general vector  $\theta$ . suppose that we also have a physical model that describes the relation between  $\theta$  and  $y$ , then we may write

$$y = \mathcal{P}\theta$$

where  $\mathcal{P}$  is a general operator that represents our physical model. The **forward problem** is then defined as to find the values of data  $y$  given image  $\theta$ .

In the image reconstruction problem, we are interested in the **inverse problem**. That is, to find the **image**  $\theta$ , given the data  $y$ . Or we could write

$$\theta = \mathcal{P}^{-1}y$$

where  $\mathcal{P}^{-1}$  denotes the inverse or a pseudo-inverse operator.

In x-ray imaging  $\theta$  represents an absorption map of object,  $y$  will correspond to the measured attenuation projection, and the (linear) operator  $\mathcal{P}$  is a so called **Radon transform** performing simple line integrals on the image. The inverse operator can be performed analytically using inverse Fourier transformation or using filter back projection.

In Optical Tomography the forward problem is to be able to compute the transmitted light intensities through, for example, the head when the optical properties of that head is given. However, the reconstruction problem is the inverse, we like to know what the optical parameters are **given** the measured intensities around the interrogated the organ. The model operator  $\mathcal{P}$  will be derived in the next chapter and we will see that it is based on the **diffusion equation** of heat transfer. An analytical form for the

inverse operator  $\mathcal{P}^{-1}$  does not exist in Optical Tomography because the problem is nonlinear.

#### Characterics of the OT inverse problem

To summarise, we may say that the inverse problem in Optical Tomography can be characterized by the following points:

- **Ill-posedness.** The diffusive nature of the photons is cause for the ill-posedness of problem. In practice this means that the obtained images have low resolution and are very sensitive to noise.
- **Large-scale.** The inverse problem typically deals with 1000 to several 10,000 parameters.
- **Non-linear.** The inverse problem in Optical Tomography is highly nonlinear. This precludes the use of analytic methods such as back-projection methods.

- **Dual-parameter.** The optical property tissue is characterized by two parameters. Simultaneous reconstruction of both parameters complicates the inversion and may induce crosstalk between the images.

Despite these mathematical and other practical difficulties, the promised advantages of a noninvasive oxygen monitor are huge and considerable efforts are still in progress.

## CONCLUSION

This Bayesian model is considerably more complex than the models but we have shown that by using careful quantitative assessment of prior knowledge we are able to choose rational values for the hyper parameters in the model. We have applied the model to two reconstruction examples from simulated and experimental data.

From the experimental images we have seen that the Bayesian images depend strongly on the assumptions that are made. If strong prior knowledge about the flatness of the background can be assumed improvement of the image contrast can be obtained.

Finally, we wish to suggest that when presenting Bayesian estimates it is important to state *precisely* what quantitative assumptions were made and what model is used. It is recommended *always* to present a ML image together with the Bayesian estimate. For the latter we recommend using the hold-out validated FML-ICM method for an objective choice of the optimum ML estimate.

## REFERENCE

1. Y. Yao, Y. Wang, Y. Pei, W. Zhu. and R. L. Barbour. "Frequency domain optical imaging of absorption and scattering distributions by a Born iterative method," *J. Opt. Soc. Am. A*, vol. 14, no. 1, pp. 325-342, 1997.
2. Gibson A, Hebden J and Arridge S R 2005 Recent advances in diffuse optical tomography *Phys. Med. Biol.* 50 R1—43
3. Guven M, Yazici B, Intes X and Chance B 2005 Diffuse optical tomography with a priori anatomical information *Phys. Med. Biol.* 50 2837-58
4. Intes X and Chance B 2005 Non-PET functional imaging techniques: optical. *The Radiologic Clinics of North America* 43(1) 221-34.
5. Kincaid K 2004 Optical Diagnostics continue migration from bench top to bedside *Laser Focus World* 130-4.
6. Intes X, Ripoll J, Chen Y, Nioka S, Yodanis C and Chance B 2003 In vivo continuous-wave optical breast imaging enhanced with Indocyanine Green *Med. Phys.* 30. 1039-47.
7. Allan M., Cote, D., Davidson, L., Dazai, J., Henkelman, R. (2007). 'Combined magnetic resonance and bioluminescence imaging of live mice', *Journal of Biomedical Optics* 12. 034018.
8. Arridge. S. (1999), 'Optical tomography in medical imaging'. *Inverse problems* 15. 41-41.
9. Arridge. S., Schweiger, M., Hiraoka, M., Delpy, D. (1993), 'A finite element approach for modeling photon transport in tissue'. *Medical Physics* 20. 299.