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A Two Unit Parallel System with Inspection, Repair and Post Repair

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Abstract – This paper deals with a two-unit active (Parallel) redundant system model with two types of repair Ist and IInd. Whenever a unit fails it requires inspection to decide whether the failed unit needs type I repair or type II repair. Inspection time, type-I repair time, type-II repair time and post repair time distributions are also negative exponential. Various measures of system effectiveness are obtained by using regenerative point technique.

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INTRODUCTION

A large number of papers including (Goel and Singh, 1985), (Gupta and Singh, 1985) with two types of repair have widely been studied in literature. They analysed a single unit multi-component system model with two types of repair (minor and overhaul) where the decision about the type of repair was taken by inspection. The purpose of this paper is to analyses a two-unit active (parallel) redundant system. Both the units of the system have two modes – Normal (N) and total failure (F). The system breaks down when both the units enter into F-mode whenever a unit fails it first inspected by the repairman who decides the needed type of repair (Type I or Type II) and then accordingly the repair of the failed unit is started. On completion of the repair, the unit is finally checked and re-repaired if required by the repairman. The failure and repair times of a unit are assumed to be independent and uncorrelated random variables. The distribution of time to failure of a unit is taken to be exponential.

By using regenerative point technique, the following economic measures of system effectiveness are obtained –

(i) Reliability of the system and MTSF.

(ii) Point wise and steady-state availabilities of the system.

(iii) Expected up time of the system during $(0, t)$.

(iv) Expected busy period of the repairman in inspection, in type-I repair, in type-II repair and in cost repair during $(0, t)$.

(v) Net expected profit incurred by the system during $(0, t)$ and in steady state.

NOTATIONS

N_0 = Unit in *N* (normal) mode and operative.

F_1 = Unit in *F* (Failure) mode and under inspection.

F_{r1} = Unit in F-mode and under repair of type-I.

F_{r2} = Unit in F-mode and under repair of type-II.

$F_{\Pr}$ = Unit in F-mode and under post repair.

F_{W1} = Unit in F-mode and waiting for inspection.

α = Constant failure rate of an operating unit.

P = Probability that a failed unit needs type I repair after inspection.

q = Probability that a failed unit needs type II repair after inspection.

$g_1(\cdot), g_2(\cdot)$ = p.d.f. of time to repair of type I and type II.

$G_1(\cdot), G_2(\cdot)$ = C.d.f. of time to repair of type I and type II respectively.

$k(\cdot), K(\cdot)$ = p.d.f. and c.d.f. of inspection time of a failed unit.

$h(\cdot), H(\cdot)$ = p.d.f. and c.d.f. of post repair time of a repaired unit.

Model

The transition diagram along with the possible transitions between the states is shown in fig. 1. The epochs of the entrance from states S_1 to S_4, S_2 to S_5 S_3 to S_6 and S_7 to S_8 are non-regenrative.

Transition Diagram

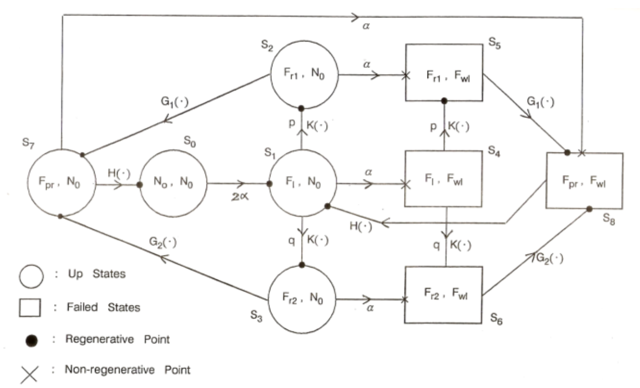

Fig. 1

The transition probability matrix (t.p.m.) is given by

$$P = (p_{ij}) = (Q_{ij}(\infty)) = Q(\infty)$$

If the system transits from S_0 to S_1 then

$$Q_{01}(t) = \int_0^t 2\alpha e^{-2\alpha u} du = 1 - e^{-2\alpha t}$$

$$Q_{12}(t) = P\int_0^t e^{-\alpha u} dK(u)$$

$$Q_{13}(t) = q\int_0^t e^{-\alpha u} dK(u)$$

$$Q_{14}(t) = \alpha\int_0^t e^{-\alpha u} \overline{K}(u) du$$

$$Q_{27}(t) = \int_0^t e^{-\alpha u} \, dG_1(u)$$

$$Q_{25}(t) = \alpha\int_0^t e^{-\alpha u} \overline{G}_1(u) du$$

$$Q_{37}(t) = \int_0^t e^{-\alpha u} \, dG_2(u)$$

$$Q_{36}(t) = \alpha\int_0^t e^{-\alpha u} \overline{G}_2(u) du$$

$$Q_{45}(t) = P\int_0^t dK(u) = PK(t)$$

$$Q_{46}(t) = qK(t)$$

$$Q_{58}(t) = \int_0^t dG_1(u) = G_1(t)$$

$$Q_{68}(t) = \int_0^t dG_2(u) = G_2(t)$$

$$Q_{70}(t) = \int_0^t e^{-\alpha u} \, dH(u)$$

$$Q_{78}(t) = \alpha\int_0^t e^{-\alpha u} \, \overline{H}(u) du$$

$$Q_{81}(t) = \int_0^t dH(u) = H(t)$$

Mean sojourn Time

m_{ij} as the mean sojourn time by the system in state s_i.

$$m_{ij} = \int t\ Q_{ij}(t) = \int t\ q_{ij}(t)\ dt$$

$$m_{01} = \frac{1}{2\alpha}$$

$$m_{12} = p \int t\, e^{-\alpha t}\ dK(t)$$

$$m_{13} = q \int t\, e^{-\alpha t}\ dK(t)$$

$$m_{14} = \alpha \int t\, e^{-\alpha t}\ \overline{K}(t) dt$$

$$m_{25} = \alpha \int t\, e^{-\alpha t}\ \overline{G}_1(t) dt$$

$$m_{27} = \int t\, e^{-\alpha t}\ d\, G_1(t)$$

$$m_{37} = \int t\, e^{-\alpha t}\ d\, G_2(t)$$

$$m_{36} = \alpha \int t\, e^{-\alpha t}\ \overline{G}_2(t) dt$$

$$m_{45} = p \int t\ d\, K(t) = P\,\theta$$

$$m_{46} = q\,\theta \qquad m_{58} = n_1 \qquad m_{68} = n_2$$

$$m_{70} = \int t\ e^{-\alpha t}\ dH(t)$$

$$m_{78} = \alpha \int t\, e^{-\alpha t}\ \overline{H}(t) dt$$

$$m_{81} = m \qquad m_{15}^{(4)} = p \int t\left(1 - e^{-\alpha t}\right) d\, K(t)$$

$$m_{16}^{(4)} = q \int t\left(1 - e^{-\alpha t}\right) d\, K(t)$$

$$m_{28}^{(5)} = \int t\left(1 - e^{-\alpha t}\right) d\, G_1(t)$$

$$m_{38}^{(6)} = \int t\left(1 - e^{-\alpha t}\right) d\, G_2(t)$$

$$m_{71}^{(8)} = \int t\left(1 - e^{-\alpha t}\right) d\, H(t)$$

Availability analysis

$$A_0 = \lim_{t \to \infty} A_0(t)$$

$$= \lim_{s \to 0} s\, A_0^*(s)$$

$$= \lim_{s \to 0} s \frac{N_2(s)}{D_2(s)}$$

Reliability and MISF.

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

$$N_1(s) = z_0^* + q_0^* z_1^* + q_{01}^* q_{12}^* z_2^* + q_{01}^* q_{13}^* z_3^* + q_{01}^*\left(q_{12}^* q_{27}^* + q_{13}^* q_{37}^*\right) z_7^*$$

$$D_1(s) = 1 - q_{01}^*\left(q_{12}^* q_{27}^* + q_{13}^* q_{37}^*\right) q_{70}^*$$

$$MTSF = \int R_0(t) dt = \lim_{s \to 0} R^*(s)$$

$$= \frac{N_1(0)}{D_1(0)}.$$

Busy period analysis of repairman

Let $B_i^1(t)$ be the probability that the repairman is busy in inspection of a failed unit at time *t*.

$$B_0^1(t) = q_{01}(t)(c)\ B_1^1(t)$$

$$B_1^1(t) = \overline{K}(t) + q_{12}(t)\,(c)\,B_2^1(t) + q_{13}(t)\,(c)\,B_3^1(t) + q_{15}^{(4)}(t)\,(c)\,B_5^1(t) + q_{16}^{(4)}(t)\,(c)\,B_6^1(t)$$

$$B_2^1(t) = q_{27}(t)\,(c)\,B_7^1(t) + q_{28}^{(5)}(t)\,(c)\,B_8^1(t)$$

$$B_3^1(t) = q_{37}(t)\,(c)\,B_7^1(t) + q_{38}^{(6)}(t)\,(c)\,B_8^1(t)$$

$$B_5^1(t) = q_{58}(t)\,(c)\,B_8^1(t)$$

$$B_6^1(t) = q_{68}(t)\,(c)\,B_8^1(t)$$

$$B_7^1(t) = q_{70}(t)\,(c)\,B_0^1(t) + q_{71}^{(8)}(t)\,(c)\,B_1^1(t)$$

$$B_8^1(t) = q_{81}(t)\,(c)\,B_1^1(t)$$

Profit Function Analysis

$P(t)$ = expected total revenue in $(0, t)$ – expected total expenditure during $(0, t)$

$$= K_0\mu_{up}(t) - K_1\mu_b^1(t) - K_2\mu_b^1(t) - K_3\mu_b^2(t) - K_4\mu_b^p(t)$$

K_0 is the revenue per unit up time.

K_1 is the cost of inspection per-unit of time.

K_2 is the cost of type-I repair per unit of time.

K_3 is the cost of type-II repair per-unit of time and K_4 is the per-unit of time cost of post repair.

$$\mu_{up}(t) = \int_0^t A_0(u)du$$

$$\mu_b^1(t) = \int_0^t B_0^1(u)du$$

$$\mu_b^1(t) = \int_0^t B_0^1(u)du$$

$$\mu_b^2(t) = \int_0^t B_0^2(u)du$$

$$\mu_b^P(t) = \int_0^t B_0^P(u)du$$

Then

$$P = K_0A_0 - K_1B_0^1 - K_2B_0^1 - K_3B_0^2 - K_4B_0^P$$

Particular Case

All the repair time distribution are also negative exponential.

$$K(t) = 1 - e^{-\eta t} \qquad G_1(t) = 1 - e^{-\beta_1 t}$$

$$G_2(t) = 1 - e^{-\beta_2 t} \qquad H(t) = 1 - e^{-\mu t}$$

So that

$$\tilde{K}(s) = \frac{\eta}{s+\eta} \qquad \tilde{G}_1(s) = \frac{\beta_1}{s+\beta_1}$$

$$\tilde{G}_2(s) = \frac{\beta_2}{s+\beta_2} \qquad \tilde{H}(s) = \frac{\mu}{s+\mu}$$

Then

$$P_{12} = \frac{P\eta}{\alpha+\eta} \qquad P_{13} = \frac{q\eta}{\alpha+\eta}$$

$$P_{14} = \frac{\alpha}{\alpha+\eta}$$

$$P_{27} = \frac{\beta_1}{\alpha+\beta_1} \qquad P_{25} = \frac{\alpha}{\alpha+\beta_1}$$

$$P_{37} = \frac{\beta_2}{\alpha+\beta_2} \qquad P_{36} = \frac{\alpha}{\alpha+\beta_2}$$

$$P_{70} = \frac{\mu}{\alpha+\mu} \qquad P_{78} = \frac{\alpha}{\alpha+\mu}$$

$$P_{15}^{(4)} = \frac{P\alpha}{\alpha+\eta} \qquad P_{16}^{(4)} = \frac{q\alpha}{\alpha+\eta}$$

$$\psi_1 = \frac{1}{\alpha+\eta} \qquad \psi_2 = \frac{1}{\alpha+\beta_1}$$

$$\psi_3 = \frac{1}{\alpha+\beta_2}$$

$$\eta_1 = \frac{1}{\beta_1} \qquad \eta_2 = \frac{1}{\beta_2}$$

$$\psi_7 = \frac{1}{\alpha+\mu} \qquad m = \frac{1}{\mu}$$

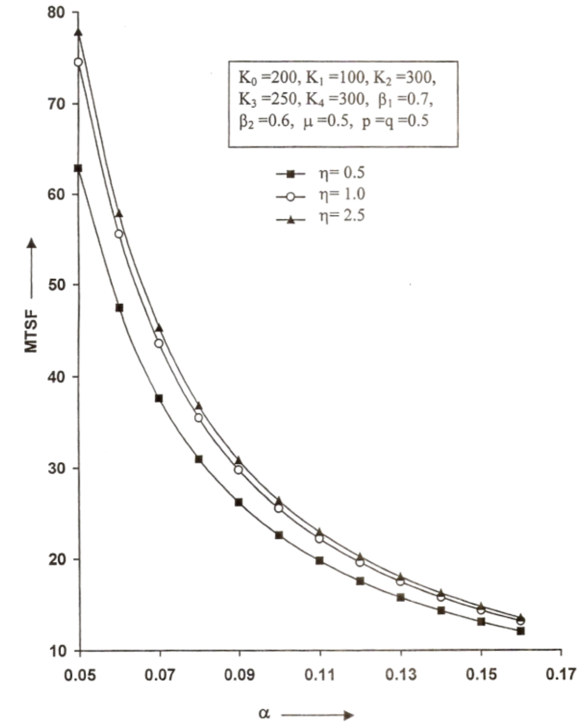

Fig. 2

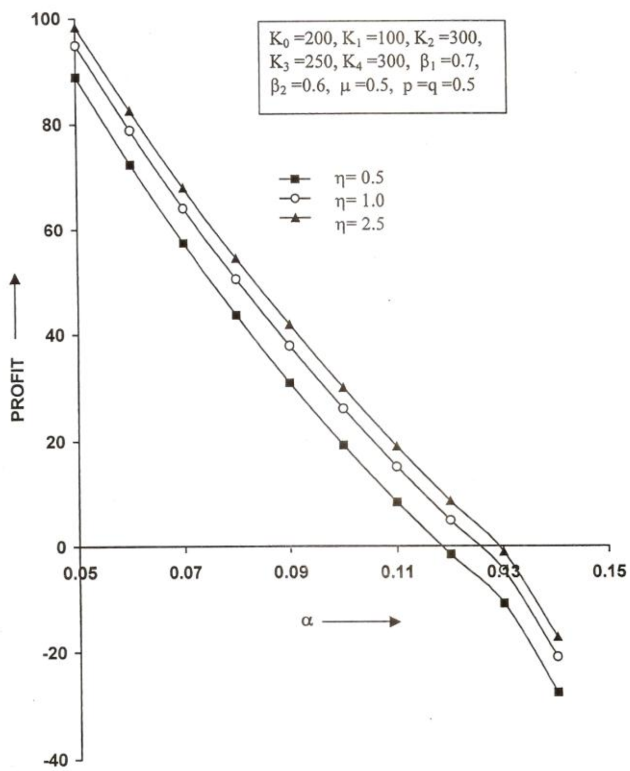

Fig. 3

CONCLUSION

In fig. 2, we observe that MTSF decreases uniformly as α increases. Moreover, it increases with the increase in η. The similar trends w.r.t. α and η are observed for the case of profit in fig. 3 but here the trends are almost linear.

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