Creep Analysis of an Isotropic Functionally Graded Cylinder

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Abstract – In this segment of the study, it is decided to investigate steady state creep in a thick-walled FGM cylinder consisting of silicon carbide particles (SiCp) embedded in an aluminum matrix and subjected to high pressure and high temperature/temperature gradient. The content of SiCp in the FGM cylinder is assumed to vary linearly, with maximum content at the inner radius and minimum content at the outer radius. The study is an attempt to investigate the relationship of particle gradient and thermal gradient either present alone or present simultaneously on the creep behaviour of internally pressurized thick-walled cylinder. A mathematical model has been developed to describe steady state creep behaviour of the FGM cylinder. The model developed is used to investigate the effect of imposing a linear gradient in the distribution of SiCp on the steady state creep response of the composite cylinder operating at a constant temperature or under a radial thermal gradient. This study is undertaken to explore the possibility of using FGM in pressure vessel

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4.1 GENERAL

Functionally graded materials (FGMs), as described earlier, consist of two or more material ingredients whose relative volume fractions and microstructure are engineered to have a continuous spatial variation in material properties. In FGMs, the volume fraction of reinforcement is gradually varied to obtain a smooth variation in material properties. Significant advances in fabrication and processing techniques during the past decade have made it possible to produce FGMs using processes that allow for great latitude in the tailoring of microstructure and material composition. FGMs, among the most important non- homogeneous composites designed for high temperature applications, show great promise in high-tech engineering fields. FGMs have been developed as ultra-high temperature resistant materials for potential applications in aircrafts, space vehicles and other structural components exposed to elevated temperature (Noda *et al*, 1998). In recent years, the problem of mechanical behaviour of FGM cylinders subjected to various mechanical and thermal loads has attracted the interest of many researchers. Creep behaviour of composites such as FGMs, possessing tailored distribution of reinforcement is of importance in view of their applications involving high temperature.

4.2 DISTRIBUTION OF REINFORCEMENT

The silicon carbide particles (SiCp) in FGM cylinder are assumed to decrease linearly from the inner (*a)* to the outer radius (*b)*. As a result, the density and creep parameters will also vary along the radius of cylinder. The content (*vol*%) of silicon carbide *V(r)* at any radius *r* is assumed to vary linearly according to relation given as,

$$V(r) = V_{\max} - \frac{(r-a)}{(b-a)}\left[V_{\max} - V_{\min}\right] = \delta_1 - \delta_2 r \tag{4.1}$$

where, $\delta_1 = (V_{\max} + a\delta_2)$ and $\delta_2 = \frac{(V_{\max} - V_{\min})}{(b-a)}$

In the above equation *Vmax* and *Vmin* are the maximum and minimum particle content respectively at the inner and the outer radii of cylinder. In order to keep average particle content (*Vavg*) in the cylinder to be constant, one has to select either *Vmax* or *Vmin*.

Using the law of mixture, the density of FGM cylinder is obtained as,

$$\rho(r) = \frac{[100 - V(r)]\rho_m + V(r)\rho_d}{100} = \rho_m + \frac{(\rho_d - \rho_m)V(r)}{100} \tag{4.2}$$

where $\rho_m = 2698.9 kg/m^3$ and $\rho_d = 3210 kg/m^3$ are respectively the densities of pure aluminum matrix and SiCp dispersoid (Metals Handbook, 1978; Clyne and Withers, 1993).

Substituting the value of *V(r)* from Eqn. (4.1) into Eqn. (4.2), we get,

$$\rho(r) = \rho_m + \frac{(\rho_d - \rho_m)(\delta_1 - \delta_2 . r)}{100} = A - B.r \qquad (4.3)$$

where,

$$A = \rho_m + (\rho_d - \rho_m)\frac{\delta_1}{100} \quad \text{and} \quad B = \frac{\delta_2(\rho_d - \rho_m)}{100}$$

The average particle content (*Vavg*) in the cylinder is expressed as,

$$V_{avg} = \frac{\int_a^b 2\pi r l V(r) dr}{\pi(b^2 - a^2) l} \qquad (4.4)$$

where *l* is length of the cylinder.

Substituting the value of *V(r)* from Eqn. (4.1) into Eqn. (4.4) and integrating, we obtain,

$$V_{min} = \frac{3V_{avg}(1-\beta^2)(1-\beta) - V_{max}(1 - 3\beta^2 + 2\beta^3)}{(2 - 3\beta + \beta^3)} \qquad (4.5)$$

Where $\beta = a/b.$

4.3 ESTIMATION OF CREEP PARAMETERS

As described in chapter 3, the creep behaviour of FGM cylinder used in this study is also described by a threshold stress based creep law having stress exponent of 5 (refer Eqn. 3.2 in chapter 3), In order to estimate the values of creep parameters *M* and □ o for various combinations of *P*, *V* and *T*, not covered in Table 3.1 (chapter 3), the regression analysis has been carried out by using Datafit Software. During the regression analysis, *P*, *V* and *T* are taken as independent variables, and *M* and □ o are selected as dependent variables. The developed regression equations are given below,

$$M(r) = 0.02876 - \frac{0.00879}{P} - \frac{14.02666}{T(r)} + \frac{0.03224}{V(r)} \qquad (4.6)$$

$$\sigma_o(r) = -0.084P - 0.023T(r) + 1.185(V(r)) + 22.207 \qquad (4.7)$$

where *V(r)*, *T(r)*, *M(r)* and □ o (*r*) respectively the particle content (*vol%*), temperature (*K*), creep parameter and threshold stress (*MPa*) at any radius *(r)* of the FGM cylinder and *P* is the particle size (*µm*).

In a FGM cylinder, with particle content varying radially as *V(r),* both the creep parameters *M(r)* and □ o (*r*) will also vary along the radial direction. In the present study, the particle size *(P)* is assumed as 1.7 *µm* while the temperature is assumed to vary radially. Therefore, for a given FGM cylinder with known particle and thermal gradients both the creep parameters will be functions of only radial distance. The values of *M (r)* and □ o (*r*) at any radius (*r*) could be estimated respectively from Eqs. (4.6) and (4.7) by substituting the values of particle content *V(r)* and operating temperature, *T(r)* at the corresponding locations.

In order to check the validity of Eqs. (4.6) and (4.7), the values of *M(r)* and □ o (*r*) are estimated from these equations for various combinations of *P*, *V* and *T* reported in Table 3.1 (chapter 3). The *M(r)* and □ o (*r*) , thus obtained, are substituted in creep law, Eqn. (3.2) to calculate the strain rates corresponding to different stress levels reported by Pandey *et al* (1992). The results obtained depict a close agreement between the theoretically estimated and experimentally observed strain rates, Figs. 4.1(a)-(c).

4.4 ESTIMATION OF THERMAL GRADIENT

In order to obtain thermal gradient in the composite cylinder, it is assumed that the inner and outer surface of the cylinder are maintained at respectively 350 *oC* and 450 *oC*. To estimate temperature at any radius, *T(r)*, the cylinder is sub- divided into ten elementary cylindrical shells, which are, fused together and have equal radial thickness but varying thermal conductivity. For individual cylindrical shell, the thermal conductivity is assumed to be constant and estimated through the rule of mixture as given below,

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \qquad (4.8)$$

where *Km* (*=247W/mK*) is the matrix conductivity (Taya and Arsenault, 1989) and *Kd* (*=100W/mK*) is the conductivity of SiC dispersoid (Clyne and Withers, 1993).

The heat flux per unit length (Q/*l*), originating due to temperature difference between the inner and outer surface of the cylinder, is given by (Holman, 1992),

$$\frac{Q}{l} = \frac{2\pi(T_i - T_o)}{\ln(r_2/r_1)/K_1 + \ln(r_3/r_2)/K_2 + ...\ln(r_n - r_{n-1})/K_n} \tag{4.9}$$

where *Ti* (= 350 *oC*) is the temperature at the inner radius, and *To* (= 450 *oC*) is the temperature at the outer radius, and *K1, K2.......Kn* are conductivities of different elementary cylindrical shells.

Knowing *Ti* and *To*, the heat flux may be calculated and thereafter the temperature of individual cylindrical shells may be estimated from the following equation,

$$T_{n+1} = T_n - \frac{Q}{2\pi K_n l}\ln(r_{n+1}/r_n) \tag{4.10}$$

In this study we have considered four different types of cylinders as mentioned below in Table 4.1. The conductivity of these cylinders at different radius is estimated from Eqn. (4.8).

Table 4.1: Description of Cylinders

Cylinder	Particle Content and other Operating Conditions
Uniform / Non-FGM(C1)	Without particle and thermal gradients [V_{max}= V_{min}=V_{avg} = 20 *vol%* ; T = T_{avg}=400 °C]. T_{avg} is the average of T_i and T_o.
Uniform / Non-FGM (C2)	With thermal gradient (TG) but without particle gradient (PG) [V_{max}= V_{min}= V_{avg} = 20 *vol%*].
FGM (C3)	With particle gradient (PG) but without thermal gradient (TG) [V_{max}=25 *vol%* ; V_{min}=16 *vol%* ; V_{avg} = 20 *vol%* ; T = T_{avg}=400 °C].
FGM (C4)	With particle and thermal gradients [V_{max}=25 *vol%* ; V_{min}=16 *vol%* ; V_{avg} = 20 *vol%*].

The radial variation of temperature obtained for various composite cylinders mentioned in Table 4.1 are plotted in Fig. 4.2.

4.5 ANALYSIS OF CREEP IN FGM CYLINDER

The analysis of creep in FGM cylinder may proceed similarly as given for uniform composite cylinder in chapter 3. However, for FGM cylinder the density and creep parameters will vary with radial distance (*r*).

Let us consider a long, closed end, thick-walled, hollow cylinder made of functionally graded Al-SiCp. The cylinder is assumed to have inner and outer radii *a* and *b* respectively and is operating under internal pressure *p* alone. Assuming coordinates axes *r, θ* and *z* respectively along the radial, tangential and axial directions of the cylinder.

The analysis carried out in this chapter is based on the following assumptions:

i. Material of the cylinder is incompressible and locally isotropic *i.e.* the properties remain constant at a given radial location but vary with radial distance.

ii. Pressure is applied gradually and is held constant during the loading history.

iii. Stresses at any point in the cylinder remain constant with time *i.e.* steady state condition of stress is assumed.

iv. Elastic deformations are small and neglected as compared to creep deformations.

Following procedure described in section 3.4 (chapter 3), we may get the following equation,

$$\sigma_\theta - \sigma_r = \frac{I_1(r)}{r^{2/n}} + I_2(r) \tag{4.11}$$

where,

$$I_1 = \left[\frac{2}{\sqrt{3}}\right]^{\frac{n+1}{n}} \left(\frac{C^{1/n}}{M(r)}\right) \quad \text{and} \quad I_2 = \frac{2}{\sqrt{3}}\sigma_0(r)$$

The symbols appearing above are already explained in chapter 3.

Using Eqn. (4.11) into equilibrium Eqn. (3.9) and integrating the resulting equation between limits *a* to *r*, we get,

$$\sigma_r = \int_a^r \frac{I_1(r)}{r^{(n+2)/n}}dr + \int_a^r \frac{I_2(r)}{r}dr + (\sigma_r)_{r=a} \tag{4.12}$$

The boundary conditions for a cylinder subjected to internal pressure are given by,

(i) At $r = a$, $\sigma_r = -p$ (4.13)

(ii) At $r = b$, $\sigma_r = 0$ (4.14)

The Eqn. (4.12) may be solved, between limits *a* to *b* and under the imposed boundary conditions given above, to obtain the value of constant C given by,

$$C = \left[\frac{p - \left(2/\sqrt{3}\right)X_1}{\left(2/\sqrt{3}\right)^{\frac{n+1}{n}} X_2} \right]^n \tag{4.15}$$

where,

$$X_1 = \int_a^b \frac{\sigma_0(r)}{r} dr \qquad \text{and} \qquad X_2 = \int_a^b \frac{1}{r^{(n+2)/n} . M(r)} dr$$

Using Eqn. (4.12) in Eqn. (4.11), the tangential stress (□ □) is obtained as,

$$\sigma_\theta = \int_a^r \frac{I_1(r)}{r^{\frac{n+2}{n}}} dr + \int_a^r \frac{I_2(r)}{r} dr + \frac{I_1(r)}{r^{2/n}} + I_2(r) - p \tag{4.16}$$

The axial stress (□ *z*) in the cylinder is the average of σ_θ and σ_r as indicated by Eqn. (3.26) in chapter 3. Therefore, □ *z* may be obtained by using Eqs. (4.12) and (4.16) as,

$$\sigma_z = \int_a^r \frac{I_1(r)}{r^{\frac{n+2}{n}}} dr + \int_a^r \frac{I_2(r)}{r} dr + \frac{I_1(r)}{2r^{2/n}} + \frac{I_2(r)}{2} - p \tag{4.17}$$

Similar to chapter 3, it may be proved that for FGM cylinder too, the following relationship holds,

$$\dot{\varepsilon}_\theta = -\dot{\varepsilon}_r = 0.87\,\dot{\varepsilon}_e \tag{4.18}$$

Therefore, the radial and tangential strain rates in FGM cylinder are also 87*%* of the effective strain rates.

4.6 RESULTS AND DISCUSSION

On the basis of analysis presented in the previous section, a computer program has been developed to obtain the steady state creep behaviour of Non- FGM and FGM cylinders operating either at a constant temperature or under a radial thermal gradient (Table 4.1) shown in Fig. 4.2.

Before discussing the results obtained in this study, it is necessary to check the validity of analysis and the software developed. To accomplish this task, the creep rates are estimated from the current analysis for an isotropic composite cylinder having 20 *vol%* of SiCp distributed uniformly (*i.e. Vmax* = *Vmin* = *Vavg* = 20 *vol%*). The values of strain rate, thus estimated, are exactly equal to those estimated in chapter 3 for a similar cylinder. Therefore, the analysis is presented in section 4.3 for FGM cylinders converges for uniform composite cylinder, which implies the validity of analysis as well as software developed in this chapter.

4.6.1 Variation of Creep Parameters

In order to study the effect of particle gradient on the distribution of temperature in FGM and Non-FGM cylinders, the temperature distribution in these cylinders has been determined through analysis scheme presented in section 4.4. The variation of temperature shown in Fig. 4.2 for cylinders C2 and C4 has similar trend but slightly different values, especially in the middle region. It is revealed from Fig. 4.2 that compared to cylinders C1 and C3, which operate at a uniform average temperature of 400 *oC*, the temperature towards the outer radius of cylinders C2 and C4 are higher but are lower towards the inner radius, due to the temperature imposed at the inner and outer radii (*i.e. Ti* = 350 *oC*, *To* = 450 *oC*) of the cylinder. The temperature in FGM cylinder C4 is slightly higher than the uniform cylinder C2, with a maximum difference of about 2 *oC* somewhere in the middle.

Figure 4.3 shows the distribution of reinforcement (SiCp) in various cylinders reported in Table 4.1. The content of SiCp decreases linearly from the inner to outer radius of the FGM cylinders C3 and C4 while in uniform (Non- FGM) cylinders C1 and C2 the content of SiCp remains constant (20 *vol%*) over the entire radius.

Figures 4.4(a)-(b) show the variation of creep parameters *M(r)* and □ o (*r*) in dimensionless form with radial distance for different composite cylinders. The dimensionless values of creep parameters are obtained by dividing the values of these parameters with the value of corresponding creep parameters obtained at the inner radius of uniform cylinder C1. The creep parameter *M(r)* and □ o (*r*) observed for uniform cylinder (C1) remains constant over the entire radius, due to a constant (20 *vol%*) amount of SiCp and constant temperature (*T* = 400 *oC*). But on imposing thermal gradient in uniform cylinder, the value of parameter *M(r)* decreases near the inner radius but increases towards the outer radius, as observed for uniform cylinder

C2. The parameter *M(r)* is higher in location having higher temperature as compared to locations having lower temperature, Fig. 4.2. The effect of imposing particle gradient, as observed in FGM cylinder C3, is similar to the effect of imposing temperature gradient (cylinder C2), though the extent of decrease and increase in *M(r)* is relatively less. The parameter *M(r)* increases on moving from the inner to outer radius of the FGM cylinder C3. The increase observed in *M(r)* may be attributed to decrease in particle content *V(r)* in FGM cylinder C3, as one move from the inner to outer radius, Fig. 4.4(a). The simultaneous presence of both particle and thermal gradients reinforce each other to further reduce *M(r)* near the inner radius but increase *M(r)* near the outer radius as observed in FGM cylinder C4. On the other hand, the threshold stress, □ o (*r*) , shown in Fig. 4.4(b) decreases linearly on moving from the inner to outer radius of cylinders C2-C4. The threshold stress is higher in regions having more amount of SiCp compared to those having lower SiCp content. Similar to creep parameter *M(r),* the variation of □ o (*r*) also becomes steeper when particle and thermal gradients are imposed, either alone or simultaneously.

4.6.2 Distribution of Stresses and Strain Rates

To investigate the effect of imposing thermal and particle gradients, either alone or simultaneously, on the creep behaviour of composite cylinders, the steady state stresses and strain rates, in dimensionless form, have been estimated for different cylinders as mentioned in Table 4.1, and the results obtained are shown in Figs. (4.5)-(4.7). The dimensionless forms of creep stresses and strain rates are obtained by dividing their values with the corresponding stress and strain rate, observed at the inner radius of uniform composite cylinder C1. The dimensions of cylinder and operating pressure used in this study are kept similar to those reported by Johnson *et al* (1961) for copper cylinder (refer Table 3.2). The radial stress, Fig. 4.5(a) remains compressive throughout the cylinder, with maximum value at the inner radius and zero at the outer radius, under the imposed boundary conditions given in Eqs. (4.13) and (4.14). The magnitude of radial stress decreases throughout the cylinder when either particle gradient or thermal gradient is imposed. The simultaneous presence of particle and thermal gradient s reinforce each other and leads to further decrease in the radial stress. The maximum variation observed in radial stress is somewhere in the middle region of composite cylinder. The tangential stress shown in Fig. 4.5(b) remains tensile throughout and is observed to increase with increasing radius for uniform cylinder C1. The imposition of thermal gradient, with higher temperature at the outer radius as compared to inner radius, in uniform cylinder leads to increase the tangential stress near the inner radius but reduces the tangential stress near the outer radius of cylinder C2 as compared to uniform cylinder C1 operating at a constant temperature. Unlike cylinder C1, the tangential stress in cylinder C2 is higher near the inner radius as compared to those observed near the outer radius. By increasing content of reinforcement (SiCp) near the inner radius, as observed in FGM cylinder C3, the tangential stress increases near the inner radius but decreases towards the outer radius when compared to the distribution of tangential stress in uniform cylinder C1. The regions having relatively more amount of reinforcement offer higher stress compared to the regions containing lesser amount of reinforcement. The simultaneous imposition of particle and thermal gradients reinforce each other to further enhance the tangential stress near the inner radius but reduce the tangential stress towards the outer radius. The axial stress in uniform cylinder C1 changes its nature from compressive to tensile, on moving from the inner to outer radius, Fig. 4.5(c). By imposing thermal gradient in uniform cylinder, the axial stress at the inner radius becomes tensile and its magnitude decreases on moving towards the outer radius of uniform cylinder C2 than that observed for uniform cylinder C1 operating at a constant average temperature. The imposition of particle gradient alone in the uniform composite cylinder decreases the value of axial stress near the inner (compressive) as well as the outer radius of the FGM cylinder C3 when compared to uniform cylinder C1. The simultaneous presence of both the gradients superimposes each other to result effect similar to that observed for tangential stress in Fig. 4.5(b). The effect of imposing thermal and particle gradients, either alone or simultaneously on the effective stress, Fig. 4.5(d), is similar to those observed for tangential stress in Fig. 4.5(b).

The strain rates depends upon the effective strain rate (□□*e*) which is again a function of stress difference (□ *e* □ □ o) , as described earlier in chapter 3.

Therefore, to investigate the effect of imposing particle and thermal gradients on the creep rates, the distribution of (□ *e* □ □ o) is depicted in Fig. 4.6. The effect of imposing particle and thermal gradients, either alone or simultaneously, on the stress difference (□ *e* □ □ o) is similar to those noticed for effective stress in Fig. 4.5(d). In spite of significantly higher value of (□ *e* □ □ o) near the inner radius of uniform cylinder C2 as compared to uniform cylinder C1, the effective strain rates in cylinder C2 decreases significantly over the entire radius than those observed in cylinder C1. The decrease observed in effective strain rate near the inner radius of cylinder C2 may be attributed to relatively lower values of parameter *M(r)*, Fig. 4.4(a), in spite of relatively higher value of (□ *e* □ □ o) in this region, Fig. 4.6, when the results are compared with those of uniform cylinder C1. However, towards the outer radius, inspite of higher value of *M(r)* in cylinder C2 than cylinder C1, the lower value of effective stress in cylinder C2 than cylinder C1 results in lower effective strain rate in uniform cylinder C2 as compared to uniform cylinder C1. The effect of imposing particle gradient on effective strain rate in the composite cylinder is similar to that observed for imposing thermal gradient. By reinforcing more amount of reinforcement near the inner radius of composite cylinder (FGM cylinder C3), the inter-particle spacing decreases, which increases the threshold stress (Li and Langdon, 1999) but decreases the creep parameter *M(r),* Fig. 4.4. Both these factors contribute in reducing the strain rates near inner radius of FGM cylinder C3, compared to those observed in non- FGM cylinder C1. But towards the outer radius, in spite of relatively higher value of parameter *M(r)* and lower threshold stress, the lower effective stress is responsible for reducing the effective strain rate in FGM cylinder C3 than the non- FGM cylinder C1. The simultaneous presence of both particle and thermal gradients reinforce each other and further reduce the effective strain rate in FGM cylinder C4 as compared to any other composite cylinders C1, C2 and C3. The radial (compressive) and tangential (tensile) strain rates in the composite cylinders are 13% lower than the corresponding effective strain rates (Eqn. 4.18), as is evident from Fig. 4.7(b). The presence of thermal and particle gradients affects these strain rates in a similar way as observed in Fig. 4.7(a) for effective strain rate.

4.6.3 Effect of Increasing Particle Gradient on Stresses and Strain Rates

This section investigates the effect of imposing increasing particle gradient from 0 *vol%* to 27 *vol%* on stresses and strain rates in the composite cylinder. The particle gradient is defined as the difference of maximum and minimum particle content in the composite cylinder. The results are estimated for FGM cylinders having maximum particle content of 25 *vol%*, 30 *vol%* and 35 *vol%* and the corresponding minimum particle content as 16 *vol%*, 12 *vol%* and 8 *vol%* respectively. For the purpose of comparison, the results are also obtained for composite cylinder having uniform average amount of SiCp (*i.e. Vmax* = *Vmin* = *Vavg* = 20 *vol%*). The cylinders are assumed to operate under a radial thermal gradient given by Eqn. (4.10). It is observed that the maximum tangent ial stress increases linearly with the increase in particle gradient, as evident from Fig. 4.8. However, the lowest tangential stress observed at the outer radius exhibits a decrease with the increase in particle gradient. By increasing particle gradient in the FGM cylinder, the density increases near the inner radius but decreases towards the outer radius and as a result of which the tangential stress increases near the inner radius but decreases towards the outer radius. Radial stress (compressive) is maximum at the inner radius and minimum at the outer radius, under the imposed boundary conditions given in Eqs. (4.13) and (4.14). The maximum axial stress, observed at outer radius, decreases with the increase in particle gradient upto 18%, but with further increase in particle gradient it increases slightly whereas the minimum axial stress increases with increasing particle gradient, Fig. 4.9. The stress inhomogenity is defined as the difference of maximum and minimum stress in the composite cylinder. The tangential stress inhomogenity increases from 21.1 *MPa* to 100.9 *MPa* with the increase in particle gradient from 0% to 27%, as shown in Fig. 4.10. On the other hand, the axial stress inhomogenity decreases from 34.5 *MPa* to 7.8 *MPa* with increase in particle gradient from 0 to 18% and eventually saturates with further increase in particle gradient beyond 18%. The radial stress inhomogenity will remain constant due to fixed values of maximum and minimum radial stresses respectively at the inner and outer radii of the cylinder under the imposed boundary conditions given in Eqs. (4.13) and (4.14).

The maximum and the minimum values of both tangential (tensile) and radial (compressive) strain rates (□□□□ □□□*r*) observed respectively at the inner and outer radii of the cylinder, decreases with the increase in particle gradient, Fig. 4.11. The extent of decrease in the maximum value of tangential/radial strain rate is significantly higher than the decrease observed in the minimum value of these strain rates, for a given increase in particle gradient. The tangential/radial strain rate in homogeneity decreases significantly with the increase in particle gradient, as shown in Fig. 4.12. Though, the inhomogenity in axial and tangential stresses are minimum corresponding to particle gradients of 0% and 35% respectively but the inhomogenity observed in strain rate is the minimum at a maximum particle gradient of 35%. Therefore, the increase in particle gradient in the FGM cylinder tends to decrease the strain rate inhomogenity. In other words, the distribution of strain rate becomes more uniform in the FGM cylinder with the increase in particle gradient and as a result, the extent of deformations in the composite cylinder will tend to reduce.

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