Study on Within the Phase-Coherence Length

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Abstracts – The results of the experiments mapping electron ow in a 2DEG showed two surprising results. The _rst was the branched nature of the ow, explained as the eject of a disordered background potential. The second was the perseverance of regular interference fringes over the entire, several-micron range of the scans. Interference fringes spaced by half of the Fermi wavelength were expected close to the point contact. At distances of several hundred nanometers, the various energies present in the electron own would remain in phase with one another. As one moves farther from the QPC, however, simple considerations would suggest that the various frequencies would disperse and any interference e_ects would be washed out. The survival of the interference fringes is seen in full quantum-mechanical simulations with thermal averaging, so we know that a complete theory will reproduce them. This full solution does not, however, tell us much about the mechanism that allows the fringes to survive.

In this chapter, we begin by explaining why one might expect the fringes to die

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INTRODUCTION

We see the survival of interference fringes beyond the thermal length in a full quantum-mechanical simulation. (A) shows the quantum-mechanical ux through the system; for a discussion of the \branched" nature of the ux, see . In (B), we have introduced a movable tip potential and plot conductance as a function of the tip position. This scan is taken at the Fermi energy. In (C), we have a thermally averaged scan of the same region. In this simulation, `T _ 600nm and the left edge of the scan is approximately 1 _m from the point contact out as we move away from the QPC. Then we present a simple model for fringe formation, appealing only to _rst-order scattering that has a very deferent dependence on thermal averaging and shows that fringes should survive to the phase-coherence length. We will also take a look at an idea, appealing to higher-order scattering, that suggests that fringes may occur beyond the phase-coherence length.

At the root of these discussions is the model that we have for the potential seen by electrons in a 2DEG. Though we found that the donor-atom contribution to the potential was the more important component for branching, it is the impurity contribution that is more important for fringing. The reason is that only the impurities give us strong scattering centers, which are necessary to get waves traveling back towards the QPC.

REVIEW OF LITERATURE

There are many dfferent quantities referred to as \thermal lengths," one of which plays a role in our understanding of the interference fringes. Before explaining our understanding for the perseverance of the interference fringes, it is worth spending some time to explain why they were unexpected. Recall that the measurements are of conductance as a function of tip position. We have shown that the signal is dependent on electrons being scattered back to the QPC, so we will consider paths with this result. There are three sources of scattering to consider: the gates, the AFM tip, and the impurities in the crystal structure. The gates are clearly strong scatterers, and we would expect the same 1For this eject, we ignore the small-angle scattering caused by donor atom density variations. It of the AFM tip as long as it creates a depletion region. The impurity scatterers, however, have comparatively small cross sections. Though we may consider multiple scattering events from the AFM and the gates, we expect any signal involving multiple scattering from impurities to be negligible compared to a single-scattering signal. There are two ways to understand the fringes near the QPC. The _rust is the one that we used in constructing our simple model. If the QPC has no open channels, so that all conductance comes from tunneling, then a wave scattered back to the QPC from the AFM can interfere with the outgoing wave and change conductance.

If there are open channels, which is the more common experimental situation, then the outgoing wave is distinguishable from the return wave. Here, we look at the interference of multiple ways of returning to the QPC. A wave scattered from the AFM tip will, in general, be partly transmitted back through the QPC and partially reected. This reected wave can scatter from the AFM again and interfere with the _rst return wave. We thus have, essentially, an open Fabret-Perot cavity and we can understand the interference fringes within that paradigm.

MATERIAL AND METHOD

Simple kinematic considerations suggest that these fringes should die out at a thermal length `T given by

`T = _h2k0=2mkT: (5.1) this length comes from a consideration of the spread of energies present in the experiments. Though they are performed at low temperatures (less than 4:7 K), the implied spread of energies is still noticeable. We found `T as the distance at which waves di_ering in energy by kT will drift out of phase by one radian over the round is unlikely that it would cause a path shorter than the phase-coherence length to return to the QPC. Trip (QPC to AFM and back). When this happens, the interference patterns from the various energies present will be su_ciently out of phase with one another that the aggregate signal would have no discernible fringe pattern. The fringes seen experimentally, however, survive well beyond this radius. We look for other paradigms of fringe formation to understand this observation.

Here we use a simple, single-scattering model to predict the fringes seen beyond the thermal length. The result depends on phase-coherent transport at each individual energy present, and therefore does not apply beyond the phase-coherence length. We will consider single-scattering events involving the AFM and the impurities that result in waves returning to the QPC, and the interference between these various paths.

Thermal Averaging It was the thermal average, a sum over the various energies present in our propagating electrons, that gave us the thermal length and the expectation that fringes would die out. In this model, we will need to perform a thermal averaging integral explicitly.

The fully correct thermal average is accomplished by an integral over energy with the derivative of the Fermi function as a weighting function. In order to simplify the mathematics of this model, we seek an approximation that is an integral over wave vector with Gaussian weighting. For the ranges of parameters in this system, such an approximation can be made to an acceptable degree of accuracy.

The thermal distribution of energies begins with the derivative of the Fermi function at the known temperature T and Fermi energy EF: f(E) = h 1 + e(E□EF)=kT i□1 (5.2) □f0(E) = H 1 + e(E□EF)=kT i□2 1 kT e(E□EF)=kT : (5.3) We wish to approximate this function by a Gaussian while preserving the normalization.

We can do so by taking

□ f0(E) _ 1 2_1=2kT e□[(E□EF)=(2kT)]2 : (5.4)

The width of this Gaussian was set by matching the second-order Taylor series about E = EF for the two functions. If we were to match the value at E = EF rather than matching the normalization, we would see that the shapes of the two curves are very similar. The normalization doesn't change this fact; it merely changes the overall constant. The majority of the error in this approximation comes from this _rst step.

To transform this exponential into a Gaussian in k, we perform further simpli_-cations. We take _

E □ EF 2kT _2 = k2 □ k2 0 4mkT=_h2 !2 (5.5)

= _h4 16m2(kT)2 (k2 □ k2 0)2 (5.6)

_ h4 16m2(kT)2 H 4k2 0(k □ k0)2 I (5.7)

= h4k2 0 4m2(kT)2 (k □ k0)2: (5.8)

We note that, using the de_nition of the thermal length in Eq.,

_h4k2 0 4m2(kT)2 = `2 T ; (5.9)

Here we show two possible weighting functions for comparison. First is the true thermal distribution function, the derivative of the Fermi function. Second is our approximation, a Gaussian in wave number. The curves are calculated for EF = 16 meV and T = 4:7 K. in terms of which we take

□ f0(E) _ 1 2_1=2kT e□(k□k0)2`2 T : (5.10)

This is the weighting function that we will use in performing the thermal average. We compare it to the original derivative of the Fermi function in though the two curves are not identical, it is reasonable to assert that the key results from the model will not be ejected. We wish to integrate over k rather than E in taking the thermal average. Given the dispersion relation E = _h2k2=2m, we have dE = (_h2k=m) dk. Again appealing to the values that will appear for k0 and `T, we can approximate this dispersion relation as linear over the range of the weighting function and take dE _ (_h2k0=m) dk. Hence, for a signal s(k; r) at _xed wave vector, we have the thermally averaged signal s(r) given by s(r) = h2k0 2_1=2mkT Z dk e□(k□k0)2`2 T s(k; r) (5.11)

= _□1=2`T Z dk e(k□k0)2`2 T s(k; r): (5.12)

The Single-Scattering Model This model is designed to have simple mathematics so that we can express an analytic result. The approximations made have no ejected on the qualitative results, and little eject on the quantitative results. We simplify the mathematics by using eikr rather than Bessel functions for the two-dimensional s-waves. We assume scattering amplitudes proportional to the scattering length for each scatterer, and a phase shift equal to the scattering length times the wave number. The quantity of interest is the ux back through the point contact as a result of the scattering. We will look for constructive or destructive interference of the returning waves at the point contact, and take that as our signal. Take a random distribution of s-wave scatterers (the impurities) at the points frig with scattering lengths faig, and assume phase-coherent transport over the roundtrip distances. Let the wave from the QPC be just r□1=2eikr, and the scattered wave from a point scatterer, measured at the QPC, be (cai=ri)eik(2ri+ai). We have called the constant of proportionality between the scattering length and amplitude c. The actual

value of this constant will depend on details of the scattering potentials irrelevant to this model. Note also that there are two factors of r □1=2

i , one for the fallo_ of the

wave illuminating the scatterer and one for the fallo_ of the scattered wave, and that the phase advances by the round-trip distance plus the phase shift. Let the tip be at a radius rt and have the scattering length at, giving a similar return wave. Finally, to simplify the notation, let us de_ne r0

The full return wave at a single energy is

X I cai ri e2ikr0 i + cat rt e2ikr0 t : (5.13)

We are interested in the returning signal, so we take the absolute square of this wave. We concentrate on the cross terms, which will give rise to the oscillations with rt. The cross terms are

s(r; k) = 2 Re" X I c2 a I a tri rte 2ik (r0i□r0t)#: (5.13)

We should take note at this point of the terms that we are neglecting. First, there are the terms independent of rt. Since in any physical system we would be detecting a change in the conductance, this constant background would simply _gure into our baseline. Second, there is the term c2a2t =r2 t , a generally expected monotonic signal independent of energy.

Now we need to thermally average this signal using the thermally averaged signal, averaged after the absolute square so that it is an incoherent sum, is s(r) = 2_□1=2`T Re

Note that we can bring the selection of the real part outside of the integral, since all other terms in the expression are real. We carry the imaginary part through the thermal average, since it makes the integral easier. Performing the resulting Gaussian integral, we have s(r) = 2_□1=2`T Re

We see the following in this result. The wave scattered from the tip interferes with the background of waves scattered from the impurities. After the thermal average, most of the resulting signal is lost. The pieces that survive the average are contributions from those scatterers that are close to the same radius2 from the QPC as is the tip. Though we can have a signal when r0 t > `T , the thermal length still plays

a role in that it determines the width of the band around r0 t that contributes to the thermally averaged signal.

Note that the fringes predicted by this model are at half the Fermi wavelength, as observed. Furthermore, the fringes will be oriented perpendicular to the direction of electron ow, also as observed.

CONCLUSION

We show examples of s(r) at two temperatures for the same distribution of scatterers. To make the signal easier to observe, we divide out the overall radial dependence of the signal strength. We determine that there is a clear r□2 dependence. However, there is a less obvious factor of r1=2 that appears as well, giving the signal strength an overall r□3=2 dependence. The r1=2 comes from 2To be completely correct, we should substitute \path length" for \radius." a density dependence of the signal. If we simply take the sum _ of N cosines of random phase, we _nd that hj_ji / N1=2. Noting that the number of scatterers in our `T - wide band increases approximately linearly with radius, we have a resulting r1=2 modification of the fringe strength.

The radial variation in fringe strength suggested by this model is, unfortunately, made difficult to observe experimentally by other variations of signal strength. Experiments are currently being planned and performed that should provide more direct evidence of this model.

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