Behavioral and Availability Analysis of Two Unit System, in which one can work in reduced capacity.

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Abstract: This paper discusses the Behavioral and Availability Analysis of Two Unit System, in which one can work in reduced capacity a Single Unit Redundant System having Imperfect Switch-over device with a single repair facility. in this chapter there is two unit model connected in series in which one can work in reduced state after first failure instead of completely failed. Thus there are two type of failure: Partially failed and completely failed. The system consists of two non-identical units 'A' & 'B' connected in series, in which 'A' can work in reduced state after failure. The unit 'A' can fail partially and hence can be in up state, partially failed state (reduced state) or totally failed state. The system can work with reduced capacity in a partially failed state. There is a single repair facility catering to the needs of both the unit upon failure. Repairs are perfect i.e. the repair facility never does any damage to the units and a repaired unit works like a new-one. The system is down if any one of unit is fails completely and nothing can further fail when the system is in failed state. The distributions of the failure times and repair times are exponential and general respectively and also different for both unit. They are also assumed to be independent of each other. The system is discussed for steady state conditions. Using the *Regenerative Point Graphical Technique (RPGT)* the following system characteristics have been evaluated to study the system performance.

- **i. Mean Time To System Failure (MTSF).**
- **ii. Total fraction of time for which the system is available.**
- **iii. The busy period of the Server doing any given job.**
- **iv. The number of the Server's visits.**

The profit analysis of the system is also carried out by using some of the system characteristics as mentioned above. Graphs are drawn to depict the behavior of the MTSF and Steady state Availability of the system for a particular case.

Key words—Reliability, Availability, Priority Maintainance, Primary Circuit, Secondary Circuit, Tertiary Circuit, Base-State, Regenerative Point Graphical Technique (RPGT), MTSF, Busy Period of Server.

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1. INTRODUCTION

The process industries are the backbone of a country for its development. The process industries must provide continuous and long term production to meet the ever increasing demand at lower costs. The reliability and availability analysis of process industries can benefit in terms of higher production, lower maintenance costs. The availability of complex systems and continuous process industries can be enhanced by considering maintenance, inspection, repairs and replacements of the parts of the failed units. A system may not be working to the fullest of its capacity in a particular state and instead it is partially available (i.e. with reduced capacity) in that state.

The researchers including Barlow et al [1], Chung et al [2] discussed the single unit system and Das et al [3], Fukuta et al [4],Kodama et al [5], Osaki et al [6] discussed two or more unit having switch-over device while, Chander et al [7], Malik et al [8] have used the Regenerative Point (RPT) and solved the transformed state equations recursively, to find $\phi_0^{\sim}(s), A_0^*(s), B_0^*(s), and\ V_0^*(s)$ corresponding to initial state '0' and then determined the parameters of the stochastic systems(under steady state conditions). Gupta, et al [9] have done the analysis of various systems by using the *'Regenerative Point Graphical Technique**(RPGT)**'* introduced by Gupta[10], for determining the Mean Time to System Failure(MTSF), Availability, Busy period of Server, number of Server's visits and number of Replacement etc. (under steady state conditions). But, the difficulty for the evaluation of key parameters of the system increases with the increase in the number of the transition states and circuits in the transition diagram of the system, it also becomes difficult to locate all the paths from the initial state to the other states and the various circuits along the different paths while using *Regenerative Point Graphical Technique***(RPGT).**

In this paper the behavioral and availability analysis of two units system in which one can work in reduced capacity is done. The mean time to system failure, availability and other key parameters of the system are evaluated using the *Regenerative Point Graphical Technique* ***(RPGT)***, discussed by Gupta [10].

In view of the above, in this paper there is two unit model connected in series in which one can work in reduced state after first failure instead of completely failed. Thus there are two type of failure: Partially failed and completely failed. The system consists of two non-identical units 'A' & 'B' connected in series, in which 'A' can work in reduced state after failure. The unit 'A' can fail partially and hence can be in up state, partially failed state (reduced state) or totally failed state. The system can work with reduced capacity in a partially failed state. There is a single repair facility catering to the needs of both the unit upon failure. Repairs are perfect i.e. the repair facility never does any damage to the units and a repaired unit works like a new-one. The system is down if any one of unit is fails completely and nothing can further fail when the system is in failed state. The distributions of the failure times and repair times are exponential and general respectively and also different for both unit. They are also assumed to be independent of each other. The system is discussed for steady state conditions. Using the *Regenerative Point Graphical Technique* ***(RPGT)*** the following system characteristics have been evaluated to study the system performance.

v. Mean Time To System Failure (MTSF).

vi. Total fraction of time for which the system is available.

vii. The busy period of the Server doing any given job.

viii. The number of the Server's visits.

The profit analysis of the system is also carried out by using some of the system characteristics as mentioned above. Graphs are drawn to depict the behavior of the MTSF and Steady state Availability of the system for a particular case.

2 ASSUMPTIONS AND NOTATIONS:

The following assumptions and notations/symbols are used:

1) The system consists of two non-identical units 'A' & 'B' connected in series, in which 'A' can work in reduced state after failure.

2) The unit 'A' can fail partially and hence can be in up state, partially failed state (reduced state) or totally failed state. The system can work with reduced capacity in a partially failed state.

3) There is a single repair facility catering to the needs of both the unit upon failure.

4) The distributions of the failure times and repair times are exponential and general respectively and also different for both unit. They are also assumed to be independent of each other.

5) Repairs are perfect i.e. the repair facility never does any damage to the units.

6) A repaired unit works like a new-one.

7) The system is down if any one of unit is fails completely.

8) Nothing can further fail when the system is in failed state.

9) The system is discussed for steady state conditions.

pr/pf : Probability/transition probability factor.

$q_{i,j}(t)$: probability density function (p.d.f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$p_{i,j}$: steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $p_{i,j} = q^*_{i,j}(0)$; where $*$ denotes Laplace transformation.

$\overline{cycle}$: a circuit formed through un-failed states.

k-cycle : a circuit (may be formed through regenerative or non-regenerative/failed states)

whose terminals are at the regenerative state k.

k-$\overline{cycle}$: a circuit (may be formed through only un-failed regenerative/non-regenerative states) whose terminals are at the regenerative state k.

$\left(i \xrightarrow{s_r} j\right)$: *r*-th directed simple path from *i*-state to j-state; r takes positive integral values for different paths from *i*-state to j-state.

$\left(\xi \xrightarrow{sff} i\right)$: a directed simple failure free path from ξ-state to i-state.

$V_{k,k}$:*pf* of the state k reachable from the terminal state k of the k-cycle.

$V_{\overline{k,k}}$: *pf* of the state k reachable from the terminal state k of the k-$\overline{cycle}$.

$R_i(t)$: reliability of the system at time t, given that the system entered the un-failed regenerative state i at t=0.

$A_i(t)$: probability that the system is available in up-state at time t, given that the system entered regenerative state i at t=0.

$B_i(t)$:probability that the server is busy doing a particular job at epoch t, given that the system entered regenerative state i at t=0.

$V_i(t)$:the expected number of visits of the server for a given job in (0,t], given that the system entered regenerative state i at t=0.

$W_i(t)$: probability that the server is busy doing a particular job at epoch t without transiting to any other regenerative state 'i' through one or more non-regenerative states, given that the system entered the regenerative state 'i' at t=0.

μ_i : mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt.$$

μ_i^1 : the total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i' at t=0.

η_i : expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0; $\eta_i = W_i^*(0)$.

f_j : fuzziness measure of the j-state.

λ_1/λ_2 : constant failure rate of the unit 'A' to a partially failed state/ from partially failed state to a totally failed state.

λ : constant failure rate of the unit 'B'.

$g(t)/G(t)$: probability density function/cumulative distribution function of the repair-time of the unit 'A' from the partially failed state.

$h(t)/H(t)$: probability density function/cumulative distribution function of the repair-time of the unit 'A' from the completely failed state.

$f(t)/F(t)$: probability density function/cumulative distribution function of the repair-time of the unit 'B'.

A/A'/a : Unit in the operative state/ partially failed state/completely failed state.

B/b : Unit in the operative state/ failed state.

The system can be in any of the following states with respect to the above symbols.

S_0 = AB

$S_1 = A'$B

S_2 = aB

S_3 = Ab

$S_4 = A'$b

States S_0, S_1, S_2, S_3 *and* S_4 are regenerative states. The possible transitions between states along with transition time c.d.f.'s are shown in Fig. 1

3 TRANSITION DIAGRAM OF THE SYSTEM:

Following the above assumptions and notations, the transition diagram of the system are shown in Fig. 1.

State	Symbol
Regenerative state/point	●
Up-state	○
Failed state	□
Degenerated/Reduced state	⬭

Table - 1

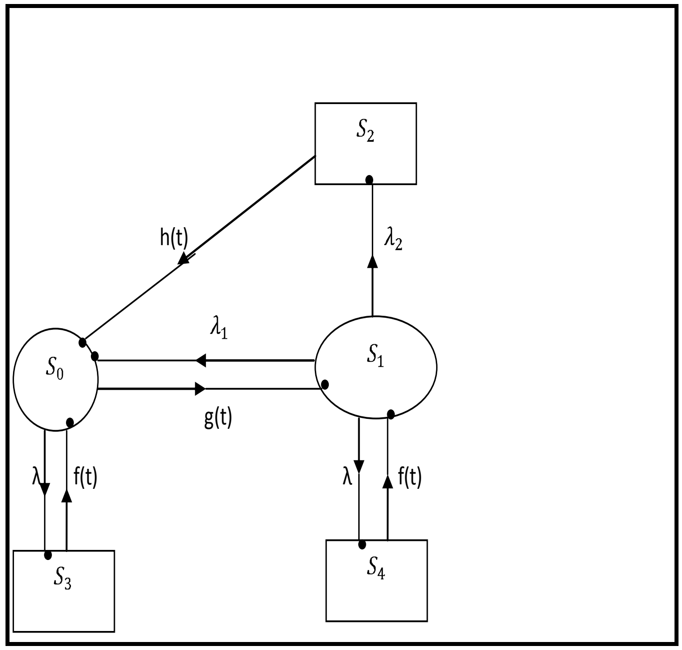

Fig. 1

4 EVALUATION OF PARAMETERS OF THE SYSTEM:

The key parameters (under steady state conditions) of the system are evaluated by determining a 'base-state' and applying RPGT. The MTSF is determined w.r.t. the initial state '0' and the other parameters are obtained by using base-state.

4.1 Determination of base-state:

From the transition diagram (Fig. 1), all the paths (P0) from one regenerative state to the other reachable states are determined and shown in Table- 2. The Primary, Secondary, Tertiary circuits at all vertices are shown in Table- 3.

Paths from State 'i' to the Reachable State 'j':P0

i	j = 0	j = 1	j = 2	j = 3	j = 4
0	{0,1,0} {0,3,0}	{0,1}	{0,1,2}	{0,3}	{0,1,4}
1	{1,0} {1,2,0}	{1,4,1} {1,0,1}	{1,2}	{1,0,3} {1,2,0,3}	{1,4}

2	{2,0}	{2,0,1}	{2,0,1,2}	{2,0,3}	{2,0,1,4}
3	{3,0}	{3,0,1}	{3,0,1,2}	{3,0,3}	{3,0,1,4}
4	{4,1,0} {4,1,2,0}	{4,1}	{4,1,2}	{4,1,0,3} {4,1,2,0,3}	{4,1,4}

Table - 2

Primary, Secondary, Tertiary circuits at a Vertex

Vertex i	**Simple Circuits**	**(CL1)**	**(CL2)**
0	{0,1,0} {0,3,0}	{1,4,1}	Nil
1	{1,4,1} {1,0,1}	{0,3,0}	Nil
2	{2,0,1,2}	{0,1,0},{0,3,0} {1,4,1}	Nil
3	{3,0,3}	{0,1,0}	Nil
4	{4,1,4}	{1,0,1}	Nil

Table- 3

In the transition diagram of Fig. 1, there are three, three, one, one and one simple circuits at the vertices 0,1,2,3 & 4 respectively. As there are three simple circuits associated each of the vertices 0 & 1.So,any of these can be the base-state of the system. Now, the distinct primary circuits along all the simple paths from the vertex '0' to all the vertices is {1,4,1}. Similarly, there are only one i.e.{0,3,0} primary circuit along the paths from the vertex '1'. Also, there are no secondary circuits from the vertex '0' and '1'. We choose the vertex '0' as a base-state.

Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State '0')

Vertex j	$\left(\mathbf{0} \xrightarrow{S_r} \boldsymbol{j}\right)$ **: (P0)**	**(P1)**	**(P2)**	**(P3)**
1	$\left(0 \xrightarrow{S_1} 1\right)$:{0,1}	{1,4,1}	Nil	Nil
2	$\left(0 \xrightarrow{S_1} 2\right)$:{0,1,2}	{1,4,1}	Nil	Nil
3	$\left(0 \xrightarrow{S_1} 3\right)$:{0,3}	Nil	Nil	Nil
4	$\left(0 \xrightarrow{S_1} 4\right)$:{0,1,4}	{1,4,1}	Nil	Nil

Table - 4

4.2 <u>TRANSITION PROBABILITIES AND THE MEAN SOJOURN TIMES</u>:

<u>Transition Probabilities:</u>

$q_{i,j}(t)$: probability density function (p.d.f.) of the first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t].

$p_{i,j}$: steady state transition probability from a regenerative state i to a regenerative state j without visiting any other regenerative state. $p_{i,j} = q^*_{i,j}(0)$; where $*$ denotes Laplace transformation.

$\boldsymbol{q_{i,j}(t)}$	$\boldsymbol{p_{i,j} = q^*_{i,j}}$**(0)**
$q_{0,1}(t) = \lambda_1 e^{-(\lambda+\lambda_1)t}$ $q_{0,3}(t) = \lambda e^{-(\lambda+\lambda_1)t}$	$p_{0,1} = \frac{\lambda_1}{\lambda+\lambda_1}$ $p_{0,3} = \frac{\lambda}{\lambda+\lambda_1}$
$q_{1,0}(t) = g(t)e^{-(\lambda+\lambda_2)t}$ $q_{1,2}(t) = \lambda_2 e^{-(\lambda+\lambda_2)t}\bar{G}(t)$	$p_{1,0} = g^*(\lambda+\lambda_2)$ $p_{1,2} = \frac{\lambda_2}{\lambda+\lambda_2}\{1-g^*(\lambda+\lambda_2)\}$
$q_{2,0}(t) = h(t)$ $q_{1,4}(t) = \lambda e^{-(\lambda+\lambda_2)t}\bar{G}(t)$	$p_{2,0} = h^*(0)$ $p_{1,4}$
$q_{3,0}(t) = f(t)$	$p_{3,0} = f^*(0)$ $= \frac{}{\lambda+\lambda_2}\{1-g^*(\lambda+\lambda_2)\}$
$q_{4,1}(t) = f(t)$	$p_{4,1} = f^*(0)$

Table - 5

It can be easily verified that;

$p_{0,1} + p_{0,3} = 1$; $p_{1,0} + p_{1,2} + p_{1,4} = 1$; $p_{2,0} = h^*(0) = 1$; $p_{3,0} = f^*(0) = 1$; $p_{4,1} = f^*(0) = 1$

Mean Sojourn Times:

$R_i(t)$: reliability of the system at time t, given that the system in regenerative state i.

μ_i :mean sojourn time spent in state i, before visiting any other states;

$$\mu_i = \int_0^\infty R_i(t)dt = R_i^*(0)$$

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0(t) = e^{-(\lambda+\lambda_1)t}$	$\mu_0 = \frac{1}{\lambda+\lambda_1}$
$R_1(t) = e^{-(\lambda+\lambda_2)t}\bar{G}(t)$	$\mu_1 = \frac{1-g^*(\lambda+\lambda_2)}{(\lambda+\lambda_2)}$
$R_2(t) = \bar{H}(t)$	$\mu_2 = -h^{*\prime}(0)$
$R_3(t) = \bar{F}(t)$	$\mu_3 = -f^{*\prime}(0)$
$R_4(t) = \bar{F}(t)$	$\mu_4 = -f^{*\prime}(0)$

Table - 6

4.3 Evaluation of Parameters:

The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated, by applying *Regenerative Point Graphical Technique* ***(RPGT)*** and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state '0' are:

$$V_{0,0} = \left[(0,3,0) + \frac{(0,1,0)}{1-L_1} + \frac{(0,1,2,0)}{1-L_1}\right] = 1$$

$$V_{0,1} = \frac{(0,1)}{1-L_1} = \frac{p_{0,1}}{1-p_{1,4}}$$

$$V_{0,2} = \frac{(0,1,2)}{1-L_1} = \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}$$

$$V_{0,3} = (0,3) = p_{0,3}$$

$$V_{0,4} = \frac{(0,1,4)}{1-L_1} = \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}$$

Where,
$1 - L_1 = 1 - \{1,4,1\} = 1 - p_{1,4}p_{4,1} = 1 - p_{1,4}$

(a). MTSF(T_0): From Fig. 1, the regenerative un-failed states to which the system can transit(initial state '0'), before entering any failed state are: i = 0,1. For 'ξ' = '0', MTSF is given by

MTSF =
$$\left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r(sff)} i)\}.\mu_i}{\prod_{k_1\neq\xi}\{1-V_{\overline{k_1,k_1}}\}}\right\}\right] \div \left[1 - \sum_{s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r(sff)} \xi)\}}{\prod_{k_2\neq\xi}\{1-V_{\overline{k_2,k_2}}\}}\right\}\right]$$

T_0 = [(0,0) μ_0+ (0,1) μ_1] $\div$ [1 – L_0] = N $\div$ D

Where, L_0 = (0,1,0) = $p_{0,1}p_{1,0}$

N = [(0,0) μ_0+ (0,1) μ_1] = $p_{0,0}\mu_0$ + $p_{0,1}\mu_1$ = μ_0 + $p_{0,1}\mu_1$

D = [1 – L_0] = 1- $p_{0,1}p_{1,0}$

(b). Availability of the system: From Fig. 1, the regenerative states, at which the system is available are: j = 0,1 and the regenerative states are i = 0 to 4. For 'ξ' = '0', the total fraction of time for which the system remains available is given by

A_0 =
$$\left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}f_j.\mu_j}{\prod_{k_1\neq\xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2\neq\xi}\{1-V_{k_2,k_2}\}}\right\}\right]$$

$A_0 = \left[\sum_j V_{\xi,j} \cdot f_j \cdot \mu_j\right] \div \left[\sum_i V_{\xi,i} \cdot \mu_i^1\right]$

$= \left[V_{0,0} \cdot f_0 \cdot \mu_0 + V_{0,1} \cdot f_1 \cdot \mu_1\right] \div \left[V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1\right]$

$= \left[f_0\mu_0 + \frac{p_{0,1}}{1-p_{1,4}} f_1 \mu_1\right] \div \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,4}}\mu_1^1 + \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}\mu_2^1 + p_{0,3}\mu_3^1 + \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}\mu_4^1\right]$

$= N_0 \div D_0$

Where,

$N_0 = \left[f_0\mu_0 + \frac{p_{0,1}}{1-p_{1,4}} f_1 \mu_1\right]$

$D_0 = \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,4}}\mu_1^1 + \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}\mu_2^1 + p_{0,3}\mu_3^1 + \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}\mu_4^1\right]$

$A_0 = N_1 \div D_1$

$N_1 = [(1- p_{1,4})\mu_0 + p_{0,1}\mu_1]; \left(f_j = 1 \;\forall\, j\right)$

$D_1 = [(1- p_{1,4})(\mu_0 + p_{0,3}\mu_3) + p_{0,1}(\mu_1 + p_{1,2}\mu_2 + p_{1,4}\mu_4)]; (\mu_j^1 = \mu_j \;\forall\, j)$

(c). Busy period of the Server: From Fig. 1, the regenerative states where Server is busy while doing repairs are: j = 1,2,3,4; the regenerative states are: i = 0 to 4. For 'ξ' = '0', the total fraction of time for which the Server remains busy is

$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}.\eta_j}{\prod_{k_1 \neq \xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1-V_{k_2,k_2}\}}\right\}\right]$

$B_0 = \left[\sum_j V_{\xi,j} \cdot \eta_j\right] \div \left[\sum_i V_{\xi,i} \cdot \mu_i^1\right]$

$= \left[V_{0,1} \cdot \eta_1 + V_{0,2} \cdot \eta_2 + V_{0,3} \cdot \eta_3 + V_{0,4} \cdot \eta_4\right] \div \left[V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1\right]$

$= \left[\frac{p_{0,1}}{1-p_{1,4}}\eta_1 + \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}\eta_2 + p_{0,3}\eta_3 + \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}\eta_4\right] \div \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,4}}\mu_1^1 + \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}\mu_2^1 + p_{0,3}\mu_3^1 + \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}\mu_4^1\right]$

$= N_{00} \div D_0$

Where, $N_{00} = \left[\frac{p_{0,1}}{1-p_{1,4}}\eta_1 + \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}\eta_2 + p_{0,3}\eta_3 + \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}\eta_4\right]$

$B_0 = N_{01} \div D_1$

$N_{01} = [(1- p_{1,4})p_{0,3}\mu_3 + p_{0,1}(\mu_1 + p_{1,2}\mu_2 + p_{1,4}\mu_4)]; (\eta_j = \mu_j \;\forall\, j)$

$D_1 = [(1- p_{1,4})(\mu_0 + p_{0,3}\mu_3) + p_{0,1}(\mu_1 + p_{1,2}\mu_2 + p_{1,4}\mu_4)]; (\mu_j^1 = \mu_j \;\forall\, j)$

(d). Expected number of Server's visits: From Fig. 1, the regenerative states where the Server visits(afresh) for repairs of the system are: j = 1,3; the regenerative states are: i = 0 to 4. For 'ξ' = '0', the expected number of server's visits per unit time is given by

$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{k_1 \neq \xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}.\mu_i^1}{\prod_{k_2 \neq \xi}\{1-V_{k_2,k_2}\}}\right\}\right]$

$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i} \cdot \mu_i^1\right]$

$= (V_{0,1} + V_{0,3}) \div \left[V_{0,0}\mu_0^1 + V_{0,1}\mu_1^1 + V_{0,2}\mu_2^1 + V_{0,3}\mu_3^1 + V_{0,4}\mu_4^1\right]$

$= \left[\frac{p_{0,1}}{1-p_{1,4}} + p_{0,3}\right] \div \left[\mu_0^1 + \frac{p_{0,1}}{1-p_{1,4}}\mu_1^1 + \frac{p_{0,1}p_{1,2}}{1-p_{1,4}}\mu_2^1 + p_{0,3}\mu_3^1 + \frac{p_{0,1}p_{1,4}}{1-p_{1,4}}\mu_4^1\right]$

$V_0 = N_{02} \div D_1$

$N_{02} = [(1- p_{1,4})p_{0,3} + p_{0,1}]$

D_1 = [(1- $p_{1,4}$)(μ_0+$p_{0,3}\mu_3$)+ $p_{0,1}(\mu_1 + p_{1,2}\mu_2 + p_{1,4}\mu_4)$];($\mu_j^1 = \mu_j \forall$ j)

5 PROFIT FUNCTION OF THE SYSTEM:

The Profit analysis of the system can be done by using the profit function:

$$P_0 = C_1 . A_0 - C_2 . B_0 - C_3 . V_0$$

Where, $C_1 =$ Revenue per unit of time the system is available.

$C_2 =$ Cost per unit time the server remains busy for the repairs.

$C_3 =$ Cost per visit of the server.

6 PARTICULAR CASE

Let us take;

$g(t) = \alpha e^{-\alpha t}$, $h(t) = \beta e^{-\beta t}$, $f(t) = \omega e^{-\omega t}$

We have,

$p_{0,1} = \frac{\lambda_1}{\lambda+\lambda_1}$, $p_{0,3} = \frac{\lambda}{\lambda+\lambda_1}$, $p_{1,0} = \frac{\alpha}{\alpha+\lambda+\lambda_2}$, $p_{1,2} = = \frac{\lambda_2}{\alpha+\lambda+\lambda_2}$, $p_{1,4} = = \frac{\lambda}{\alpha+\lambda+\lambda_2}$

, $p_{2,0}$= 1, $p_{3,0} = 1$, $p_{4,1} = 1$

$\mu_0 = \frac{1}{\lambda+\lambda_1}$, $\mu_1 = \frac{1}{\alpha+\lambda+\lambda_2}$, $\mu_2 = \frac{1}{\beta}$, $\mu_3 = \frac{1}{\omega}$, $\mu_4 = \frac{1}{\omega}$

By using these results, we get the following:

MTSF(T_0) = $\frac{\alpha+\lambda+\lambda_2+\lambda_1}{\lambda(\alpha+\lambda+\lambda_2)+\lambda_1(\lambda+\lambda_2)}$

Availability(A_0) = $\frac{\omega\beta(\alpha+\lambda_1+\lambda_2)}{\beta(\alpha+\lambda_2)(\omega+\lambda)+\lambda_1(\omega\beta+\lambda\beta+\omega\lambda_2)}$

Busy Period of the Server(B_0) = $\frac{\beta\lambda(\alpha+\lambda_2)+\lambda_1(\omega\beta+\lambda\beta+\omega\lambda_2)}{\beta(\alpha+\lambda_2)(\omega+\lambda)+\lambda_1(\omega\beta+\lambda\beta+\omega\lambda_2)}$

Expected No. of Visits by the Server(V_0) = $\frac{\omega\beta[(\lambda+\lambda_1)(\alpha+\lambda_2)+\lambda\lambda_1]}{\omega\omega_1(\omega+\lambda+\lambda_1)+\lambda\lambda_1(\omega+\omega_1+\lambda)}$

7. ANALYTICAL DISCUSSION:

The following tables, graphs, and conclusions are obtained for:

λ = 0.01; λ_2= 0.005; $\omega = \beta = 0.80.$

7.1 MTSF vs. Repair Rate:

The MTSF of the system is calculated for different values of the Failure Rate (λ_1) by taking λ_1 = 0.005, 0.006, 0.007, 0.008, 0.009 and 0.01 and for different values of the Repair Rate (α) by taking α = 0.80, 0.85, 0.90, 0.95 and 1.0. The data so obtained are shown in Table 6 and graphically in Fig. 2.

λ_1	T_0 (α=0.80)	T_0 (α=0.85)	T_0 (α=0.90)	T_0 (α=0.95)	T_0 (α=1.0)
0.00	**99.696**	**99.713**	**99.729**	**99.743**	**99.755**
0.006	99.636	99.656	99.675	99.692	99.707
0.007	99.576	99.600	99.622	99.641	99.658
0.008	99.516	99.544	99.568	99.590	99.610
0.009	99.456	99.487	99.515	99.540	99.562
0.01	99.397	99.432	99.462	99.490	99.514

Table-6

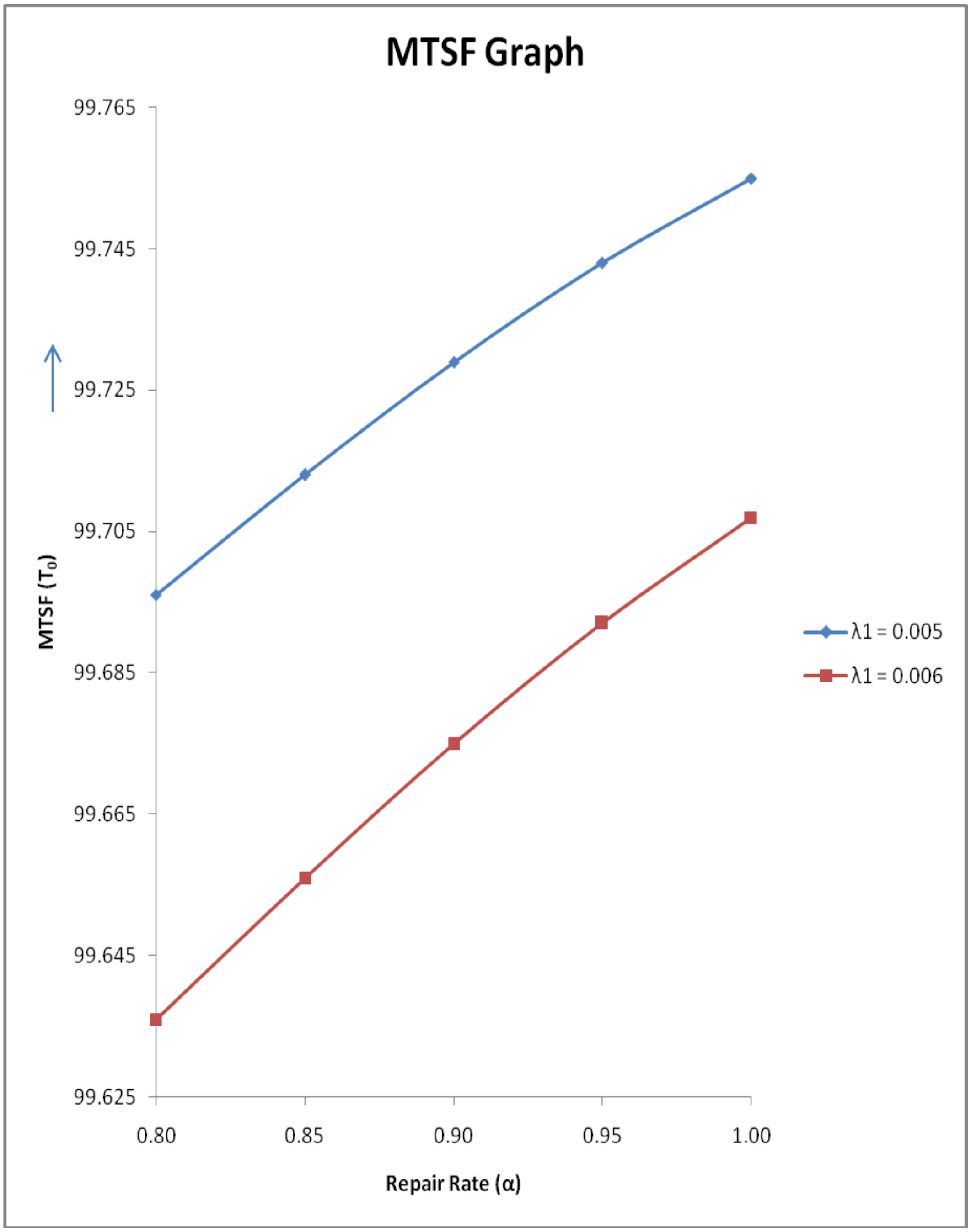

Fig. 2

Table 2 shows the behavior of the MTSF(T_0) vs. the Repair Rate(α) of the Unit of the System for different values of the Failure Rate(λ_1). It is concluded that MTSF increases with increase in the values of the Repair Rate(α). Further it can be concluded from the Fig. 2 that values of MTSF(T_0) shows the expected trend for different values of Failure Rate($\lambda = 0.005 \,\&\, 0.006$), as T_0 increases with the increase in the values of Repair Rate(α).

7.2 Availability(A_0) vs. Repair Rate(α):

The Availability of the system is calculated for different values of the Failure Rate (λ_1) by taking λ_1 = 0.005, 0.006, 0.007, 0.008, 0.009 and 0.01 and for different values of the repair rate (α) by taking α = 0.80, 0.85, 0.90, 0.95 and 1.0. The data so obtained are shown in Table -7 and graphically in Fig.3.

λ_1	A_0 (α=0.80)	A_0 (α=0.85)	A_0 (α=0.90)	A_0 (α=0.95)	A_0 (α=1.0)
0.005	0.987616	0.987619	0.987621	0.987622	0.987624
0.006	0.987609	0.987611	0.987614	0.9876216	0.987618
0.007	0.987601	0.987604	0.987607	0.987610	0.987612
0.008	0.987594	0.987597	0.987601	0.987604	0.987606
0.009	0.987586	0.987590	0.987594	0.987597	0.987600
0.01	0.987579	0.987583	0.987587	0.987591	0.987594

Table-7

Table 7 shows the behaviour of the Availability (A_0) vs. the Repair Rate (α) of the Unit of the System for different values of the Failure Rate (λ_1). It is concluded that Availability increases with increase in the values of the Repair Rate (α).

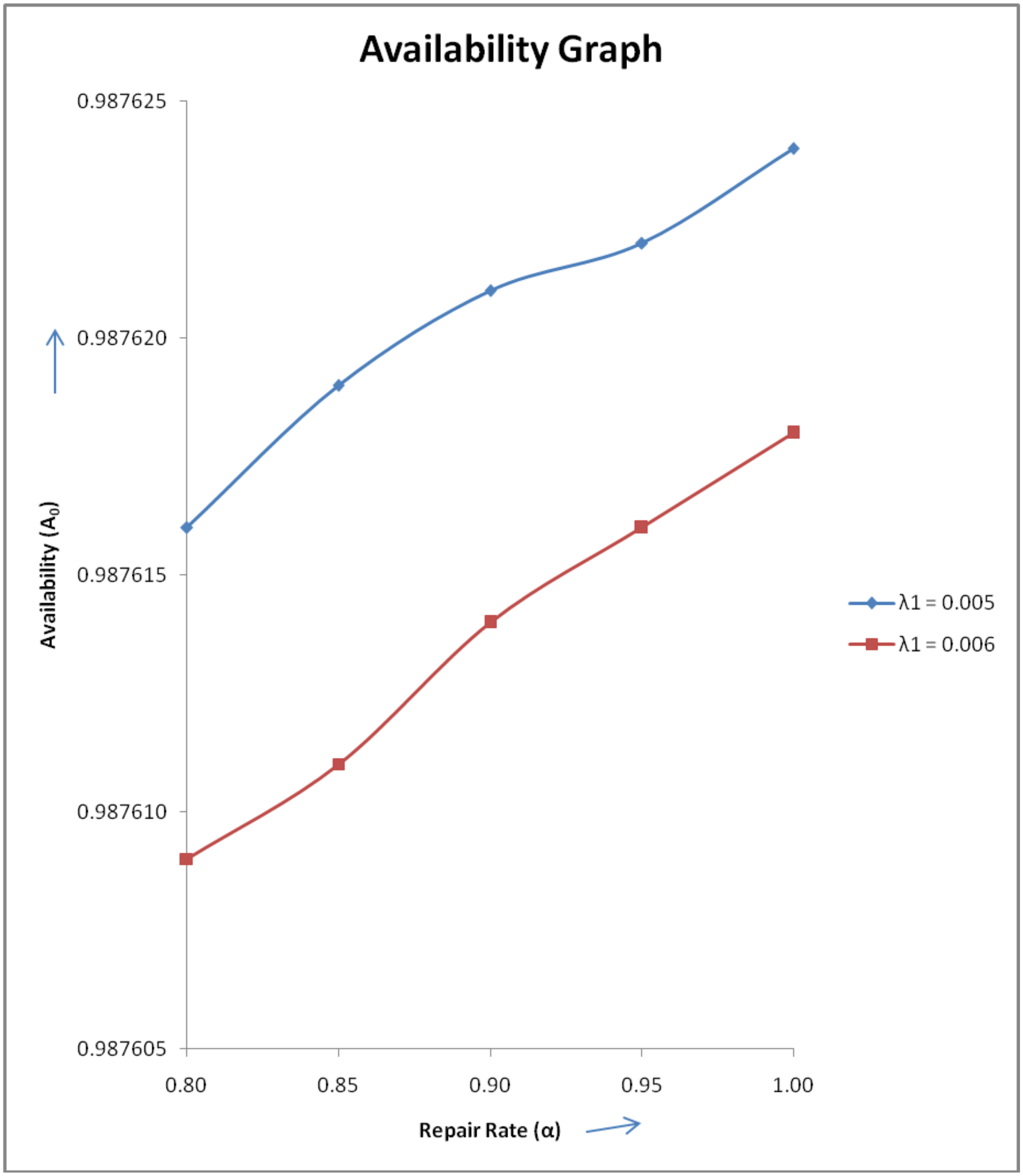

Fig. 3

Further it can be concluded from the Fig. 3 that values of Availability (A_0) shows the expected trend for different values of Failure Rate($\lambda_1 = 0.005\ \&\ 0.006$), as A_0 increases with the increase in the values of Repair Rate(α).

8. CONCLUSION:

From the Graphs and Tables, we see that as the Repair Rate(μ) increases, Availability of the System is increase, which should be. The study can be extended for two or more Unit system having Perfect and Imperfect Switch-Over devices. In future, Researchers can evaluated the parameters, when Repair rate and Failure rate are variable and also discuss the cost and profit benefit analysis. Further results can also be apply to find the Waiting Time of Units and Number of Server's visits. Any state can be taken as the Base-state to evaluate the various parameters.

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