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Optical Waveguide Theory

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ABSTRACT: ***In sections 1.2 and 1.3, the basic concepts in waveguide theory have been introduced. The Eigen value equation for a waveguide, (equation 1.12), has been derived from the wave equation by imposing suitable boundary conditions. This equation provides constraints for the number of discrete modes allowed in the guide. In section 1.4, the excitation of guided modes using the prism-coupling and end-coupling techniques has been assessed. Despite the greater practical difficulties associated with the prism-coupling method, it can provide more information on the mode properties. For example it can select individual modes and can be used to evaluate the propagation constants of the modes and refractive index of the guiding layer.***

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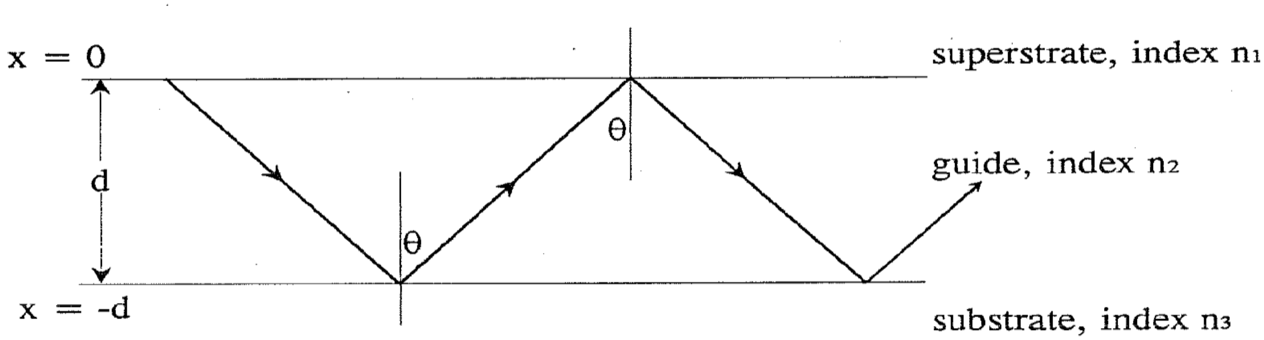
INTRODUCTION

In this paper, the fundamentals of planer waveguide theory will be introduced. Firstly, a simple ray optical picture will be used to describe the optical confinement of light in a waveguide by total internal reflection and introduce the concept of guided modes. Next, the discrete nature of these guided modes and the emergence of TE and TM polarization states in which they may propagate will be derived from Maxwell's equations. The theory will be based on an asymmetric step- index waveguide since all the thin films investigated in this thesis have been deposited on substrates with a significantly lower refractive index. The paper will then go on to cover the techniques that were used to characterize the optical waveguides. Measurements of the propagation losses and refractive indices require the coupling of radiation into a guided mode and so suitable coupling methods.

RAY-OPTICAL PICTURE

Consider a coherent light beam propagating along an optical waveguide via a series of total internal reflections at the guide superstrate and guide substrate interfaces, as shown in figure 3.1 (a) The angle θ in this figure, corresponding to the angle between the incident wavefront and the normal to the interface, must have a value greater than the critical angle at both the upper and lower interfaces for total internal reflection to occur. In this case, light is confined in the waveguide corresponding to a guided mode.

(a)



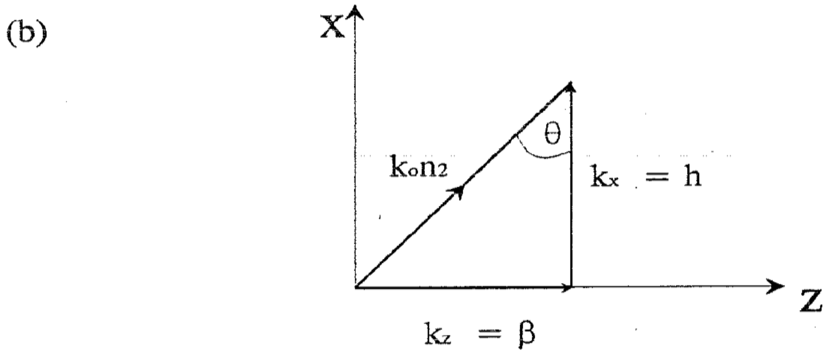


Figure 1.1 (a) The propagation of a light beam in a planar optical waveguide by total internal reflection and (b) the vector diagram for the wave propagation constant, k0n2.

A very important parameter to be considered next is the propagation constant β, which is used to characterize waveguide modes. In figure 3.1 (b), the plane wave propagation constant is defined as kon2 where; k0 = 2π / λ, (λ is the wavelength of light in free space), and n2 is the refractive index of the guide. Thus the corresponding propagation constants along the x and, z directions are:

$$k_x = k_0 n_2 \cos\theta = h$$
$$k_z = k_o n_2 \sin\theta = \beta \qquad (1.1)$$

Intuitively, one can imagine a guided mode propagating along the z direction of a medium with an index of n2sin θ, which is often referred to as the effective refractive index, (N = β/ko) of the mode and it is defined as the ratio of propagation constant of that mode to the propagation constant of vacuum. Therefore,

$$\beta = k_o N; \qquad N = n_2 \sin\theta \qquad (1.2)$$

Recall that θ can only have values greater than the critical angles at the guide-superstrate and guide-substrate interfaces This implies that the guided modes can only be supported in the range,

$$n_3 \leq N \leq n_2 \qquad (1.3)$$

Where, n3 is the refractive index of the substrate; in general, n3 > n1 where, n1 is the index of the superstrate so that the latter does not affect the range of N.

Solution of the Wave Equation:

A waveguide mode can be defined as a set of electromagnetic fields which maintain their transverse spatial distribution while travelling along a direction of propagation. By applying the geometry of figure 3.1, a plane wave propagating along the z direction with a propagation constant β can be described by the electromagnetic fields,

$$\underline{E}(x,y,z,t) = \underline{E}(x,y)\exp i(\omega t - \beta z) \qquad (1.4a)$$

$$\underline{H}(x,y,z,t) = \underline{H}(x,y)\exp i(\omega t - \beta z)$$
(1.4b)

Where; E and H are the electric and magnetic fields respectively and the angular frequency $\omega = 2\pi c/\lambda$ (c is the light velocity in free space). The planar geometry of the waveguide implies there is no field variation in the y direction and therefore $\partial/\partial y = 0,$ (also $\partial/\partial Z$ =i β) and $\partial/\partial t = i\omega.$ Substitution of equations 3.4a, b into Maxwell's equations for an isotropic, lossless dielectric medium.

$$\nabla \times \underline{E} = -\mu \frac{\partial \underline{H}}{\partial t}$$
(1.5a)

$$\nabla \times \underline{H} = -\varepsilon \frac{\partial \underline{E}}{\partial t}$$
(1.5b)

Where; ε and μ are the dielectric permittivity and magnetic permeability of the medium respectively, producing the results:

TE: $$Ey = -\frac{\omega\mu}{\beta} Hx \qquad \frac{\partial E_Y}{\partial x} = -i\omega\mu Hz$$
(1.6a)

TM: $$Hy = \frac{\omega\varepsilon}{\phi} E_x \qquad \frac{\partial Hy}{\partial x} = -\frac{\omega\varepsilon}{i} Ez$$
(1.6b)

Equations 3.6 (a) and (b) correspond to two different modes with orthogonal polarization states. Equation 3.6 (a) is the transverse electric (TE) mode which consists of the field components Ey, Hx and Hz so that the electric field is polarised along the y-direction, which is normal to the propagation direction. Similarly equation 3.6b corresponds to the transverse magnetic (TM) mode where the magnetic field is polarised along the y- direction. The solutions for the TE and TM modes are basically similar and so we will consider only the TE case. Taking EY in the form,

$$E_Y(x,z,t) = \xi_y(x) \exp i(\omega t - \beta z) \tag{1.7}$$

the functions ξ_y are chosen to model a system where the field distribution has an exponential decay as x approaches ± ∞ and a standing wave in the range -d $\leq X \leq 0,$ (d is the thickness of the guide). Therefore we hypothesise fields of the following form in the superstrate, guide and substrate respectively:

$$\xi_y(x) = \begin{cases} A\exp(-qx) & 0 \leq x < \infty \\ A\left[\cos(hx) - \frac{q}{h}\sin(hx)\right] & -d \leq x \leq 0 \\ A\left[\cos(hd) + \frac{q}{h}\sin(hd)\right]\exp\left[p(x+d)\right] & -\infty < \mathrm{x} \leq -d \end{cases} \tag{3.8}$$

Each of these fields obeys the wave equation for an isotropic, charge free medium, expressed as,

$$\nabla^2 Ey(x,z,t) + k_0^2 n_j^2 E_Y(x,z,t) = 0$$
(1.9)

Where j has values of 1,2,3 for the superstrate, guide and substrate respectively. The derivation of the wave equation from Maxwell's equations has been included in many textbooks, [Lee, 86a, b] and so has been omitted here.

By substituting equation 3.7 into 3.9, we obtain expressions for the wave numbers q, h and p in the superstrate, guide and substrate respectively,

$$q = \sqrt{\beta^2 - n_1^2 k_0^2}$$
(1.10a)

$$h = \sqrt{n_2^2 k_0^2 - \beta^2}$$
(1.10b)

$$p = \sqrt{\beta^2 - n_3^2 k_0^2}$$
(1.10c)

Now we must consider imposing boundary conditions such that a single Eigen value equation can be derived. If we consider that ξ and $\partial \xi y / \partial X$ are continuous at x = -d, then equation 1.8 yields the equation.

$$\tan(hd) = \frac{h(p+q)}{h^2 - pq} \tag{1.11}$$

To find all the solutions representing all the possible waveguide modes, this guidance condition may be written as,

$$\tan(hd + m\pi) = \frac{h(p+q)}{h^2 - pq}$$

(1.12)

Where; m = 0,1,2.... denotes the mode number. Therefore the propagation constant of a waveguide mode, (and therefore N since β = k0N), can be evaluated from equation 1.12 provided the values of n1, n2, n3 and d are known.

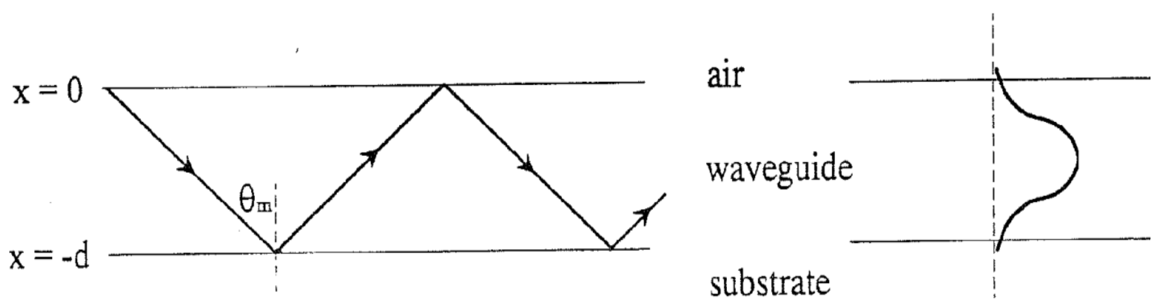
Recall from equation 3.3 that the effective refractive index of a guiding mode must lie in the range, n3 $\leq$ N $\leq$ n2. Now equation 1.12 provides constraints so that only a discrete number of guiding modes can exist for fixed values of n1, n2, n3, d and ω. In other words, only rays with certain bounce angles, θm, can propagate as guided modes because the plane wave must undergo constructive interference as it reflects from the boundaries of the guiding layer. The zeroth order mode corresponds to a ray propagating with a bounce angle, $\theta_{m=0}$ closest to 900 and so with the highest effective index. For higher order modes, θm decreases until it approaches the critical angle at the guide/substrate interface so that N $\rightarrow$ n3. This is known as cut-off where the light is no longer confined in the guiding layer but leaks into the substrate at x = -d, and thus limits the number of higher order modes that may propagate The field distribution is now referred to as a substrate mode. Figure 1.2 displays the mode profiles before and beyond the cut-off regime. The maximum number of modes, M, that can propagate in the guide can be determined because for constructive interference to occur, the phase shift, (the product of h and d), must equal M π. When N $\rightarrow$n3, equation 3. l0b can be expressed as:

$$h = k_0\sqrt{n_2^2 - n_3^2} \qquad (1.13)$$

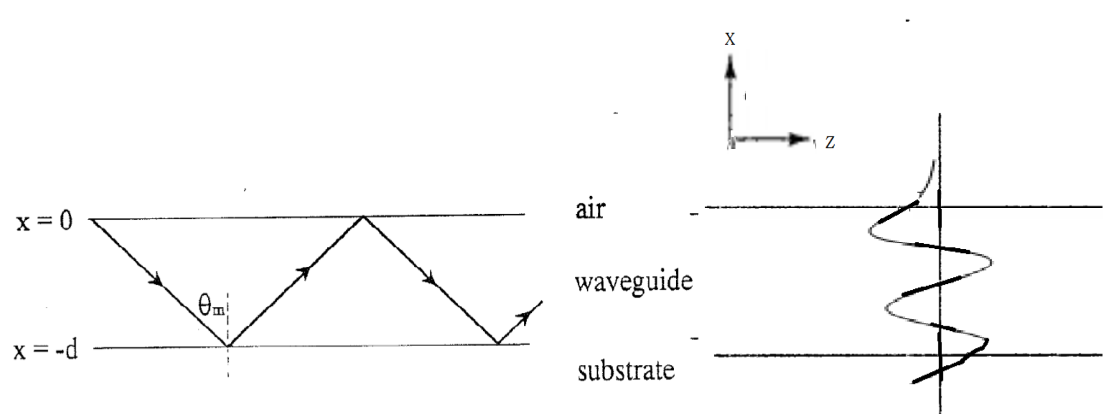
Since M π = hd, it follows that the maximum number of modes that a guide can support,

$$M = \frac{2d}{\lambda}\sqrt{n_2^2 - n_3^2} \qquad (1.14)$$

Therefore to fabricate single-mode waveguides, the values of d, n2 and n3 need to be carefully chosen' such that 1 < M < 2, (for a fixed wavelength of propagating light).



(a)



(b)

Figure 1.2 Profile of a confined mode (a) before cut-off corresponding to a waveguide mode and (b) after cut-off corresponding to a substrate mode.

COUPLING TECHNIQUES:

To evaluate the propagation characteristics and optical properties of a waveguide, it is important to possess knowledge of the various techniques that can be employed to excite a guided mode. Two coupling techniques that were extensively used in the project were prism-coupling and end-coupling and each will be discussed below. Alternative coupling techniques such as grating-coupling and fibre butt-coupling were not applied in the waveguide characterization experiments and so have not been included.

PRISM-COUPLING:

Laser radiation can be coupled into a waveguide using a high-index prism as shown in figure 1.3. The prism is pressed onto the surface of the waveguide by means of spring- loaded clamps so that a very small air gap, (typically the order of half a wavelength, is maintained between the prism and the guide.

Consider the case when ϕ, (the angle between the incident light and normal to the prism base), exceeds the critical angle at the prism-air interface. The light is totally internally reflected at the base of the prism and a stationary wave is formed in the prism. In the air gap, there exists an evanescent field as shown in figure 1.2. The interaction between this evanescent field with the evanescent field of a guided mode results in the coupling of incident light into the mode. However, this can only occur when the phase-matching condition is satisfied such that the propagation constant of the light passing through the prism is equal to the propagation constant of the guided mode. That is,

$$n_p \sin\phi = n_2 \sin\theta m \qquad (1.15)$$

Where np, is the refractive index of the prism, and as before, n2 is the index of the guide and θm is the bounce angle in the guide. A requirement of the phase-matching condition

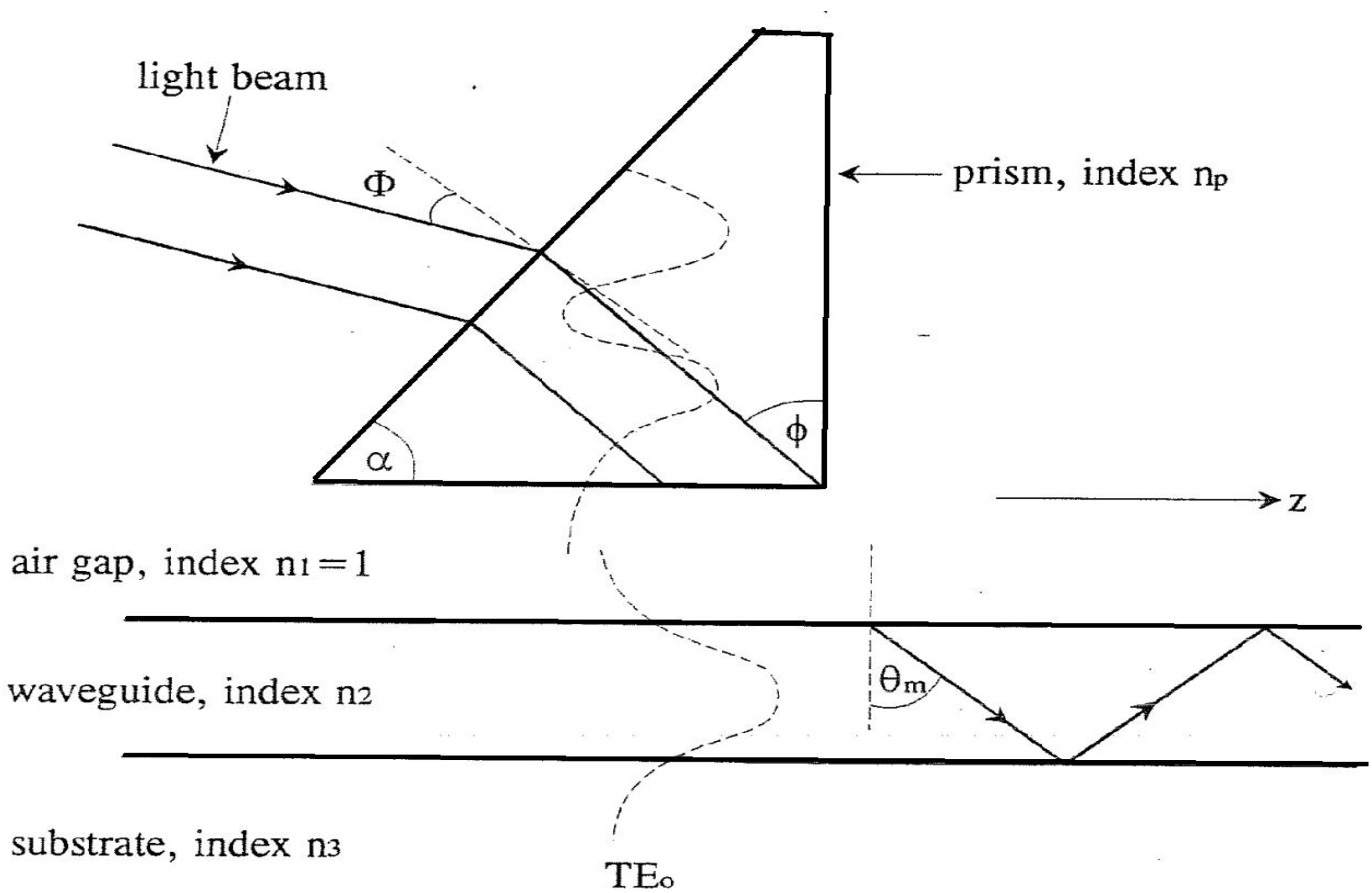


Figure 1.3 Schematic view of the coupling of radiation into a waveguide mode using a prism

is that np$\geq$ n2 so that expensive high-index prisms are required. The prisms used in our experiments were made from Rutile (TiO2), which has refractive indices, (n0 = 2.584 and ne = 2.872 at λ = 633nm), higher than any of the indices of the waveguides investigated. The attractive feature of the prism-coupling technique is that each guided mode can be individually accessed by altering the angle ϕ. A second prism can be used to couple the propagating light out of the guide in order to image the output beam onto a screen. This output appears as a bright streak of light and is often referred to as an m-line. This experimental arrangement, where an output prism is introduced, is known as "bright- mode prism-coupling' and can be used for evaluating the propagation loss in the guide. An alternative arrangement involving only one prism can be used to determine the propagation constants of the modes as well as the refractive index and thickness of the layer. This set-up is known as 'dark-mode prism-coupling".

DARK-MODE PRISM-COUPLING:

The experimental arrangement is shown in figure 1.4. An isosceles prism is pressed onto the surface of the waveguide with a converging laser beam incident on its base. The angle between the incident light and prism normal on entering the prism, Φ, is changed until the phase-matching condition is satisfied, and radiation is coupled into a guided mode. The light that has not coupled into the mode is totally internally reflected at the base of the prism and imaged onto a screen. The vertical dark line observed on the screen corresponds to the "missing" light which has coupled into the guide. If the waveguide is multimode, several m-lines are observed on the screen because the converging beam contains a range of angles, and hence the coupling condition may be satisfied for several modes simultaneously. Since all the modes can be coupled in this way, it is possible to determine the number of modes that the guide can support and compare with the theoretical prediction from equation 1.14.

In general, the dark-mode prism-coupling assembly is mounted on a rotatable micromanipulator, so that Φ can be directly measured. This can be easily related to the internal angle, ϕ, by applying Snell's law,

$$\sin\Phi = n_p \sin(\phi - \alpha) \qquad (1.16)$$

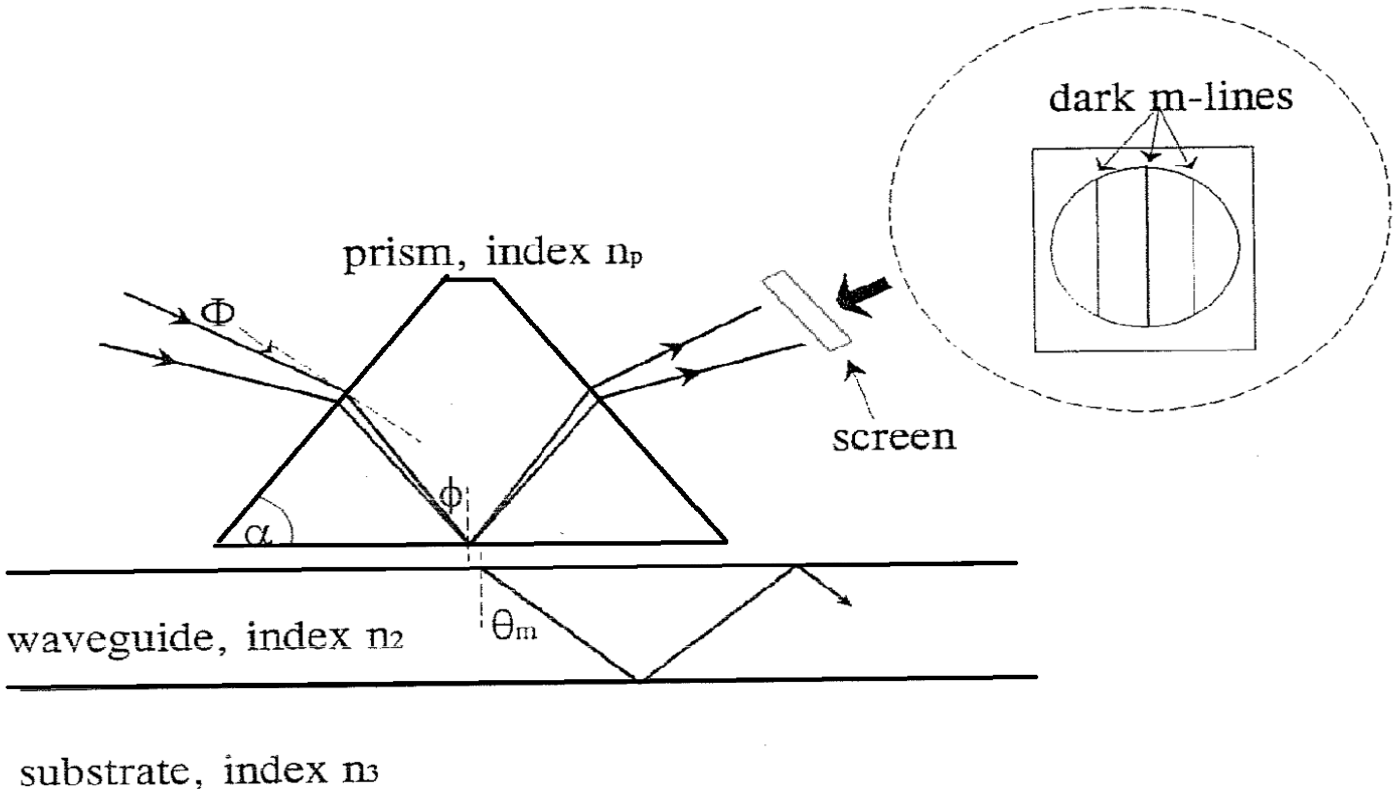


Figure 1.4 Experimental arrangement used for dark-mode prism-coupling

Where; α is the base angle of the prism. Since the effective refractive index of the mth mode, Nm = n2 sin $\theta m = np \sin\phi,$ we can eliminate the unknown parameter ϕ from equation 1.16, and by rearranging, obtain the expression:

$$N_m = n_p \sin\left[\sin^{-1}\left(\frac{\sin\Phi}{n_p}\right) + \alpha\right]$$

(1.17)

Thus we can obtain a value for the effective refractive index of each mode since the values of np and α are known and Φ can be directly measured. In addition, the propagation constants can be inferred since $\beta_m = k_0 N_m$. Once the values of β for two modes, (β_m and β_{m+1} say), can be calculated as described above, the refractive index n2, and thickness d, of the guide can be inferred, (provided the indices of the substrate and superstrate are known). This is accomplished by substituting β_m and β_{m+1} into the guidance condition of the waveguide, (equation 1.12), to produce two equations which can be solved simultaneously to provide a value for n2 and d. The prism-coupling technique was successfully used for characterizing the Ga-La-S waveguides and the GGG waveguides. Perhaps the main problem associated with this technique is that it is necessary to firmly press the prism onto the waveguide so that the air gap is small enough for the evanescent field to couple into the guide. This can easily lead to the fracturing of fragile waveguides or damage to the expensive prisms. One may also have assumed a low coupling efficiency to be another major disadvantage because only the evanescent field of the incident light is coupled into the guide. However this is not true because the incident light is continuously coupled into the guide at all the reflection points on the prism base, and these contributions are in phase, this is why coupling efficiencies as high as 90% have been reported.

END-COUPLING:

End-coupling, which is probably the easiest method of launching light into a guided mode is shown schematically in figure 1.5. The light is focused into one end of the guide using a microscope objective lens, and propagates along the whole length to emerge as a highly divergent beam from the opposite end. The general procedure is to image the output onto a screen using a second objective lens. For planar waveguides one should observe a very bright horizontal line on the screen under correct launch conditions. A major difference between end-coupling and prism-coupling is that all the waveguide modes are simultaneously excited by the former technique. However even the end-coupling technique can be adapted so that light in the fundamental mode of a multimode

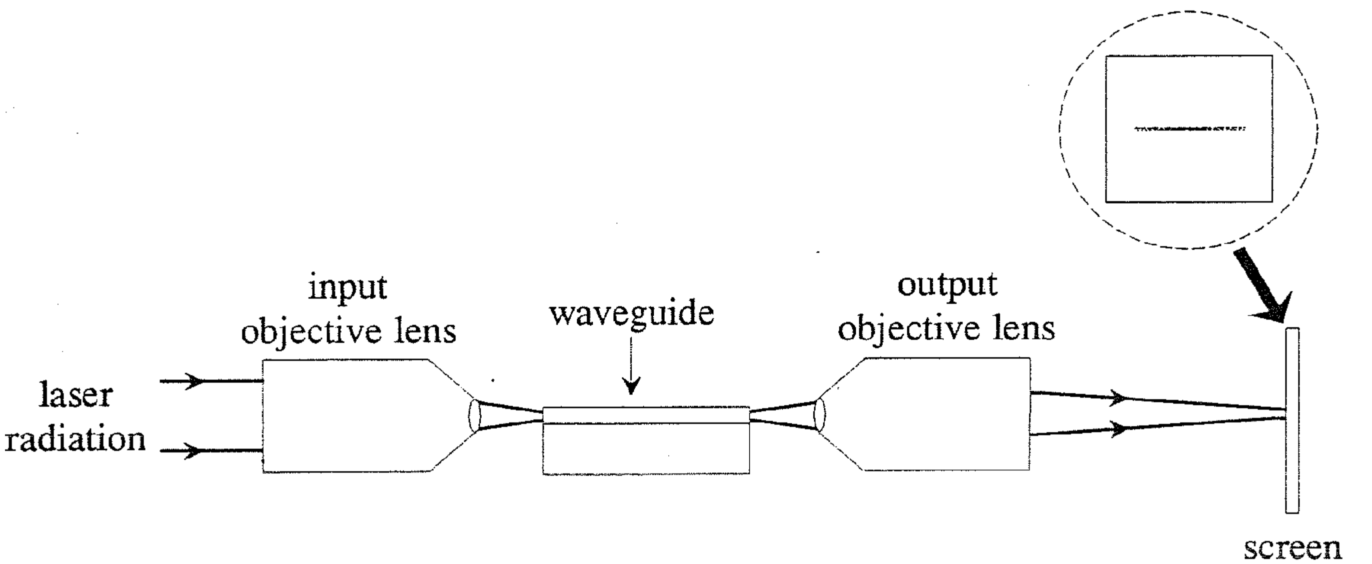


Figure 1.5 Experimental arrangements for end-launching into an optical waveguide

waveguide can be maximized at the expense of the other modes. This is accomplished by carefully matching the waist of the input beam to the spot size of the waveguide mode.

Despite the Fresnel reflection losses at both end faces of the waveguide, high coupling efficiencies can be obtained. The scattering losses at the end faces should be very low provided they have been polished to an optically smooth finish. Nonetheless, end- coupling is not perfectly suited for the waveguides investigated in this study because the films were generally very thin, (typically $\approx 1\mu m$), so the coupling efficiencies were poor.

PROPAGATION LOSSES:

The propagation loss in a waveguide will always be greater than losses in the bulk material. This is most often due to waveguide imperfections inadvertently introduced during the fabrication process, such as poor surface smoothness and intrinsic defects in the material. Scattering losses from these imperfections limit the device applications of such waveguides. The propagation loss, L, (units dB/cm), is defined as,

$$L = \left(\frac{10}{x}\right)\log\frac{P_i}{P0(x)} \qquad (1.18)$$

Where; Pi and P0(x) are the transmission powers into and out of the guide respectively after propagating a distance x. The value for L is a measure of the total loss, which takes account of all the different loss mechanisms encountered in a waveguide, of which the scattering loss is only one. For example mode conversion loss from the excited mode to other guided modes can also occur due to waveguide defects. Also, absorption losses due to impurities in the original material may be significant.

The two most common techniques of measuring propagation losses in waveguides are the prism-sliding method and the cut-back method. The principle behind both techniques is very similar, namely to provide data on the output power P0(x) as a function of x. The propagation loss can then be determined from the gradient of a plot of log P0(x) versus x so that the input coupling efficiency is eliminated from any calculations.

The prism-sliding technique, shown in figure 1.6, requires the prism coupling assembly. The input prism is pressed onto the surface of the waveguide and fixed in the same position for the duration of the experiment so that the input coupling efficiency is constant. Sometimes when a mode has been excited, a bright streak is observed on the surface of the waveguide due to out-of-plane scattering. Very faint streaks imply only modest scattering so that the losses in the guide should be low whereas brighter streaks correspond to strong scattering and high-loss waveguides.

To obtain a quantitative measurement of the loss, it is necessary to clamp an output prism

PRISM-SLIDING METHOD:

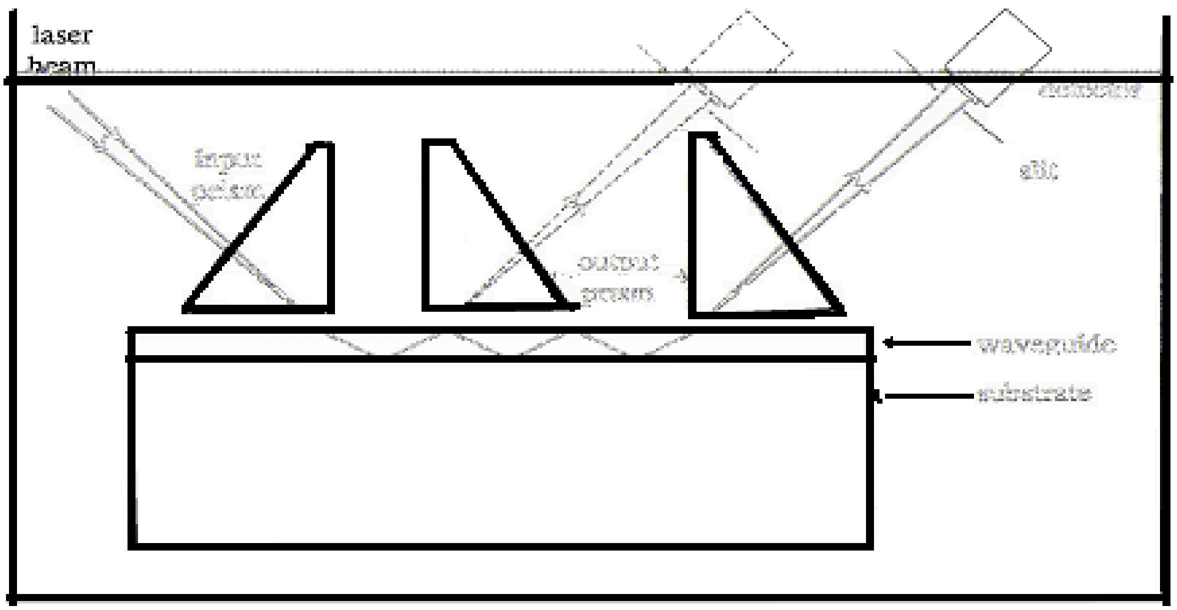


Figure 1.6 Experimental arrangement for measuring waveguide losses using prism-sliding method.

On to the guide, (as shown in figure 1.6), so that the propagating light is coupled out to emerge as a bright m-line. It is necessary to position the output prism at several distances along the propagating length so that the output power P0(x) can be measured as a function of x. To perform this experiment correctly, the output prism needs to be moved towards the input prism so that any degradation of the waveguide surface, caused by the output prism, will not affect the loss measurement. An important requirement for an accurate loss measurement is that the output coupling efficiency must be constant during the experiment. However, when the output prism is moved from one position to the next, it is difficult to ensure that the pressure applied during the clamping process is always constant and this is certainly the major drawback associated with this technique. Nonetheless, once the skills necessary to successfully perform this experiment are mastered, the prism-sliding technique can provide measurements of losses down to 0. 1dB/cm with errors of ± 0.01dB/cm, another advantage of the technique is that individual guided modes can be excited so that information on mode- conversion losses can be obtained.

CUT-BACK METHOD:

To evaluate the propagation losses in a waveguide, the cut-back method employs the end- coupling procedure. The waveguide is cut into pieces of different length and the transmission through each piece is calculated using the set up shown in figure 1.5. One can express the transmission, T(x), through the output objective as,

$$T(x) = \frac{P_O(x)}{P_i} \cdot \frac{1}{TobjiT_{Obj.O}} \cdot \frac{1}{(1-R_i)(1-R_O)}$$

(1.19)

Where; Tobj.i and Tobj,o are the transmissions of the input and output objectives respectively and Ri and R0 are the Fresnel reflection coefficients of the guide end faces. Once again, Pi and P0 are the input and output transmission measurements. The input coupling efficiency is eliminated when values of T(x) are obtained for guides of different length and once again, the waveguide loss is evaluated from a plot of logT(x) versus x. The cut-back method has several disadvantages to the prism-sliding method. The main drawbacks are that the waveguide is destroyed and the process of polishing the edges of each piece can be rather laborious. Also end-coupling of light simultaneously excites all the modes in the waveguide so that the mode-conversion component of the total loss can not be separated. Nonetheless, the cut-back method can produce more accurate results because the output coupling efficiency will always remain the same. It is also applicable to waveguides which have been produced with a superstrate that is not air.

REFERENCES:-

- Drucker, P. F. "Toward the New Organisation": Hesselbein, F., Goldsmith, M. and Beckhard, R. (1997): The Organization of the Future; Jossey-Bass Publisher.
- Ewusi-Mensah, K and Przasnyski, Z.H. (1994) Factors contributing to the abondonment of information systems development projects Journal of Information Technology (1994),9,185-201
- Ewusi-Mensah, K and Przasnyski, Z.H. (1995) Learning from abandoned information systems development projects Journal of Information Technology (1995), 10.
- Franke, U.J. (1999) The virtual web as a new entrepreneurial approach to network organization Entrepreneurship & Regional Development, 11 (1999), 203-229
- Franken P.A., Hill A.E., Peters C.W. and Weinreich G. “Generation of optical harmonics”. Phys. Review Lett. 7(4), 118-119, 15 August 2001.
- “Fundamentals of Fiber Optics in telecommunication and sensor systems”. Edited book by Bishnu P. Pal, New age international (P) Limited, Publishers. Delhi. Reprint 2009.
- G. B. Benedek and K. Fritsch, Phys. Rev. Letter pp.149, 647 (1966).
- Glass, R.L. Evolving a new theory of project success Communication of The ACM 42 Technical report 2000-004, March2000, ISSN 1404-3203.
- Hatch, M. J. (1997) Organization Theory Modern, Symbolic, and Postmodern.
- Hong, K-K. and Kim, Y-G (2001) The critical success factors for ERP implementation: an organizational fit perspective Information & Management.
- IEEE-610.12 (1991) IEEE Standard Glossary of Software Engineering terminology.
- Influence of free carrier refraction due to linear absorption on Z-scan study of porous Si, K.S. Bindra, Optics Communication. 246, 421, 2005.
- ITU-T Recommendations G.650 (10/2000), Definition and test methods for the relevant parameters of single-mode fibres, International Telecommunication Union, 2001.
- J.J.Veselka, D.A. Herr, T.O.Murphy, L.L.Buhl, and S.K.Korotky “Integrated high speed Ti:NbO3 $\Delta\beta$ reversal switching circuits”. Presented at top meet. Integrated and guided wave optics. Santa Fe, NM, Mar, 1988, pap WD2.
- Jiang, J. J., Klein, G. and Discenza, R. (2001) Perception differences of software success: provider and user views of system metrics The Journal of systems.
- Kearns, G.S. and Leder, A.L. (2000) The effect of strategic alignment on the use of IS-based resources for competitive advantage Journal of Strategic Information.
- Kotonya, G and Sommerville, I. (1997) Requirements Engineering ISBN 0-471-Krumbholz, M. and Maiden, N. (2001).
- Lang, S., Masoner, M.and Nicolau, A. (2000) An empirical examination of the influence of

organisational constraints on information systems development.

- International Journal of Accounting Information Systems 2 (2001) 75-102 L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media (Pergamon, New York, 1960)

- L. Prigent and J. P. Hamaide, IEEE Photon. Technol. Lett. 5, 1092 (1993)