Behavioral Analysis of Single Unit System with Server Failure

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ABSTRACT: ***The reliability model for a single unit system with server failure is developed in which unit fails completely either directly from normal mode or via partial failure. A single server is provided immediately to the system which is subject to failure during inspection and repair of the unit. Repair of the unit is done at its partial failure and complete failure. If repair of the unit is not feasible, it is replaced by new one in order to avoid unnecessary expenses on repair. Priority is given to the treatment of the server upon failure over repair of the unit. The repair of the unit, treatment of the server and switch devices are considered as perfect. Using the Regenerative Point Graphical Technique (RPGT) the following system characteristics have been evaluated to study the system performance.Mean Time To System Failure (MTSF).,Total fraction of time for which the system is available,The busy period of the Server doing any given job,the number of the Server's visits. The profit analysis of the system is also carried out by using some of the system characteristics as mentioned above. Graphs are drawn to depict the behavior of the MTSF and Steady state Availability of the system for a particular case.***

***Key words*—Reliability, Availability, Priority Maintainance, Primary Circuit, Secondary Circuit, Tertiary Circuit, Base-State, Regenerative Point Graphical Technique (RPGT), MTSF, Busy Period of Server,expected number of servers visits,expected number of treatments given to server.**

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1 INTRODUCTION

The process industries are the backbone of a country for its development. The process industries must provide continuous and long term production to meet the ever increasing demand at lower costs. The reliability and availability analysis of process industries can benefit in terms of higher production, lower maintenance costs. The availability of complex systems and continuous process industries can be enhanced by considering maintenance, inspection, repairs and replacements of the parts of the failed units. A system may not be working to the fullest of its capacity in a particular state and instead it is partially available (i.e. with reduced capacity) in that state.

The researchers including Barlow et al [1], Chung et al [2] discussed the single unit system and Das et al [3], Fukuta et al [4],Kodama et al [5], Osaki et al [6] discussed two or more unit having switch-over device while, Chander et al [7], Malik et al [8] have used the Regenerative Point (RPT) and solved the transformed state equations recursively, to find $\phi_0^{\sim}(s), A_0^*(s), B_0^*(s), and\ V_0^*(s)$ corresponding to initial state '0' and then determined the parameters of the stochastic systems(under steady state conditions). Gupta, et al [9] have done the analysis of various systems by using the *'Regenerative Point Graphical Technique**(RPGT)**'* introduced by Gupta[10], for determining the Mean Time to System Failure(MTSF), Availability, Busy period of Server, number of Server's visits and number of Replacement etc. (under steady state conditions). Jindal[11] analysed a single unit Redundant system having perfect switch-over device.But, the difficulty for the evaluation of key parameters of the system increases with the increase in the number of the transition states and circuits in the transition diagram of the system, it also becomes difficult to locate all the paths from the initial state to the other states and the various circuits along the different paths while using *Regenerative Point Graphical Technique* **(RPGT).**

In reliability analysis of repairable systems, it is usually assumed that the repair facility neither fails nor deteriorates and operating unit enters directly into complete failed state. But, in practice, a repair facility is subject to failure while performing jobs due to some causes including mishandling of the system, electric shock and carelessness. However, the repair may resume the job after taking some treatment.

In view of the above and considering the fact that single-unit systems are frequently used in many sphere of life due to their inherent reliability and common man's affordability, here reliability model for a single unit system with server failure is developed in which unit fails completely either directly from normal mode or via partial failure. A single server is provided immediately to the system which is subject to failure during inspection and repair of the unit. Repair of the unit is done at its partial failure and complete failure. If repair of the unit is not feasible, it is replaced by new one in order to avoid unnecessary expenses on repair. Priority is given to the treatment of the server upon failure over repair of the unit. The repair of the unit, treatment of the server and switch devices are considered as perfect.

All random variables are assumed as independent and uncorrelated, the distributions of the failure time of the unit and server follows negative exponential while that of inspection time, repair time and treatment time of the server are taken as arbitrary. To carry out cost-benefit analysis, expressions for various measures of system effectiveness such as mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period of the server, expected number of inspection by the server, expected number of treatments given to the server, expected number of visits by the server and profit function are derived using the *Regenerative Point Graphical Technique* ***(RPGT).*** The following system characteristics have been evaluated to study the system performance.

- **i.** Mean Time To System Failure(MTSF).
- **ii.** Total fraction of time for which the system is available.
- **iii.** The busy period of the Server doing any given job.
- **iv.** The number of the Server's visits.

The Tables and the Graphs are drawn to study the effect of the various system parameters on MTSF and availability of the system. The analytical analysis is done and conclusions are made by taking particular cases.

2 ASSUMPTIONS AND NOTATIONS

The following assumptions and notations/symbols are used:

1) The system consists of single-unit, considering the idea of server failure while performing inspection and repair of the unit which may fail completely either directly from normal mode or via partial failure.
2) There is a single server who reaches the system immediately to do inspection and repair.
3) The distributions of the failure time of the unit and server follow negative exponential whereas inspection time and repair time of the unit and treatment time of the server are distributed arbitrarily.
4) The repair of the unit and treatment given to the server are considered as perfect.
5) A repaired unit works like a new-one.
6) The system is discussed for steady state conditions.
7) Priority is given to the treatment of the server upon failure over repair of the unit.

8) If repair of the unit is not feasible after inspection, it is replaced by new one.

9) Failure time, repair time and treatment time are statistically independent.

pr/pf : Probability/transition probability factor.

$q_{i,j}(t)$: probability density function (p.d.f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$q_{i,j.k}(t)$: probability density function (p.d.f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].

$p_{i,j}$: steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $p_{i,j}$ = $q^*_{i,j}(0)$; where $*$ denotes Laplace transformation.

$p_{i,j.k}$: steady state transition probability from a regenerative state i to a regenerative state j while visiting non-regenerative state $\underline{k}$ once. $p_{i,j.k}$ = $q^*_{i,j.k}(0)$; where $*$ denotes Laplace transformation.

$\underline{k}$: k-state is a non-regenerative state.

$(i,j)/(i,\underline{k},j)$: (i,j) = $p_{i,j}$; $(i,\underline{k},j)$ =$p_{i,j.k}$; (i,j,k) = $(i,j).(j,k)$; $(i,j,\underline{k},l)$ = $(i,j).)/(j,\underline{k},l)$

$\overline{cycle}$: a circuit formed through un-failed states.

k-cycle : a circuit (may be formed through regenerative or non-regenerative/failed states)

whose terminals are at the regenerative state k.

k-$\overline{cycle}$: a circuit (may be formed through only un-failed regenerative/non- . regenerative states)whose terminals are at the regenerative state k.

$\left(i \xrightarrow{s_r} j\right)$: *r*-th directed simple path from *i*-state to j-state; r takes positive integral values for different paths from *i*-state to j-state.

$\left(\xi \xrightarrow{sff} i\right)$: a directed simple failure free path from ξ-state to i-state.

$V_{k,k}$:*pf* of the state k reachable from the terminal state k of the k-cycle.

$V_{\overline{k,k}}$: *pf* of the state k reachable from the terminal state k of the k-$\overline{cycle}$.

$R_i(t)$: reliability of the system at time t, given that the system entered the un-failed regenerative state i at t=0.

$A_i(t)$: probability that the system is available in up-state at time t, given that the system entered regenerative state i at t=0.

$B_i(t)$: probability that the server is busy doing a particular job at epoch t, given that the system entered regenerative state i at t=0.

$V_i(t)$: the expected number of visits of the server for a given job in (0,t], given that the system entered regenerative state i at t=0.

$W_i(t)$: probability that the server is busy doing a particular job at epoch t without transiting to any other regenerative state 'i' through one or more non-regenerative states, given that the system entered the regenerative state 'i' at t=0.

μ_i : mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt.$$

μ_i^1 : the total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i' at t=0.

η_i : expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0; $\eta_i = W_i^*(0)$.

f_j : fuzziness measure of the j-state.

O : The unit is operative and in normal mode.

SG : The server is good.

$\lambda/\lambda_1/\lambda_2$: constant failure rate of the unit from normal mode to complete failure/ normal mode to partial failure/ partial failure mod to complete failure.

$_PU_i$/ $_PU_r$/ $_PW_i$/ $_PW_r$: The unit is partially failed and under inspection/ under repair/ waiting for . inspection/ waiting for repair.

ω : Constant failure rate of the server.

p/q : Probability that repair of the unit at partial failure is not feasible/feasible.

$_FU_r$/ $_FW_r$: The unit is completely failed and under repair/ waiting for repair. SFU_t/ SFU_T : The server is failed and under treatment/ under treatment continuously from the previous state.

$g(t)/G(t)$: Probability density function/cumulative distribution function of the repair-time of the completely failed unit.

$g_1(t)/G_1(t)$: Probability density function/cumulative distribution function of the repair-time of the partially failed unit.

$h_1(t)/H_1(t)$: Probability density function/cumulative distribution function of the inspection time of the unit at partial failure.

$f(t)/F(t)$: Probability density function/cumulative distribution function of the treatment time of the server.

(s)/© : Symbol for Stieltjes convolution/ Laplace convolution.

~/* : Symbol for Laplace Stieltjes Transform(LST)/ Laplace Transform(LT).

'(desh) : Symbol for derivative of the function.

The system can be in any of the following states with respect to the above symbols.

S_0 = O S_1= $_PU_r(SG)$

S_2 = $_FU_r(SG)$

S_3 = $_PW_r(SFU_t)$

S_4 =$_FW_r(SFU_t)$ S_5 = $_FW_r(SFU_T)$

The transition states S_0, S_1, S_2, S_3, S_4 are regenerative and state S_5 is non-regenerative.. The possible transitions between states along with transition time c.d.f.'s are shown in Fig.1.

3 TRANSITION DIAGRAM OF THE SYSTEM

Following the above assumptions and notations, the transition diagram of the systems are shown in Fig.1

State	Symbol
Regenerative state/point	•
Up-state:	○
Failed state:	□
Degenerated/Reduced state	⬭

Table-1

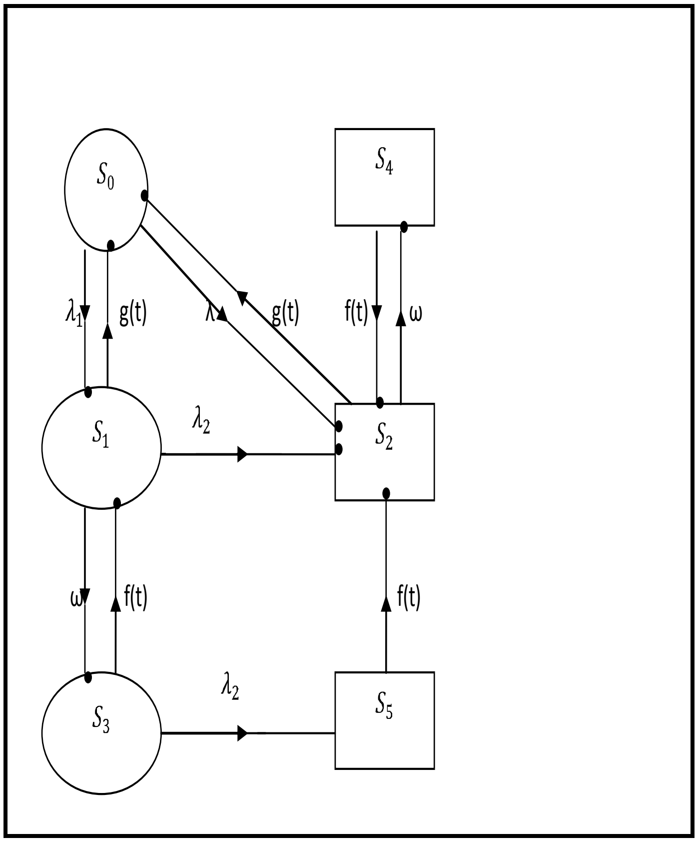

Fig 1

4 EVALUATION OF PARAMETERS OF THE SYSTEM:

4.1 Analysis of System:

The key parameters (under steady state conditions) of the system are evaluated by determining a 'base-state' and applying RPGT. The MTSF is determined w.r.t. the initial state '0' and the other parameters are obtained by using base-state.

4.1.1 Determination of base-state:

From the transition diagram (Fig.1), The Primary, Secondary, Tertiary circuits at all vertices are shown in Table-2.

Primary, Secondary, Tertiary circuits at a Vertex

Vertex i	Simple circuits	(CL1)	(CL2)	(CL3)
0	{0,1,0} {0,2,0} {0,1,2,0} {0,1,3, 5,2,0 }	{1,3,1 } {2,4,2 } {1,3,1 }	Nil	Nil
1	{1,0,1} {1,3,1} {1,2,0,1} {1,3, 5,2,0,1 }	{0,2,0 } {2,0,2 } {2,4,2	{2,4,2 }	Nil
2	{2,0,2} {2,4,2} {2,0,1,2} {2,0,1,3, 5,2 }	{0,1,0 } {0,1,0 } {1,3,1	{1,3,1 } {1,3,1 }	Nil
3	{3,1,3} {3, 5,2,0,1,3 }	{1,0,1 } {2,0,2 } {2,4,2	{0,2,0 } {0,1,0 }	Nil
4	{4,2,4}	{2,0,2	{0,1,0	{1,3,1

Table-2

In the transition diagram of fig. 1, there are four, four, four, two and one simple circuits at the vertices 0,1,2,3 & 4 respectively. As there are four simple circuits associated each of the vertices 0, 1 & 2. So, any of these can be the base-state of the system. Now, the distinct primary circuits along all the simple paths from the vertex '0' to all the vertices are: {1,3,1},{2,4,2}. There are no secondary and tertiary circuits along the paths from the vertex '0'. Therefore, there are only four primary circuits along all the paths from the vertex '0'.but, the distinct primary circuits along all the simple paths from the vertex '1' to all the vertices are: {0,2,0},{2,4,2}.there are only one distinct secondary circuit along all the simple paths from the vertex '1'i.e. {2,4,2}.there is no tertiary circuits from the vertex '1'. And similarly there are four, two and one simple, primary and secondary circuits respectively from the vertex '2'. Since, there is largest number(four) of circuits at the vertex '0' with less number of primary, secondary and tertiary circuits, therefore, '0' is a base-state.

Primary, secondary, tertiary circuits w.r.t. The simple paths (base-state '0')

Vertex j	$\left(0 \xrightarrow{S_r} j\right)$: (P0)	(P1)	(P2)	(P3)
1	$\left(0 \xrightarrow{S_1} 1\right)$:{0,1}	{1,3,1}	Nil	Nil
2	$\left(0 \xrightarrow{S_1} 2\right)$:{0,2} $\left(0 \xrightarrow{S_2} 2\right)$:{0,1,2} $\left(0 \xrightarrow{S_3} 2\right)$:{0,1,3, 5, 2}	 {1,3,1},{2,4,2} {1,3,1},{2,4,2}	Nil	Nil
3	$\left(0 \xrightarrow{S_1} 3\right)$:{0,1,3}	{1,3,1}	Nil	Nil
4	$\left(0 \xrightarrow{S_1} 4\right)$:{0,2,4} $\left(0 \xrightarrow{S_2} 4\right)$:{0,1,2,4} $\left(0 \xrightarrow{S_3} 4\right)$:{0,1,3, 5, 2,4}	{2,4,2} {1,3,1},{2,4,2} {1,3,1},{2,4,2}	Nil Nil Nil	Nil Nil Nil

Table-3

4.1.2 transition probabilities and the mean sojourn times:

Transition probabilities:

$Q_{i,j}(t)$: probability density function (p.d.f.) Of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$P_{i,j}$: steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $P_{i,j} = q^*_{i,j}(0)$; where $*$ denotes laplace transformation.

$q_{i,j}(t)$	$p_{i,j} = q^*_{i,j}(0)$
$q_{0,1}(t) = \lambda_1 e^{-(\lambda+\lambda_1)t}$ $q_{0,2}(t) = \lambda e^{-(\lambda+\lambda_1)t}$	$p_{0,1} = \frac{\lambda_1}{\lambda+\lambda_1}$ $p_{0,2} = \frac{\lambda}{\lambda+\lambda_1}$
$q_{1,0}(t) = g_1(t)e^{-(\omega+\lambda_2)t}$ $q_{1,2}(t) = \lambda_2 e^{-(\omega+\lambda_2)t}\bar{G}_1(t)$	$p_{1,0} = g_1^*(\omega+\lambda_2)$ $p_{1,2} = \frac{\lambda_2}{\omega+\lambda_2}\{1-g_1^*(\omega+\lambda_2)\}$
$q_{2,0}(t) = g(t)e^{-\omega t}$ $q_{2,4}(t) = \omega e^{-\omega t}\bar{G}(t)$	$p_{2,0} = g^*(\omega)$ $p_{2,4} = 1-g^*(\omega)$
$q_{1,3}(t) = \omega e^{-(\omega+\lambda_2)t}\bar{G}_1(t)$ $q_{3,1}(t) = f(t)e^{-\lambda_2 t}$	$p_{1,3} = \frac{\omega}{(\omega+\lambda_2)}\{1-g_1^*(\omega+\lambda_2)\}$ $p_{3,1} = f^*(\lambda_2)$
$q_{4,2}(t) = f(t)$ $q_{3,2.5}(t) = \{\lambda_2 e^{-\lambda_2 t}\bar{F}(t) \quad f(t)\}$	$p_{4,2} = f^*(0)$ $p_{3,2.5} = \{f^*(\lambda_2) \quad f^*(0)\}$

Table-4

It can be easily verified that;

$p_{0,1} + p_{0,2} = 1$; $p_{1,0} + p_{1,2} + p_{1,3} = 1$; $p_{2,0} + p_{2,4} = 1$; $p_{3,1} + p_{3,2.5} = 1$; $p_{4,2} = f^*(0) = 1$

Mean sojourn times:

$R_i(t)$: reliability of the system at time t, given that the system in regenerative state i.

μ_i :mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt = R_i^*(0).$$

$R_i(t)$	$\mu_i = R_i^*(0).$

$R_0(t)=e^{-(\lambda+\lambda_1)t}$	$\mu_0=\frac{1}{\lambda+\lambda_1}$
$R_1(t)=e^{-(\omega+\lambda_2)t}\bar{G}_1(t)$	$\mu_1=\frac{1-g_1^*(\omega+\lambda_2)}{(\omega+\lambda_2)}$
$R_2(t)=e^{-\omega t}\bar{G}(t)$	$\mu_2=\frac{1-g^*(\omega)}{\omega}$
$R_3(t)=e^{-\lambda_2 t}\bar{F}(t)$	$\mu_3=\frac{1-f^*(\lambda_2)}{\lambda_2}$
$R_4(t)=\bar{F}(t)$	$\mu_4=-f^{*\prime}(0)$

Table-5

4.1.3 <u>evaluation of parameters:</u>

The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated, by applying *regenerative point graphical technique**(rpgt)*** and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state '0' are:

$V_{0,0}=\left[\frac{(0,1,0)}{1-L_1}+\frac{(0,2,0)}{1-L_2}+\frac{(0,1,2,0)}{\{1-L_1\}\{1-L_2\}}+\frac{(0,1,3,\underline{5},2,0)}{\{1-L_1\}\{1-L_2\}}\right]$ = 1

$V_{0,1}=\frac{(0,1)}{1-L_1}=\frac{p_{0,1}}{1-p_{1,3}p_{3,1}}$

$V_{0,2}=\left[\frac{(0,2)}{1-L_2}+\frac{(0,1,2)}{\{1-L_1\}\{1-L_2\}}+\frac{(0,1,3,\underline{5},2)}{\{1-L_1\}\{1-L_2\}}\right]=\frac{1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}}{p_{2,0}(1-p_{1,3}p_{3,1})}$

$V_{0,3}=\frac{(0,1,3)}{1-L_1}=\frac{p_{0,1}p_{1,3}}{1-p_{1,3}p_{3,1}}$

$V_{0,4}=\left[\frac{(0,2,4)}{1-L_2}+\frac{(0,1,2,4)}{\{1-L_1\}\{1-L_2\}}+\frac{(0,1,3,\underline{5},2,4)}{\{1-L_1\}\{1-L_2\}}\right]=\frac{p_{2,4}(1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1})}{p_{2,0}(1-p_{1,3}p_{3,1})}$

Where, $1-L_1=1-\{1,3,1\}=1-p_{1,3}p_{3,1}$

$1-L_2$ = $1-$ {2,4,2} = $1-p_{2,4}p_{4,2}$ = $1-p_{2,4}=p_{2,0}$

(a). Mtsf(T_0): from fig.1, the regenerative un-failed states to which the system can transit(initial state '0'), before entering any failed state are: i = 0,1,3. For 'ξ' = '0', mtsf is given by

Mtsf = $\left[\Sigma_{i,s_r}\left\{\frac{\left\{pr\left(\xi\xrightarrow{s_r(sff)}i\right)\right\}.\mu_i}{\Pi_{k_1\neq\xi}\{1-V_{\overline{k_1,k_1}}\}}\right\}\right]\div\left[1-\Sigma_{s_r}\left\{\frac{\left\{pr\left(\xi\xrightarrow{s_r(sff)}\xi\right)\right\}}{\Pi_{k_2\neq\xi}\{1-V_{\overline{k_2,k_2}}\}}\right\}\right]$

T_0 = [(0,0) μ_0+ $\frac{(0,1)}{1-L_1}\mu_1$+ $\frac{(0,1,3)}{1-L_1}\mu_3$] ÷ [1 – $\frac{(0,1,0)}{1-L_1}$] = n ÷ d

where, $1-L_1=1-\{1,3,1\}=1-p_{1,3}p_{3,1}$

n = [(0,0) μ_0+ $\frac{(0,1)}{1-L_1}\mu_1$+ $\frac{(0,1,3)}{1-L_1}\mu_3$] = $[p_{0,0}(1-L_1)\mu_0+p_{0,1}(\mu_1+p_{1,3}\mu_3)]\div 1-L_1$

= $[\mu_0(1-p_{1,3}p_{3,1})+p_{0,1}(\mu_1+p_{1,3}\mu_3)]\div 1-p_{1,3}p_{3,1}$

d = [1 – $\frac{(0,1,0)}{1-L_1}$] =[$1-L_1-p_{0,1}p_{1,0}$] ÷ $1-L_1$

= $[1-p_{1,3}p_{3,1}-p_{0,1}p_{1,0}]\div 1-p_{1,3}p_{3,1}$

mtsf= $\frac{\mu_0(1-p_{1,3}p_{3,1})+p_{0,1}(\mu_1+p_{1,3}\mu_3)}{1-p_{1,3}p_{3,1}-p_{0,1}p_{1,0}}$

(b). Availability of the system: from fig.1, the regenerative states, at which the system is available are: j

= 0,1,3 and the regenerative states are i = 0 to 4. For 'ξ' = '0', the total fraction of time for which the system remains available is given by

$$A_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\} f_j \cdot \mu_j}{\prod_{k_1 \neq \xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\} \cdot \mu_i^1}{\prod_{k_2 \neq \xi}\{1-V_{k_2,k_2}\}}\right\}\right]$$

$$A_0 = \left[\sum_j V_{\xi,j} \cdot f_j \cdot \mu_j\right] \div \left[\sum_i V_{\xi,i} \cdot \mu_i^1\right]$$

$$= [V_{0,0} \cdot f_0 \cdot \mu_0 + V_{0,1} \cdot f_1 \cdot \mu_1 + V_{0,3} \cdot f_3 \cdot \mu_3] \div [V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1]$$

$$= [f_0\mu_0 + \frac{p_{0,1}}{1-p_{1,3}p_{3,1}} f_1\,\mu_1 + \frac{p_{0,1}p_{1,3}}{1-p_{1,3}p_{3,1}} f_3\mu_3]$$

$$\div \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,3}p_{3,1}}\mu_1^1 + \frac{1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}}{p_{2,0}(1-p_{1,3}p_{3,1})}\mu_2^1 + \frac{p_{0,1}p_{1,3}}{1-p_{1,3}p_{3,1}}\mu_3^1 + \frac{p_{2,4}(1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1})}{p_{2,0}(1-p_{1,3}p_{3,1})}\mu_4^1\right]$$

$$= N_0 \div D_0$$

Where,

$$N_0 = [f_0\mu_0 + \frac{p_{0,1}}{1-p_{1,3}p_{3,1}} f_1\,\mu_1 + \frac{p_{0,1}p_{1,3}}{1-p_{1,3}p_{3,1}} f_3\mu_3]$$

$$D_0 = \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,3}p_{3,1}}\mu_1^1 + \frac{1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}}{p_{2,0}(1-p_{1,3}p_{3,1})}\mu_2^1 + \frac{p_{0,1}p_{1,3}}{1-p_{1,3}p_{3,1}}\mu_3^1 + \frac{p_{2,4}(1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1})}{p_{2,0}(1-p_{1,3}p_{3,1})}\mu_4^1\right]$$

$$A_0 = N_1 \div D_1$$

$$N_1 = p_{2,0}[\mu_0(1-p_{1,3}p_{3,1}) + p_{0,1}(\mu_1 + p_{1,3}\mu_3)]$$

where; $(f_j = 1 \,\forall\, j)$

$$D_1 = p_{2,0}\mu_0(1-p_{1,3}p_{3,1}) + p_{2,0}p_{0,1}(\mu_1 + p_{1,3}\mu_3) + (1-p_{1,3}p_{3,1}-p_{0,1}p_{1,0})(\mu_2 + p_{2,4}\mu_4)$$;($\mu_j^1 = \mu_j \,\forall$ j)

(c). Busy period of the server: from fig.1, the regenerative states where server is busy while doing repairs are: j = 1,2; the regenerative states are: i = 0 to 4.for 'ξ' = '0', the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\} \cdot \eta_j}{\prod_{k_1 \neq \xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\} \cdot \mu_i^1}{\prod_{k_2 \neq \xi}\{1-V_{k_2,k_2}\}}\right\}\right]$$

$$B_0 = \left[\sum_j V_{\xi,j} \cdot \eta_j\right] \div \left[\sum_i V_{\xi,i} \cdot \mu_i^1\right]$$

$$= [V_{0,1}\eta_1 + V_{0,2}\eta_2] \div [V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1]$$

$$[\frac{p_{0,1}}{1-p_{1,3}p_{3,1}}\eta_1 + \frac{1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}}{p_{2,0}(1-p_{1,3}p_{3,1})}\eta_2]$$

$$\div \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,3}p_{3,1}}\mu_1^1 + \frac{1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}}{p_{2,0}(1-p_{1,3}p_{3,1})}\mu_2^1 + \frac{p_{0,1}p_{1,3}}{1-p_{1,3}p_{3,1}}\mu_3^1 + \frac{p_{2,4}(1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1})}{p_{2,0}(1-p_{1,3}p_{3,1})}\mu_4^1\right]$$

$$= N_{00} \div D_0$$

Where, $N_{00} = [\frac{p_{0,1}}{1-p_{1,3}p_{3,1}}\eta_1 + \frac{1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}}{p_{2,0}(1-p_{1,3}p_{3,1})}\eta_2]$

$$B_0 = N_{01} \div D_1$$

$$N_{01} = [p_{2,0}p_{0,1}\mu_1 + (1-p_{1,3}p_{3,1}-p_{0,1}p_{1,0})\mu_2]$$;($\eta_j = \mu_j \,\forall$ j)

$$D_1 = p_{2,0}\mu_0(1-p_{1,3}p_{3,1}) + p_{2,0}p_{0,1}(\mu_1 + p_{1,3}\mu_3) +$$

$(1 - p_{1,3}p_{3,1} - p_{0,1}p_{1,0})(\mu_2 + p_{2,4}\mu_4)$

;($\mu_j^1 = \mu_j \; \forall$ j) **(d). Expected number of treatments given to the server:** from fig.1, the regenerative states where the treatment is give to the server are: j = 3 & 4, which is also continuous to the non-regenerative state 5; the regenerative states are: i = 0 to 4. For 'ξ' = '0', the expected number of server's visits per unit time is given by

$$T_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{k_1 \neq \xi}\{1 - V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1 - V_{k_2,k_2}\}}\right\}\right]$$

$$T_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i} . \mu_i^1\right]$$

$$= [V_{0,3} + V_{0,4} + V_{0,2.5}] \div [V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1]$$

where, $V_{0,2.5} = \frac{(0,1,3,\underline{5},2)}{\{1 - L_1\}} = \frac{p_{0,1}p_{1,3}p_{3,2.5}}{1 - p_{1,3}p_{3,1}}$

$$= \left[\frac{p_{0,1}p_{1,3}}{1 - p_{1,3}p_{3,1}} + \frac{p_{2,4}(1 - p_{1,3}p_{3,1} - p_{1,0}p_{0,1})}{p_{2,0}(1 - p_{1,3}p_{3,1})} + \frac{p_{0,1}p_{1,3}p_{3,2.5}}{1 - p_{1,3}p_{3,1}}\right] \div D_0$$

$$T_0 = N_{02} \div D_1$$

$N_{02} = p_{2,0}p_{0,1}p_{1,3}(1 + p_{3,2.5}) + p_{2,4}(1 - p_{1,3}p_{3,1} - p_{0,1}p_{1,0})$ and D_1 is already defined.

(e). Expected number of server's visits: from fig.1, the regenerative states where the server visits(afresh) for repairs of the system are: j = 1,2; the regenerative states are: i = 0 to 4. For 'ξ' = '0', the expected number of server's visits per unit time is given by

$$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{k_1 \neq \xi}\{1 - V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1 - V_{k_2,k_2}\}}\right\}\right]$$

$$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i} . \mu_i^1\right]$$

$$= [V_{0,1} + V_{0,2}] \div [V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1]$$

$$= \left[\frac{p_{0,1}}{1 - p_{1,3}p_{3,1}} + \frac{1 - p_{1,3}p_{3,1} - p_{1,0}p_{0,1}}{p_{2,0}(1 - p_{1,3}p_{3,1})}\right] \div D_0$$

Where, D_0 is already defined.

$$V_0 = N_{03} \div D_1$$

$N_{03} = p_{0,1}p_{2,0} + (1 - p_{1,3}p_{3,1} - p_{1,0}p_{0,1})$ and D_1 is already defined.

4.1.4 <u>profit function of the system</u>:

The profit analysis of the system can be done by using the profit function:

$$P_0 = C_1.A_0 - C_2.B_0 - C_3.T_0 - C_4.V_0$$

Where, $C_1 =$ revenue per unit of time the system is available.

$C_2 =$ cost per unit time the server remains busy for the repairs.

$C_3 =$ cost per unit time treatment given to the server.

$C_4 =$ cost per visit of the server.

5 PARTICULAR CASE

Let us take;

$g(t) = \alpha e^{-\alpha t}$, $g_1(t) = \alpha_1 e^{-\alpha_1 t}$, $f(t) = \beta e^{-\beta t}$, $h_1(t) = \gamma e^{-\gamma t}$

We have

$p_{0,2} = \frac{\lambda}{\lambda+\lambda_1}$, $p_{1,0} = \frac{\alpha_1}{\alpha_1+\omega+\lambda_2}$, $p_{1,2} = \frac{\lambda_2}{\alpha_1+\omega+\lambda_2}$, $p_{1,3} = \frac{\omega}{\alpha_1+\omega+\lambda_2}$, $p_{2,0} = \frac{\alpha}{\alpha+\omega}$, $p_{2,4} = \frac{\omega}{\alpha+\omega}$

$p_{3,1} = \frac{\beta}{\beta+\lambda_2}$, $p_{4,2} = 1$

$p_{0,1} = \frac{\lambda_1}{\lambda+\lambda_1}$, $p_{3,2,5} = \frac{\lambda_2}{\beta+\lambda_2}$

$\mu_0 = \frac{1}{\lambda+\lambda_1}$, $\mu_1 = \frac{1}{\alpha_1+\omega+\lambda_2}$, $\mu_2 = \frac{1}{\alpha+\omega}$, $\mu_3 = \frac{1}{\beta+\lambda_2}$, $\mu_4 = \frac{1}{\beta}$

By using these results, we get the following:

Mtsf(T_0) = $\frac{(\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\lambda_1(\beta+\lambda_2)+\omega\lambda_1}{(\lambda+\lambda_1)(\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)-\alpha_1\lambda_1(\beta+\lambda_2)}$

Availability(A_0) =

$$\frac{\alpha\beta[(\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\lambda_1(\beta+\omega+\lambda_2)]}{[\alpha\beta+(\beta+\omega)(\lambda+\lambda_1)](\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\alpha\beta\lambda_1(\beta+\omega+\lambda_2)-\alpha_1\lambda_1(\beta+\lambda_2)(\beta+\omega)}$$

Busy period of the server(B_0)

$$\frac{\beta[(\lambda+\lambda_1)(\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\lambda_1(\beta+\lambda_2)(\alpha-\alpha_1)]}{[\alpha\beta+(\beta+\omega)(\lambda+\lambda_1)](\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\alpha\beta\lambda_1(\beta+\omega+\lambda_2)-\alpha_1\lambda_1(\beta+\lambda_2)(\beta+\omega)}$$

Expected number of treatments given to the server(T_0)

=

$$\frac{\beta\omega[(\lambda+\lambda_1)(\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\alpha\lambda_1(\beta+2\lambda_2)-\alpha_1\lambda_1(\beta+\lambda_2)]}{[\alpha\beta+(\beta+\omega)(\lambda+\lambda_1)](\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\alpha\beta\lambda_1(\beta+\omega+\lambda_2)-\alpha_1\lambda_1(\beta+\lambda_2)(\beta+\omega)}$$

Expected no. Of visits by the server(V_0)

=

$$\frac{\beta[(\alpha+\omega)\{(\lambda+\lambda_1)(\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)-\alpha_1\lambda_1(\beta+\lambda_2)\}+\alpha\lambda_1(\beta+\lambda_2)(\alpha_1+\omega+\lambda_2)]}{[\alpha\beta+(\beta+\omega)(\lambda+\lambda_1)](\alpha_1\beta+\lambda_2\beta+\alpha_1\lambda_2+\omega\lambda_2+\lambda_2^2)+\alpha\beta\lambda_1(\beta+\omega+\lambda_2)-\alpha_1\lambda_1(\beta+\lambda_2)(\beta+\omega)}$$

6 ANALYTICAL DISCUSSION:

6.1

The following tables, graphs, and conclusions are obtained for:

$\lambda_1 = \lambda_2 = 0.005$; $\alpha_1 = 0.80$; $\beta = 0.80$; $\omega = 0.005$

A). Mtsf vs. Failure rate:

The mtsf of the system is calculated for different values of the failure rate (λ) by taking λ = 0.005, 0.006, 0.007, 0.008, 0.009 and shows a particular behavior for treatment rate(β) of the server. The data so obtained are shown in table 6 and graphically in fig.2.

Failure rate(λ)	**Mtsf(T_0)**
0.005	200.0000
0.006	166.8393
0.007	143.1111
0.008	125.2918
0.009	111.4186
0.01	100.3115

Table 6

Table 6 shows the behavior of the mtsf (t_0) vs. The failure rate (λ) of the unit of the system .it is concluded that mtsf decreases with increase in the values of the failure rate (λ).

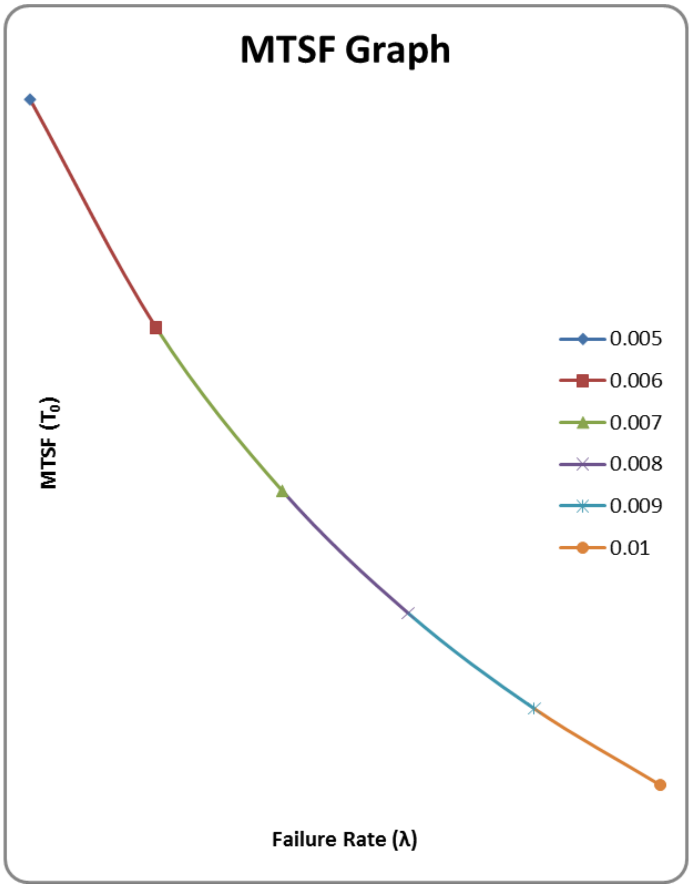

Fig. 2

Further it can be concluded from the Fig.2 that values of MTSF(T_0) shows the expected trend for different values of Failure Rate(λ), as T_0 decreases with the increase in the values of Failure Rate (λ).

b). Availability(A_0) vs. The Repair Rate(α):

The Availability of the system is calculated for different values of the Repair Rate (α) by taking α= 0.80, 0.85, 0.90, 0.95 and 1.00 and shows a particular behavior for Treatment Rate (β) of the server. The data so obtained are shown in Table 7 and graphically in Fig.3.

Repair Rate(α)	Availability(A_0)
0.80	0.99375
0.85	0.99411
0.90	0.99444
0.95	0.99473
1.00	0.99499

Table-7

Table 7 shows the effect of the Repair Rate (α) upon the Availability (A_0) of the system for some particular value of Treatment Rate (β). It is observed that there is substantial positive change in the values of Availability, as it increases with increase in the values of the Repair Rate (α).

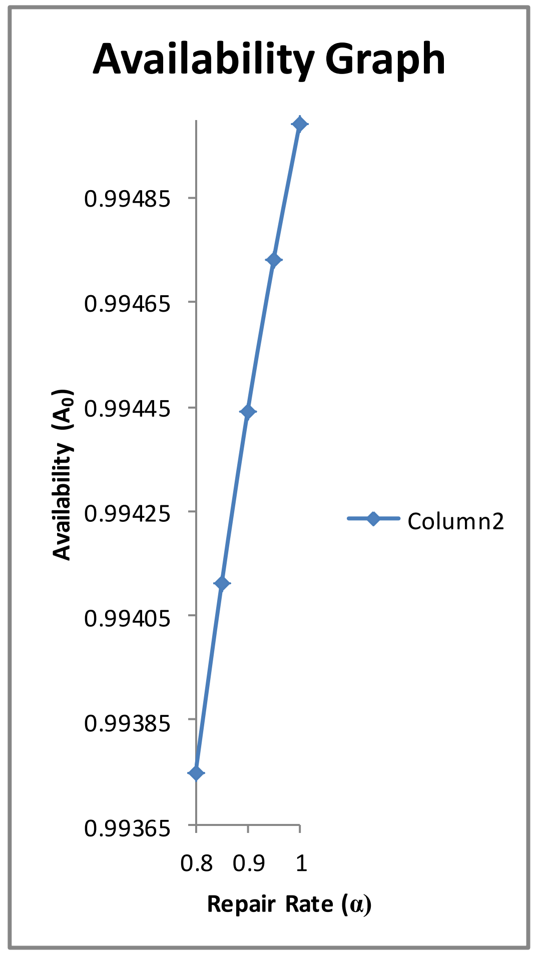

Fig. 3

Further it can be concluded from the Fig. 3 that values of Availability (A_0) shows the expected trend for different values of Repair Rate (α), as A_0 increases with the increase in the values of Repair Rate (α).

7 CONCLUSION:

From the Graphs and Tables, we see that as the Repair Rate(μ) increases, Availability of the System is increase, which should be. The study can be extended for two or more Unit system having Perfect and Imperfect Switch-Over devices. In future, Researchers can evaluated the parameters, when Repair rate and Failure rate are variable and also discuss the cost and profit benefit analysis. Further results can also be apply to find the Waiting Time of Units and Number of Server's visits. Any state can be taken as the Base-state to evaluate the various parameters.

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