

Analytical Study on Cordial and 3-Equitable Labeling For A Lot of Superstar Connected Equity Graphs

Mr. Bokade Swapnil Janba

Research Scholar, CMJ University, Shillong, Meghalaya, India

Abstract – We introduce here cordial and 3-equitable labeling for the graphs got by joining summit vertices of two stars to another vertex. We amplify the aforementioned comes about for k duplicates of stars. Another idea of marking called the marked feature sincere naming is presented and researched for way graphs, cycle graphs, star- $K_{1,n}$, Bistar- $B_{n,n}$, and Some general comes about on marked feature cordial labeling are mulled over.

INTRODUCTION

The discrete science is that part of arithmetic which manages methodical medication and comprehension of discrete structures and process and experienced in our day by day life, which are regularly intrinsically very intricate in nature yet apparently justifiable.

This character of discrete arithmetic is teem with fervor, infrequently surprisingly, to a regular man. The labeling of discrete structures is additionally a field which control the same trademark. The issues rolling out from the investigation of an assortment of labeling plans of the components of a graph, or of any discrete structure is a potential range of challenge as it cuts crosswise over extensive variety of controls of human comprehension. graph labeling issues are truly not of later beginning. Case in point, shading the vertices of a graph rolled out in association with the four shade theorem, which stayed for a long time known by the name four color guess, took more than 150 years for its result in 1976. Lately, new connections have developed wherein the labeling of the vertices additionally edges of a given graph by components of certain subsets of the set of genuine numbers R .

In the late of 1960's an issue in radio space science let to the duty of unquestionably the distinctions of sets of numbers happening on the positions of radio antennae to the connections of the lay-out arrangements of the antennae under obligations of the optimal lay-outs to output the unmistakable districts of the divine arch rapidly made its path to detail more short scientific issues on graph labelings. The idea of b-valuation was presented by

Alexander Rosa in 1967. In 1972 S.W.Golomb freely found b- valuations and renamed them as smooth labeling. He too sharp out the significance of examining agile graphs in attempting to settle the mind boggling issue of breaking down the finish graph by isomorphic duplicates of a given tree of the same request. The Ringle-Kotzig-Rosa guess and numerous renowned chips away at dexterous graph gave the purpose behind distinctive routes for labeling of graph structures.

DEFINITION

We start with modest, limited, joined, undirected chart $G = (V, E)$. In the present work $K_{1,n}$ mean the star. Vertex relates to K_1 is called a pinnacle vertex. For all other terminology and documentations we accompany Harary. We will give concise abstract of definitions which are handy for the present examinations.

Definition : Consider two stars $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ then $G = \langle K_{1,n}^{(1)} : K_{1,n}^{(2)} \rangle$ is the graph obtained by joining apex vertices of stars to a new vertex x .

Note that G has $2n + 3$ vertices and $2n + 2$ edges.

Definition : Consider k copies of stars namely $K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}, \dots, K_{1,n}^{(k)}$ Then the $G = \langle K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} : \dots : K_{1,n}^{(k)} \rangle$ is the

graph obtained by joining apex vertices of each $K_{1,n}^{(p-1)}$ and $K_{1,n}^{(p)}$ to a new vertex x_{p-1} where $2 \leq p \leq k$.

Note that G has $k(n+2) - 1$ vertices and $k(n+2) - 2$ edges.

Definition : If the vertices of the graph are appointed qualities subject to certain conditions is regarded as graph labeling.

Above all enthralling graph labeling issues have three vital qualities.

1. a set of numbers from which the marks are picked.
2. a decide that appoints a quality to every edge.
3. a condition that the aforementioned qualities should fulfill.

For part review on graph labeling one can point Gallian. Unlimited measure of expositive expression is accessible on diverse sorts of graph labeling. Consistent with Beineke and Hegde graph labeling serves as a boondocks between number hypothesis and structure of graphs.

Marked graph have mixed bag of provisions in coding hypothesis, especially for rocket direction codes, outline of exceptional radar sort codes and convolution codes with optimal autocorrelation lands. Marked graph plays indispensable part in the investigation of X-Ray crystallography, correspondence organize and to verify optimal circuit layouts. An item investigation of mixture of provisions of graph labeling is given by Bloom and Golomb.

Definition : A paired vertex marking of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is agreeable if it allows cordial labeling.

The thought of agreeable marking was presented by Cahit.

Numerous specialists have considered heartfeltness of graphs. e.g. Cahit demonstrated that tree is cordial. In the same paper he demonstrated that K_n is cordial if and just in the event that $n \leq 3$. Ho et al. demonstrated that unicyclic graph is cordial unless it is C_{4k+2} .

Andar et al. talked over heartfeltness of numerous shells. Vaidya et al., have likewise examined the heartfeltness of different graphs.

3-EQUITABLE LABELING OF GRAPHS

Definition : Let $G = (V(G), E(G))$ be a graph. A mapping $f: V(G) \rightarrow \{0, 1, 2\}$ is called ternary vertex labeling of G and $f(v)$ is called name of the vertex v of G under f .

For an edge $e = uv$, the instigated edge labeling $f^*: E(G) \rightarrow \{0, 1, 2\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1), v_f(2)$ be the amount of vertices of G having names 0, 1 and 2 separately under f and let $e_f(0), e_f(1), e_f(2)$ be the amount of edges having marks 0, 1 and 2 individually under f .

Definition : A ternary vertex labeling of a graph G is called 3-equitable labeling in the event that if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, 0 \leq i, j \leq 2$.

A graph which concedes 3-equitable labeling is called 3-equitable graph. Some Known Results

- Cahit, demonstrated that
- C_n is 3-equitable if and just if n is not consistent to $3(\text{mod}6)$.
- An Eulerian graph with $q \equiv 3(\text{mod}6)$ is not 3-equitable where q is the number of edges.
- All caterpillars are 3-equitable.
- (Conjecture) A triangular cactus with n pieces is 3-equitable if and just if n is even.
- Every tree with fewer than five close vertices has a 3-equitable labeling.
- Seoud and Abdel Maqsooud demonstrated that
- A graph with p vertices and q edges in which each vertex has odd degree is not 3-equitable if $p \equiv 0(\text{mod}3)$ and $q \equiv 3(\text{mod}6)$.
- All fans aside from $P_2 + K_1$ are 3-equitable.
- P_n^2 is 3-equitable for all n aside from 3.
- $K_{m,n}$ ($3 \leq m \leq n$) is 3-equitable if and just if $(m, n) = (4, 4)$.
- Bapat and Limaye demonstrated that Helms H_n (where $n \geq 4$) are 3-equitable.

- Youssef demonstrated that $W_n = C_n + K_1$ is 3-equitable for all $n \geq 4$.

In the instantaneous segment we will furnish short record of outcomes explored by us in the vicinity of 3-equitable labeling of certain graphs.

PATH UNION OF GRAPHS AND CORDIAL LABELING

Definition : Let G be a graph and $G_1, G_2, \dots, G_n, n \geq 2$ be n duplicates of graph G . At that point the graph got by adding an edge from G_i to G_{i+1} (for $i = 1, 2, \dots, n-1$) is called way union of G .

Shee and Ho presented above idea. They too demonstrated that way union of Petersen graph, trees, wheels and unicyclic graphs are friendly.

We have explored cardinal labeling for way union of limited number of duplicates of cycle with harmony, cycle with twin harmonies and cycle with triangle.

Theorem : The way union of limited number of duplicates of cycle C_n with one harmony is welcoming, where harmony shapes a triangle with edges of the cycle.

Verification: Let G be the way union of cycle C_n with one harmony and $G_1, G_2, \dots, G_k$ be k duplicates of cycle C_n with one harmony, where $|G_i| = n$, for every i . Gave us a chance to signify the continuous vertices of graph G_i by $\{u_{i1}, u_{i2}, \dots, u_{in}\}$, for $i = 1, 2, \dots, k$. Let u_{i1}, u_{i2}, u_{i3} shapes a triangle with harmony e . Let $e_i = u_{i3}u_{(i+1)1}$ be the edge joining G_i and G_{i+1} , for $i = 1, 2, \dots, k-1$. To demarcate labeling capacity $f: V(G) \rightarrow \{0, 1\}$ we acknowledge accompanying cases.

Case 1: $n \equiv 0 \pmod{4}$

In this case we define labeling as:

$$f(u_{ij}) = 0; \text{ if } j \equiv 0, 3 \pmod{4}$$

$$= 1; \text{ if } j \equiv 1, 2 \pmod{4}, \text{ when } i \text{ is even, } 1 \leq i \leq k, 1 \leq j \leq n.$$

$$f(u_{ij}) = 0; \text{ if } j \equiv 1, 2 \pmod{4}$$

$$= 1; \text{ if } j \equiv 0, 3 \pmod{4}, \text{ when } i \text{ is odd, } 1 \leq i \leq k, 1 \leq j \leq n.$$

Case 2: $n \equiv 1 \pmod{4}$

In this case we define labeling as:

When $i \equiv 0, 1 \pmod{4}$

$$f(u_{ij}) = 0; \text{ if } j \equiv 1, 2 \pmod{4}$$

$$= 1; \text{ if } j \equiv 0, 3 \pmod{4}, 1 \leq i \leq k, 1 \leq j \leq n.$$

When $i \equiv 2, 3 \pmod{4}$

$$f(u_{ij}) = 0; \text{ if } j \equiv 0, 3 \pmod{4}$$

$$= 1; \text{ if } j \equiv 1, 2 \pmod{4}, 1 \leq i \leq k, 1 \leq j \leq n.$$

Case 3: $n \equiv 2 \pmod{4}$

In this case we define labeling as:

$$f(u_{in-1}) = 1, f(u_{in}) = 0 \text{ and}$$

$$f(u_{ij}) = 0; \text{ if } j \equiv 1, 2 \pmod{4}$$

$$= 1; \text{ if } j \equiv 0, 3 \pmod{4}, 1 \leq i \leq k, 1 \leq j \leq n-2.$$

Case 4: $n \equiv 3 \pmod{4}$

In this case we define labeling as:

When $i \equiv 0, 1 \pmod{4}$

$$f(u_{i1}) = 0 \text{ and}$$

$$f(u_{ij}) = 0; \text{ if } j \equiv 0, 3 \pmod{4}$$

$$= 1; \text{ if } j \equiv 1, 2 \pmod{4}, 1 \leq i \leq k, 2 \leq j \leq n.$$

When $i \equiv 2, 3 \pmod{4}$

$$f(u_{i1}) = 1 \text{ and}$$

$$f(u_{ij}) = 0; \text{ if } j \equiv 1, 2 \pmod{4}$$

$$= 1; \text{ if } j \equiv 0, 3 \pmod{4}, 1 \leq i \leq k, 2 \leq j \leq n.$$

The labeling pattern defined above covers all possible arrangement of vertices.

In each case, the graph G under consideration satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ as shown in following Table. i.e. G admits cordial labeling.

Let $n = 4a + b, k = 4c + d$ where $n, k \in \mathbb{N}, n \geq 4$.

b	d	vertex conditions	edge conditions
0	0,1,2,3	$v_f(0)=v_f(1)$	$e_f(0)+1=e_f(1)$
1	0,2	$v_f(0)=v_f(1)$	$e_f(0)=e_f(1)+1$
	1	$v_f(0)=v_f(1)+1$	$e_f(0)=e_f(1)$
	3	$v_f(0)+1=v_f(1)$	$e_f(0)=e_f(1)$
2	0,1,2,3	$v_f(0)=v_f(1)$	$e_f(0)=e_f(1)+1$
3	0,2	$v_f(0)=v_f(1)$	$e_f(0)+1=e_f(1)$
	1	$v_f(0)=v_f(1)+1$	$e_f(0)=e_f(1)$
	3	$v_f(0)+1=v_f(1)$	$e_f(0)=e_f(1)$

CONCLUSION

Labeled graph is the theme of momentum investment for numerous analysts as it has expanded provisions. We talk about here cardial labeling and 3-impartial marking of some star identified diagrams. This methodology is novel and donate two new diagrams to the hypothesis of agreeable charts and also 3-equitable graphs.

The determined marking example is exhibited by method of rich delineations which furnishes better comprehension of the determined effects. The outcomes reported here are new and will include new extent in the hypothesis of agreeable and 3-equitable graphs.

We have committed four new comes about for cardial labeling and 3-evenhanded marking each. Some new groups of heartfelt and 3-equitable graphs are likewise acquired. We have likewise talked over implanting and NP-finish issues in the connection of both the labelings. To research some more cheerful and 3-equitable graphs which remains invariant under different diagram operations is a potential territory of examination.

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