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A Comparative Analysis on Several Crucial Benefits upon Triangular Sum Graphs

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Abstract - In this paper we demonstrate that each tree could be inserted as a triangular sum graph, which may give a stage send in the guess "each tree is a triangular whole diagram" . Additionally we demonstrate that each cycle might be inserted as an incited subgraph of a triangular sum graph, giving some identified effects. We name some groups of triangular sum graphs.

Let G = (V,E) be a diagram with p vertices a q edges. A chart G is stated to allow a triangular aggregate naming assuming that its vertices could be named by non-negative whole numbers such that incited edge marks got by the entirety of the names of close vertices are the first q triangular numbers. A chart G which affirms a triangular entirety marking is called a triangular entirety chart. In the present work we research certain classes of diagrams which does not concede a triangular entirety marking. Likewise we demonstrate that certain classes of diagrams might be installed as an impelled subgraph of a triangular sum graph. This work is a decent creation of diagram hypothesis and combinatorial number hypothesis.

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INTRODUCTION AND DEFINITION

Hedge and Shankaran present triangular sum labeling notion, they state that a chart _ is stated to affirm a triangular sum labeling, in the event that its vertices might be marked by nonnegative whole numbers so the qualities on the edges, acquired as the entirety of the names of their closure vertices, are the first _ triangular numbers.

They give an essential condition for an Eulerian diagram to affirm a triangular sum labeling, they indicate that certain groups of diagrams affirm a triangular total marking, and they guess that each tree allows a triangular sum labeling. Likewise they indicate that certain groups of charts could be installed as instigated subgraphs of triangular sum graphs. At last they close stating "as each chart can't be implanted as an affected subgraph of a triangular sum graphs, it is intriguing to implant groups of GRAPHS as an affected subgraph of a triangular sum graphs ". We demonstrate that each tree might be inserted as an affected subgraph of a triangular sum graphs, additionally we demonstrate that each cycle might be inserted as an affected subgraph of a triangular sum graphs giving some identified outcomes. We name certain groups of triangular sum graphs, and demonstrate that certain families might be inserted as an instigated subgraphs of triangular sum graphs.

We start with basic, limited, associated, undirected and non-inconsequential chart G = (V,E), where V is called the situated of vertices and E is called the set of edges. For different chart theoretic documentations and terminology we accompany Horrible and Yellen and for number speculation we accompany Burton. We will give short summery of definitions which are helpful for the present examinations.

Definition : If the vertices of the graph are doled out qualities, subject to certain conditions is reputed to be chart naming.

For portion overview on graph naming one can point Gallian . Immense measure of expositive expression is accessible on distinctive sorts of diagram marking and more than 1000 examination papers have been produced so far in most recent four decades. Most enthralling marking issues have three essential fixings.

• a set of numbers from which vertex marks are picked.

• a decide that relegates a worth to every edge.

• a condition that the aforementioned qualities should fulfill.

The present work is expected to examine one such marking regarded as triangular sum labeling.

Definition : A triangular number is a number got by including all positive whole numbers less than or equivalent to a given positive number n. In the event that nth triangular number is indicated by T_n then $T_n = \frac{1}{2}n(n+1)$. It is simple to watch that there does not exist continuous numbers which are triangular numbers.

Definition : A triangular sum labeling of a graph G is a coordinated capacity f : V → N (where N is the situated of all non-negative whole numbers) that affects a bijection f^+ : $E(G) \rightarrow \{T_1, T_2, \ldots, T_n\}$ of the edges of G characterized by $f^+(uv) = f(u) + f(v), \forall e = uv \in E(G)$. The diagram which affirms such marking is called a triangular sum graph. This idea was presented by Hegde and Shankaran . In the same paper they got a fundamental condition for an Eulerian chart to concede a triangular entirety marking. In addition they examined certain classes of charts which might be implanted as a prompted subgraph of a triangular sum graph. In the present work we examine certain classes of charts which does not confirm a triangular sum labeling.

Definition : The *helm* graph H_n is the graph obtained from a wheel $W_n = C_n + K_1$ by attaching a pendant edge at each vertex of C_n.

Definition : The graph $G = < W_n : W_m >$ is the graph obtained by joining apex vertices of wheels W_n and W_m to a new vertex x. (A vertex corresponding to $K1$ in $W_n = C_n + K_1$ is called an apex vertex.)

Definition : A chord of a cycle C_n is an edge joining two non-adjacent vertices of cycle C_n.

Definition : Two chords of a cycle are said to be twin chords if they form a triangle with an edge of the cycle C_n.

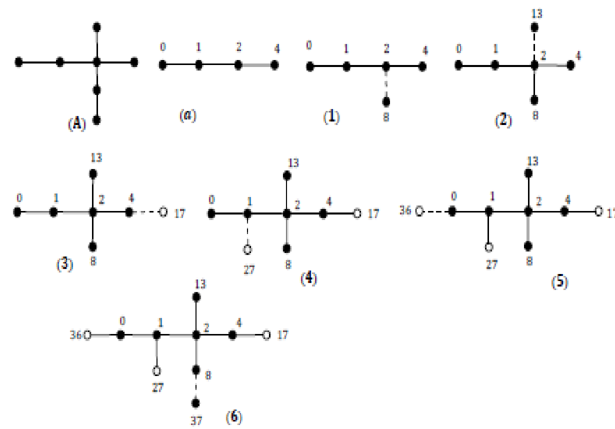
Embedding graphs as induced subgraphs of triangular sum graphs

Definition : If we change, in Definition, the limitation of the qualities of the edges to be any q triangular numbers then we depict the chart, which accedes this sort of naming as a diagram which might be installed as an instigated subgraph of a triangular sum graph, since this chart could be inserted as a part of a greater chart which is a triangular sum graph.

Theorem : Every tree might be installed as an affected subgraph of a triangular sum graph.

Evidence: For each tree we can begin from any way of this tree, which is demonstrated to be a triangular sum graph. Presently by applying the past Lemma we can include fitting vertices to get the definitive tree with probability to be a subgraph of a greater tree.

Case: In Figure we will apply Theorem, on the tree (A): first we name the way P_4, as indicated in (a) then we apply Lemma six times in the following 6 steps to get the tree (A) inserted as an affected subgraph of a triangular sum graph.



Observation : In *Table* we give the first 90 triangular numbers, each of them is given as its remainder divided by 9 and as its multiple of the number 9:

T_i	T_i mod 9	$\lfloor T_i/9 \rfloor$	T_i	T_i mod 9	$\lfloor T_i/9 \rfloor$	T_i	T_i mod 9	$\lfloor T_i/9 \rfloor$
T_1	1	0	T_{31}	1	55	T_{61}	1	210
T_2	3	0	T_{32}	6	58	T_{62}	0	217
T_3	6	0	T_{33}	3	62	T_{63}	0	224
T_4	1	1	T_{34}	1	66	T_{64}	1	231
T_5	6	1	T_{35}	0	70	T_{65}	3	238
T_6	3	2	T_{36}	0	74	T_{66}	6	245
T_7	1	3	T_{37}	1	78	T_{67}	1	253
T_8	0	4	T_{38}	3	82	T_{68}	6	260
T_9	0	5	T_{39}	6	86	T_{69}	3	268
T_{10}	1	6	T_{40}	1	91	T_{70}	1	276
T_{11}	3	7	T_{41}	6	95	T_{71}	0	284
T_{12}	6	8	T_{42}	3	100	T_{72}	0	292
T_{13}	1	10	T_{43}	1	105	T_{73}	1	300
T_{14}	6	11	T_{44}	0	110	T_{74}	3	308
T_{15}	3	13	T_{45}	0	115	T_{75}	6	316
T_{16}	1	15	T_{46}	1	120	T_{76}	1	325
T_{17}	0	17	T_{47}	3	125	T_{77}	6	333
T_{18}	0	19	T_{48}	6	130	T_{78}	3	342
T_{19}	1	21	T_{49}	1	136	T_{79}	1	351
T_{20}	3	23	T_{50}	6	141	T_{80}	0	360
T_{21}	6	25	T_{51}	3	147	T_{81}	0	369
T_{22}	1	28	T_{52}	1	153	T_{82}	1	378
T_{23}	6	30	T_{53}	0	159	T_{83}	3	387
T_{24}	3	33	T_{54}	0	165	T_{84}	6	396
T_{25}	1	36	T_{55}	1	171	T_{85}	1	406
T_{26}	0	39	T_{56}	3	177	T_{86}	6	415
T_{27}	0	42	T_{57}	6	183	T_{87}	3	425
T_{28}	1	45	T_{58}	1	190	T_{88}	1	435
T_{29}	3	48	T_{59}	6	196	T_{89}	0	445
T_{30}	6	51	T_{60}	3	203	T_{90}	0	455

Comments:

1. We acknowledge that all amounts of the shape $1+9T_i, i=1,2,\dots$ are triangular numbers again of the shape $T_{3i+1}, i=1,2,\dots$. (It is straightforward to demonstrate this for each)

2. We see that for each there are two sequential amounts of remnants 0 the point when partitioned by 9, T_{9i-1}, T_{9i} consistent with this tenet: $T_{9i}-T_{9i-1}=9i, i=1,2,\dots.3$. Also there exist four successive triangular amounts of the structure: $T_{9i-2}, T_{9i-1}, T_{9i}, T_{9i+1}$ with remnants 1,0,0,1 individually when isolated by 9,such that $T_{9i-1}-T_{9i-2}=9i-1, i=1,2,\dots,$ and
$T_{9i+1}-T_{9i}=9i+1, i=1,2,\dots$.

Theorem : Every cycle could be implanted as an actuated subgraph of a triangular sum graph.

Verification: Even cycles: In Figure we display an even cycle to be marked as accompanies:

TRIANGULAR SUM LABELING

Triangular number : A triangular number is a number got by including all positive whole numbers less than alternately equivalent to a given positive whole number n. Assuming that n^{th} triangular number is indicated by T_n then $T_n=\frac{1}{2}n(n+1)$. It is simple to watch that there does not exist continuous numbers which are triangular numbers.

Triangular whole chart : A triangular sum labeling of a graph G is a balanced capacity $f:V\rightarrow N$ (where N is the situated of all non-negative whole numbers) that actuates a bijection $f^+:E(G)\rightarrow\{T_1,T_2,\cdots,T_q\}$ of the edges of G outlined by $f^+(uv)=f(u)+f(v)$, $\forall e=uv\in E(G)$.

The chart which allows such naming is called a triangular sum graph.

Some existing outcomes : This thought was presented by Hegde and Shankaran and they demonstrated that _ Path P_n, Star $K_{1,n}$ are triangular sum graphs.

- Any tree got from the star $K_{1,n}$ by swapping every edge by a way is a triangular total diagram.
- The lobster T got by joining the focuses of k duplicates of a stat to another vertex w is a triangular sum graph.
- The complete n-ary tree T_m of level m is a triangular sum graph.
- The complete chart K_n is triangular entirety if and just if $n\leq 2$.

They too indicated that

- If G is an Eulerian (p,q)-diagram acceding a triangular sum labeling then $q\not\equiv 1$ (mod 12).
- The dutch windmill DW(n)(n duplicates of K_3 offering a regular vertex) is not a triangular total diagram.
- The complete chart K_4 could be installed as an instigated subgraph of a triangular sum graph.
- In a paper by Vaidya et al. it has been indicated that
- In any triangular entirety diagram G the vertices with names 0 and 1 are dependably neighboring.
- In any triangular sum graph G, 0 and 1 can't be the vertex names in the same triangle held in G.
- In any triangular sum graph G, 1 and 2 can't be the vertex names of the same triangle held in G.
- The rudder diagram Hn is not a triangular sum graph.
- If each edge of a chart G is an edge of a triangle then G is not a triangular sum graph.

CONCLUSION

As each graph is not a triangular sum graph it is extremely engaging to research charts or diagram families which are not triangular sum graphs yet they could be installed as an instigated subgraph of a triangular sum graph. We demonstrate that cycle, cycle with one harmony and cycle with twin harmonies might be implanted as an instigated subgraph of a triangular sum graph.

As each diagram does not allow a triangular sum labeling, it is extremely fascinating to explore classes of diagrams which are not triangular sum graphs and to implant classes of diagrams as an instigated subgraph of a triangular sum graph.

We explore numerous classes of diagrams which does not affirm triangular sum labeling. In addition we demonstrate that cycle, cycle with one harmony and cycle with twin harmonies might be implanted as an instigated subgraph of a triangular sum graph. This work donate numerous new come about to the speculation of diagram marking.

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