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PSEUDO-THICK MODELING OF SELF- GRAVITATING DISCS AND THE FRAMING OF FLAT MASS PROPORTION PAIRS

Pseudo-Thick Modeling Of Self-Gravitating Discs and the Framing of Flat Mass Proportion Pairs

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Abstract – We exhibit expository models for the nearby structure of self-controlled self-gravitating accretion disc that are liable to practical cooling. This methodology might be utilized to expect the mainstream advancement of self-floating discs (which can functionally be contrasted and future radiation hydrodynamical re-enactments) and to characterize different physical administrations as a role of range what's more identical enduring state growth rate. We indicate that fragmentation is unavoidable, given sensible rates of infall into the disc, once the disc grows to radii >70 au (on account of a sun powered mass midway article). Owing to the outward redistribution of disc material by gravitational torques, we additionally anticipate fragmentation at >70 au even on account of flat rakish force centers which at first fall to a much more modest sweep. We indicate that 70 au is near the average double detachment and suggest that such postponed fragmentation, at the focus that the disc extends to >70 au, guarantees the production of flat mass proportion buddies that can keep away from significant further development and subsequent advancement towards unit mass ratio. We in this way suggest this as a guaranteeing component for handling flat mass proportion parallels, which, while plentiful observationally, are severely underproduced in hydrodynamical models.

Recent numerical simulations of self-gravitating protostellar disks have suggested that gravitational instabilities can lead to the production of substellar companions. In these simulations, the disk is typically assumed to be locally isothermal; i.e., the initial, axisymmetric temperature in the disk remains everywhere unchanged. Such an idealized condition implies extremely efficient cooling for outwardly moving parcels of gas. While we have seen disk disruption in our own locally isothermal simulations of a small, massive protostellar disk, no longlived companions formed as a result of the instabilities. Instead, thermal and tidal effects and the complex interactions of the disk material prevented permanent condensations from forming, despite the vigorous growth of spiral instabilities. In order to compare our results more directly with those of other authors, we here present three-dimensional evolutions of an older, larger, but less massive protostellar disk. We show that potentially longlived condensations form only for the extreme of local isothermality, and then only when severe restrictions are placed on the natural tendency of the protostellar disk to expand in response to gravitational instabilities.

INTRODUCTION

Since the early endeavors of Paczynski (1978) and Lin & Pringle (1987), there has been respectable investment in depicting the activity of self-gravity in gradual addition discs through adaption of the standard plan of gooey disc hypothesis. In this methodology, the net impact of self-gravitating modes in discs is to give a phenomenological consistency, in much the sameway that the movement of the magneto-rotational flimsiness (MRI) is regularly talked over as a source of a pseudo-thickness. To be sure, self-gravity is a prime competitor 'thickness component' in the early phases of development of protostellar discs, both since the MRI is unrealistic to be viable in thick locales of cool, pitifully ionized discs

(Gammie 1996) and additionally in light of the fact that it is difficult to maintain a strategic distance from the inference – in a situation where the star shapes from material channelled by the disc – that at promptly times the disc's self-gravity ought to be significant.

There are evident focal points of having the capacity to model the impact of disc self-gravity (or for sure, the MRI) utilizing a pseudo-gooey remedy, most strikingly the way that one can utilize numerous components of the mechanical assembly of 'α' accumulation disc speculation (Shakura & Sunyaev 1973) to register the structure and development of self-floating discs. Case in point, one may process

the structure of an unfaltering state disc (as a role of accumulation rate: see Rafikov 2009) or else, given

a disc's surface thickness and temperature circulation, can assess the disc's advancement utilizing the gooey dispersion comparison.

The investigation of this methodology has been empowered by some clear shallow likenesses between the activity of thick torques and those generated by self-gravitating features in the disc. Non-axisymmetric modes prepare torques that redistribute rakish force inside the disc and the work done by the aforementioned torques is disseminated in the disc, furnishing a net change of mechanical life into high temperature which allows the driving of an accumulation stream.

In the aforementioned regards, in this manner, the activity of self-gravity is comparable to that of thickness and it is alluring to portray it thusly. In this case, the main contrast between a self-gravitating disc and a traditional gooey disc is that the size of the gravitational pseudo-consistency is not known a priori (cf. Bertin 1997), however rather self-conforms so as to keep the disc in a state of negligible gravitational steadiness. This picture is again constant with an extent of reproductions of self-gravitating gas discs, which exhibit such self-regulation (e.g. Lodato & Rice 2004; Boley et al. 2006; Mayer et al. 2007; Stamatellos et al. 2007; Cossins, Lodato & Clarke 2009): the abundancy of winding headlines self-conforms in order to hotness the disc to a purpose of minor gravitational steadiness.

On the other hand, this expansive comparability between a percentage of the impacts of consistency, and of self-gravitating modes, is not in itself sufficient for one to receive a pseudo-thick depiction likewise, one likewise needs an increasingly exact prerequisite to be fulfilled, in particular that, as in the gooey scenario, the rate of work done on the rush mainly is completely specified by the neighborhood torque. As sharp out by Balbus & Papaloizou (1999), there is an elective scenario where the force extricated from the stream is not vitally scattered mainly however is rather transported in a spreading wave, to be scattered at some other spiral area. Though this does not modify the worldwide power offset (i.e. the rate of mechanical life lost by the growth stream is still equivalent to the aggregate rate of vigor scattering, incorporated over the disc), it evidently averts a basic association between nearby torques and the nearby thermodynamic state of the disc (see Lodato & Bertin 2001 for a talk of how such transport could influence the range created by self-gravitating discs).

Numerical re-enactments nonetheless moreover permit one to test the pseudoviscous speculation straight, by measuring both the torques in the disc and the nearby vigor scattering rate, then afterward analyzing this association with that needed on account of nearby scattering.

(Verifiably, this was first undertaken by Gammie 2001, granted that, as focused by Balbus & Papaloizou, the type of the border conditions in shearing box recreations guarantees area in any case.) All the more prominently, a mixed bag of worldwide re-enactments, both smoothed molecule hydrodynamics (SPH) and framework-based counts, indicate that the association between torques and life dispersal rate is without a doubt comparative to what would apply in the gooey case (Lodato & Rice

2004, 2005; Boley et al. 2006). This basically neighborhood conduct seems, by all accounts, to be set by the prerequisite that the stream speed of the gas into winding arms is imperceptibly supersonic, which counteracts waves from proliferating far from corotation (Cossins et al. 2009). Such a contention suggests that in increasingly great discs, which are more sweltering in a state of negligible gravitational strength, the higher sound speed might as well permit waves to proliferate further from corotation, and that non-neighborhood impacts are wanted to be essential. Reproductions validate this tendency, in that deviations from neighborhood conduct are more proclaimed for increasingly great discs (Lodato & Rice 2005; Cossins et al. 2009). Neighborhood vigor dispersal notwithstanding seems, by all accounts, to be a sufficient close estimation on account of discs with masses up to numerous tenths of the centermost article mass (Lodato & Rice 2005; Cossins et al. 2009).

Protostellar disk instabilities have gained renewed interest as a possible mechanism for the formation of substellar companions to stars. As formulated by Kuiper (1951) and later Cameron (1978), instabilities in the solar nebula could lead directly to the early and rapid (orbital timescale) formation of clumps of material that eventually evolve into Jovian planets. The mechanism has the virtue of a short timescale, which has been perhaps the most serious problem for the currently favored core-accretion model for the formation of Jupiter (e.g., Mizuno 1980; Pollack 1984; Pollack et al. 1996). While many researchers have shown that protostellar disks are susceptible to a variety of instabilities (see references in Pickett et al. 1998, hereafter PCDLI), the conditions under which they lead to companion formation have yet to be defined. Numerical simulations that suggest the possibility of instability-induced companion formation have been conducted under rather idealized assumptions about the thermal conditions of the disks, and so the ultimate fate of fragments that do form has not been reliably determined.

The past several years have witnessed remarkable progress in the study of brown dwarfs (Basri 2000). To date, more than 200 of such objects have been discovered. Most are found in isolation; only a small fraction orbit around stars, typically at separations of tens of AU and beyond. Despite the impressive pace of discovery, investigation into their origins has only just begun. The isolated brown dwarfs could be formed in small multiple systems through cloud

fragmentation like stars but ejected through interaction with other (heavier) members before gaining enough mass for hydrogen burning (Reipurth & Clarke 2001). The brown dwarf companions could in principle be produced in (rotationally supported) circumstellar disks through gravitational fragmentation, although it is unclear whether the conditions for fragmentation can arise naturally during star formation (Durisen 2001). In this Letter, we advance a new scenario for forming substellar companions, through the gravitational fragmentation of magnetically, rather than rotationally, supported disks. Such disks are expected to develop around protostars formed out of strongly magnetized clouds (Li & McKee 1996, hereafter LM96), as envisioned in the standard picture of isolated low-mass star formation (Shu, Adams, & Lizano 1987).

Conceptually, the formation of magnetically supported disks parallels that of the more familiar rotationally supported disks. Once gravity has initiated the dynamic collapse of a rotating magnetized dense core of a molecular cloud, both the magnetic flux and the angular momentum associated with rotation are trapped by the collapsing flow and carried to the vicinity of the protostar. However, both must eventually be stripped almost completely from the matter before it enters the star; otherwise, the stellar magnetic field strength and rotation rate would be many orders of magnitude higher than actually observed. These are, respectively, the magnetic flux and angular momentum problem of star formation. The stripping of angular momentum is thought to take place primarily in a rotationally supported disk by friction between adjacent annuli of matter moving at different speeds. The mechanism for magnetic flux stripping is less clear. LM96 proposed that it occurs as the field lines decoupled from the stellar matter expand against the collapsing inflow, reversing the inward motion of charged particles and the magnetic field tied to them. The bulk neutral material can still slip through, but only at a much reduced speed because of frequent collisions with the already stopped charged particles.

The slowdown causes the infalling matter to pile up and settle along field lines into a flattened structure—a magnetically supported circumstellar disk (see also Ciolek & Königl 1998 and Contopoulos, Ciolek, & Königl 1998).

THERMAL EQUILIBRIUM SOLUTIONS

Through our calculation of $v(\Sigma, R)$, we can assign to each pair of parameters (Σ and R) the accretion rate that would correspond to this solution if the disc was in a steady state. In the case of a disc where the torque vanishes at the origin, we have

$$\dot{M}_{ss} = 3\pi v \Sigma.$$

We emphasize that our local solutions do not require the disc to be in a steady state and that we use \dot{M}_{ss} only as a convenient way of parametrizing our solutions: in general, the actual accretion rate is related to \dot{M}_{ss} via

$$\dot{M} = \dot{M}_{ss} + 2R \frac{\partial \dot{M}_{ss}}{\partial R}.$$

We can now classify all points in the parameter space of radius versus steady state accretion rate and draw regime boundaries in this space (see the Appendix for analytic expressions for solutions in the various regimes and for the location of regime boundaries).

The regions between the bold lines represent the range of parameter space for which the disc is expected to be in the self-regulated, self-gravitating regime: in other words, angular momentum transport is dominated by gravitational torques, but the disc is not expected to fragment. For most of this region, the thermal input to the gas is also dominated by heating associated with gravitational modes. The exception to this is the lower right of this region (low accretion rates, large radius) where the gas is mainly heated by an assumed interstellar radiation field. Although the gas is assumed to be isothermal at 10K in this region, it is deemed not to fragment because the cooling time-scale is long enough that perturbations behave quasi-adiabatically.

The topology of the boundaries of this region is set both by the fragmentation boundary (right-hand boundary) and the interface with regions where the MRI dominates the angular momentum transfer (left-hand boundary). A notable feature of the former is the vertical boundary at ~ 70 au which coincides with regions of the disc where the opacity is dominated by ice grains. In this case $\kappa \propto T^2$, and therefore, for an optically thick disc in a state of marginal gravitational instability, the cooling time-scale is simply a function of radius (i.e. independent of accretion rate, provided one remains in the ice cooling regime). Consequently, the fragmentation boundary (which can be cast either in terms of a critical value of α or, equivalently, in terms of the ratio of cooling time-scale to dynamical time-scale) is encountered at a fixed radius, a feature first noted by Rafikov (2005) (see also Matzner & Levin 2005; Stamatellos et al.

2007). Outside this radius, therefore, a self-gravitating disc is always subject to fragmentation, regardless of the accretion rate.

SELF-GRAVITATING SUPPORTED DISKS

MAGNETICALLY

We think about the least difficult instance of an axisymmetric non-selfgravitating disk and determine a condition for the disk to come to be self-floating around a solitary, secluded star M_* ; conceivable nonaxisymmetric impacts will be remarked on in the direction of the finish of the segment. We evaluate the disk lands in a manner comparative to LM96 yet incorporate the attractive layering, which controls the plate thickness, and the attractive tension compel, which rules the attractive force inclination in the outspread power parity:

$$\frac{GM_*}{r^2} \approx \frac{B_z B_r}{2\pi\Sigma}.$$

We show below that the disk column density Σ is roughly independent of the cylindrical radius r . To counter the larger stellar gravity at a smaller radius, the field strength must increase inward, roughly as r^{-1} . For such a distribution, the radial and vertical field components are comparable on the disk (assuming a potential field outside as usual). They are

$$B_z \approx B_r \approx (2\pi G \Sigma M_*)^{1/2}/r.$$

The magnetic flux enclosed within a disk of radius R is

$$\Phi_d = \int_0^R 2\pi B_z r dr \approx 2\pi R (2\pi G \Sigma M_*)^{1/2}.$$

Since this flux is mostly that stripped from the stellar mass M_* (LM96), we have $\Phi_d \approx \Phi_* = \epsilon(2\pi G^{1/2})M_*$, where $\epsilon(<1)$ is the flux-to-mass ratio of the dynamically collapsing envelope that feeds the star-disk system in units of the critical value $2\pi G^{1/2}$. Eliminating Φ_d from equation (3), we obtain the following estimate of the disk mass:

$$M_d \approx \pi R^2 \Sigma \approx 0.5\epsilon^2 M_*.$$

which is simply the mass needed to “weigh down” the magnetic flux released from the star and prevent it from escaping. Even though the star-disk system remains magnetically supercritical, the disk (which contains a small fraction of the mass of the system but most of its flux) is subcritical, by a factor of $\sim 2/\epsilon$. Calculations of the evolution of strongly magnetized clouds driven by ambipolar diffusion suggest that ϵ is close to $\frac{1}{2}$ (e.g., Nakamura & Li 2002). This value would yield a disk mass of $\sim 10\%$ of the stellar mass, or some 10^2 Jupiter masses for solar mass stars. Therefore, there appears to be enough matter in the disk to form one brown dwarf near the hydrogen-burning limit, a few less massive brown dwarfs, or several massive planets.

IMPLICATIONS FOR BROWN DWARF COMPANIONS

If the substellar companions are formed from the fragmentation of magnetically supported disks, they should have initial masses between the opacity-limited Jeans mass, which is in the (upper) planetary mass range, and the disk mass, typically comparable to that of a massive brown dwarf. Further mass increase by accretion from the residual envelope is possible but probably not by much because of the rapid decline of the overall accretion rate after the Class 0 phase, as a result of, e.g., the powerful protostellar wind unbinding much of the envelope.

If substantial mass accretion does occur, the substellar objects may be turned into (very) low mass stellar companions. A prediction of our scenario is that the substellar companions should form at distances of a few hundred AU from the central star, set by the typical size of the self-gravitating disk. The orbits could shrink after formation for several reasons: First, the disk material out of which the objects form was mainly magnetically supported and thus sub-Keplerian. Indeed, the slowdown of radial infall in the disk, coupled with the presence of a strong magnetic field, allows magnetic braking to operate more efficiently, which could remove a large fraction of the disk angular momentum, similar to the case of magnetically subcritical clouds (Basu&Mouschovias 1994). Second, the substellar objects could transport some of their orbital angular momentum to the residual envelope by raising tides in it. In addition, since the disk contains more than one Jeans mass, more than one object could form from its fragmentation, with the ejection of all but one as a likely outcome of gravitational interactions. The final orbit of the remaining object could tighten by a large factor and tends to be highly eccentric (Papaloizou & Terquem 2001). Under extreme conditions, these interactions could in principle explain the smaller separations of Gliese 86B (~ 19 AU; Els et al. 2001) and HR 7672B (~ 14 AU; Liu et al. 2002). To send a brown dwarf even closer in, say, to the ~ 3 AU distance of HD 168443c (Marcy et al. 2001), would be even more difficult, consistent with the well-known dearth of brown dwarf companions within ~ 4 AU of solar-type FGK stars—the so-called brown dwarf desert. For the same reason, our scenario may produce (massive) extrasolar giant planets on relatively wide orbits of tens to hundreds of AU but unlikely those within a few AU of their host stars detected through radial velocity surveys. These close-in planets are presumably formed in rotationally supported disks, possibly through the conventional core-nucleation mechanism. Their orbital evolution may be affected, however, by the presence of substellar companions at larger distances.

FRAGMENTATION

In self-gravitating discs that are optically thick, with opacity provided by ice grains, the cooling time-scale is a function of radius only (Rafikov 2005) and is independent of temperature. This implies a particular radial location r_{frag} for disc fragmentation in this regime, independent of accretion rate. As has been noted by several previous authors (Matzner & Levin 2005; Rafikov 2005; Stamatellos et al. 2007) r_{frag} is around 70 au for solar mass stars; inward of r_{frag} , fragmentation can only be expected for high accretion rates ($>10^{-4} M_{\odot} \text{ yr}^{-1}$).

Obviously, therefore, one expects disc fragmentation in the case of cores whose specific angular momentum is high enough for significant infall beyond ~ 70 au. In terms of the factor β_J (the ratio of the rotational kinetic energy of the core to its break-up value) this implies $\beta_J > 0.01$ for solar mass cores collapsing from a Jeans scale at temperature ~ 10 K. However, our results furthermore imply that fragmentation at r_{frag} is inevitable for *any* plausible initial core rotation rate because of the possibility of outward transfer of angular momentum (and mass) even in the case where the maximum radius of infall (r_{inf}) is restricted to radii $\ll r_{\text{frag}}$. All that is required in this case is that there is enough angular momentum and mass in the infalling material for it to be self-gravitating at $>r_{\text{frag}}$, following radial redistribution by gravitational torques. Since the specific angular momentum in a Keplerian disc scales as $R^{1/2}$, then we require that if a disc spreads so that mass M_{frag} ends up at radius larger than r_{frag} , then we must have $M_{\text{disc}} r_{\text{inf}}^{1/2} > M_{\text{frag}} r_{\text{frag}}^{1/2}$. Given the typical parameters of self-gravitating discs at $r > r_{\text{frag}}$, we require that M_{frag} is at least around 10 per cent of the central object mass. Since the entire central object mass is initially in the disc, we then require that the disc initial infall radius is at least around $0.01 r_{\text{frag}}$. This radius is extremely small (less than 1 au) and would correspond to an implausibly low initial core rotation rate ($\beta_J < 10^{-4}$). We thus conclude that, because of the efficient outward angular momentum transfer in self-gravitating discs, disc fragmentation at $>r_{\text{frag}}$ is inevitable in just about any core with a realistic initial angular momentum content. This conclusion is not substantially changed if we relax the assumption that the MRI is effective only beyond 100 au. If instead we postulate that the disc is MRI active down to radius r_{frag} then we could in principle avoid fragmentation if the MRI took over as an angular momentum transfer mechanism once material reached r_{frag} .

Our numerical experiments, involving the integration of equation (10) for a variety of disc infall histories, indicate that this condition is never met unless the rate of infall is very low ($<10^{-7} M_{\odot} \text{ yr}^{-1}$; see discussion following equation 9). Collapse models for low mass cores (e.g. Vorobyov & Basu 2005, and references therein) however suggest rates that are an order of

magnitude higher than this and these higher values ($\sim 10^{-6} M_{\odot} \text{ yr}^{-1}$) are corroborated by modelling of line profiles in such cores (e.g. Tafalla et al. 2000).

The two important elements to emerge from this discussion are

(i) the thermodynamics of self gravitating discs only permits fragmentation at $r > r_{\text{frag}} \sim 70 M^{1/3}$ au and

(ii) (ii) virtually any core with non-negligible angular momentum will produce a disc that will expand, due to the action of gravitational torques, to radii $> r_{\text{frag}}$. Both these points are relevant to binary star formation and it is tempting to make the association between r_{frag} and the median binary separation (which is observed to be similar to this e.g. Duquennoy & Mayor 1991; Fischer & Marcy 1992).

CONCLUSION

Our derivation of analytic expressions for the properties of self-gravitating self-regulated discs has allowed us to derive useful diagrams illustrating different physical regimes as a function of steady state accretion rate and radius. These delineate the fragmentation boundary and illustrate that for all but the lowest mass stars, one expects non-fragmenting discs to be well described by the local (pseudo-viscous) approach employed here. These plots also demonstrate, as noted by several previous authors, that fragmentation is only expected at radius $> r_{\text{frag}} \sim 70 (M/M_{\odot})^{1/3}$ au provided that infall from the parent core is at less than $\sim 10^{-4} M_{\odot} \text{ yr}^{-1}$. Thus, it is only in high mass cores $> 10 M_{\odot}$, where such high infall rates are deduced (Cesaroni et al. 2007)] that one would expect fragmentation at smaller radius. On the other hand, fragmentation at $>r_{\text{frag}}$ is pretty much inevitable for any plausible initial core rotation rates:

the only scenario in which such fragmentation could be avoided is in the case both that the MRI extends in to r_{frag} and if the mass infall rate from the parent core is very low ($<10^{-7} M_{\odot} \text{ yr}^{-1}$).

We point out that our analytic expressions provide a useful framework for comparison with future hydrodynamic collapse calculations that incorporate the necessary cooling physics.

89744B and Gl 618.1B (Kirkpatrick et al. 2001; Wilson et al. 2001). Those ejected could account for, at least in part, the isolated brown dwarfs and possibly the free-floating planetary mass objects that

may have been uncovered in γ Orionis (Zapatero Osorio et al. 2000) and elsewhere.

An esthetically appealing feature of our scenario is that the brown dwarf companions are produced in essentially the same manner as the isolated low-mass stars envisioned in the standard picture, through the magnetic diffusion-driven fragmentation of a strongly magnetized, self-gravitating medium. Two common products of the fragmentation in the stellar case are circumstellar disks and binaries. By analogy, we expect disks and binaries to result from the fragmentation in the substellar case as well. A surrounding disk could explain the infrared excess observed in the young brown dwarf in the GG Tau quadruple system (White et al. 1999), and brown dwarf pairs have been detected (Basri 2000). Dynamical ejection from the 102 AU scale distance from a Sun-like low-mass star should leave intact the inner part of the disk (within a few AU of the brown dwarf) or tightly bound pairs. The former is consistent with the inference of disks around young isolated brown dwarfs (Muench et al. 2001) and the latter with the dearth of isolated brown dwarf binaries with separations greater than 10 AU (Reid et al. 2001).

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