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Chebyshev Inequality and Minimax Distribution Free Procedure in Mixed Inventory Model with Effective Investment to Reduce Lead Time and Setup Cost with Imprecise Demand

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Abstract – In this paper, Fuzzy Economic Order Quantity (FEOQ) model is studied to determine the optimal order quantity, discount backordered cost and lead-time. Whole of the study is performed in fuzzy environment. Even today, most of the researchers are ignoring this concept, just for the sake of simplicity of their models. But due to globalization and cut throat competition, it is the need of the hour to study the inventory model in fuzzy environment. So that they can accurately analyze inventory parameters and hence increase the goodwill of the organization in market. This paper is a part of my Ph.D. thesis and included in my thesis as Chapter-4

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4.1 INTRODUCTION:

Now-a-days managers of every organization recognize the fact that by managing risks associated in their business, they can successfully manage their inventories. One of the most common risks that have attracted significant attention of managers is supply uncertainty, especially the risk associated with direct material supplies and deliveries. This type of risk has escalated as fortune of 500 companies has sourced a great proportion of products from areas of the globe with low labor costs, such as China an India. Most of the companies recognize the significance of response time as a competitive weapon and have used time as means of differentiating themselves in the market place.

Lead-Time is the time difference between the placement of the order and receipt of it. In the competitive business environment, if a decision-maker can apply some means to reduce the lead-time, it would result in efficient service from the firms' point of view and it also improved the customer satisfaction. The lead-time reduction should be considered as a very significant variable in inventory control. Although, the lead-time can either be a constant or a variable, it was often treated as a prescribed parameter in most of the inventory model and consequently not controllable. Generally, it consists of the following components: order preparation/ processing time, order transit time to the supplier, supplier lead-time, items transit time from the supplier, and the time taken from the order receipt to its availability on the shelf. By optimizing these variables the lead-time can be reduced to its minimum level. This task can be achieved by inventing extra crashing cost. By shortening lead-time, the normal safety stock requirement can be reduced, and it is turn can minimize the out-of-stocks losses, improve the customer service level; and all these factors provide competitive advantages in business. Very few researchers have modified the traditional inventory models by incorporating reduction of lead-time by an extra crashing concept.

Generally, when the system is out of stock, cost and operations of inventory depends on what happens to demand, especially, for products with high sales value and/or high direct profitability, the cost of lost demand is high. The intention of a good modeler should always be to explore the possibility of improving the current system behaviors so as to minimize the total cost of maximize the total profit. Therefore, in addition to the traditional methods of using safety stock, certain possible ways that may prevent the loss caused by stock outs should be tried. There are many factors that affect the customer's willingness in accepting the backorder. It is well known fact that for some most desirable products or fashionable goods such as certain brand gum shoes and clothes, customers prefer to wait, in order to satisfy their demands. Another factor can be the price discount offered on the stock out items. It is observed that higher the price discount from the retailer, higher the advantages of the customer, and hence a large number of backorder ratio may result.

Table-4.1: Major Characteristics of Inventory Models by Selected Researcher:

References	Demand		Setup Cost		Solution Procedure		Shortage	Decision Variables
	Constant	Fuzzy	Constant	Reducible	Minimax distribution free procedure	Chebyshev inequality		
Liao and shyu (1991)	Stochastic Demand		*				With Shortage	Lead-Time
Ben-Daya and Raouf (1994)	*		*				Partial Backorder	Lead-Time, Order Quantity
Ouyang et al. (1996)	*		*		*		Mixture of Backorder	Lead-Time, Order Quantity
Ouyang and Wn. (1998)	*		*		*		Mixture of Backorder	Lead-Time, Order Quantity
Hariga and Ben-Daya (1999)	*		*		*		Mixture of Backorder	Lead-Time, Order Quantity, Recorder Point
Ouyang and Chang (2002)	*		*		*		Fuzzy lost Sale	Lead-Time, Order Quantity, Recorder Point
Ouyang et al. (2003)	*		*		*		Discounted Backorder	Length of a review period, Backorder price discount
Cheng et al. (2004)	*		*		*		Discounted Backorder	Order Quantity, Backorder discount, Lead time
Uthayakumar (2008)	*			*	*		Discounted Backorder	Order Quantity, Backorder Price discount
Lin (2008)	*		*		*		Discounted Backorder	Length of review period Backorder rate, Lead-time
Lin (2009)	*			*	*		Discounted Backorder	Order quantity, Ordering cost, Backorder price discount, lead-time, Number of shipments
Lo (2009)	*		*		*		Discounted Backorder	Lead-time, Order quantity, Back-order discount, Safety factor
Annadurai and Uthayakumar (2010)	*			*	*		Mixture of Backorder	Order quantity, Lead-time, Setup cost
In Present Chapter		*		*	*		Discounted Backorder	Order Quantity, Backorder discount, Lead-time

The above survey and survey performed in chapter-2 revealed that most of the inventory practitioners used Minimax-distribution free procedure to solve their problems. A Chebyshev approach for solving the problems has been present by El-Gindy et al. (1995). This technique is based on the expansion of the controllable variable in Chebyshev series with unknown coefficients. Very few inventory practitioners apply Chebyshev approach in inventory control system. In literature it was found that Maity et al. (2007) used that approach for solving as constraint optimization problem. Jaggi and Arneja (2011) developed a continuous review inventory model in crisp environment by assuming that demand distribution during lead-time is unknown and solved the model by two different approaches: Minimax-distribution free approach and Chebyshev inequality.

In the chapter, a mixed inventory model has been developed by applying the fuzzy set theory to deal with the impreciseness of demand rate. By doing so the proposed inventory model captures the inventory situation better. In fact, the application of fuzzy set concepts on EOQ inventory models have been proposed by many authors. However, their studies were almost concentrated on the simple EOQ models with restrictive assumptions. Most of them assumed lead-time as a constant and took shortages either completely backorder or totally lost. Because of that, those models have very few applications in inventory system. To increase the applicability of proposed model, a combination of backorder and lost sale with backorder price discount is considered. Crashing cost is also considered in this chapter to reduce the lead-time and setup cost. Cost expression is fuzzy due to the impreciseness in demand. Equivalent crisp expression is obtained by employing signed distance and centroid method. The backorder price discount, the lead-time and the order quantity are taken as decision variables (whereas the probability distribution of lead-time demand is unknown but its first two moments are known). Chebyshev inequality and Minimax-distribution free procedure are employed to minimize the annual cost.

4.2 ASSUMPTIONS AND NOTATIONS:

To develop in inventory model in this chapter, the following notations and assumptions are used.

4.2.1 Assumptions:

1. Time horizon is finite.

2. Shortages allowed and backlogged partially.

3. The reorder point r = expected demand during lead-time + safety stock, where safety stock = k* (standard deviation of lead-time) i.e., r = DL + k $\sigma\sqrt{L}$ where k is the safety factor.

4. The lead-time L_j consists of n mutually independents components. The j^{th} component has a minimum duration a_j and normal distribution b_j, and crashing cost per unit time c_j, c_j have arranged such that $c_1 < c_2 < .... < c_n$ for the sake of convenience, since it is clear that the reductions of lead-time should be first on component l, because it has the minimum unit crashing cost and then on component, 2, and so on.

5. Let $L_0 = \sum_{j=1}^{n} b_j$ and L_r be the length of lead – time of retailer with component 1,2,3, crashed to their minimum duration, then L_r can be expressed as $L_r \sum_{j=1}^{n} b_j - \sum_{j=1}^{r} (b_j - a_j)$.

The lead-time crashing cost per cycle is C(L) for a given L $\epsilon [L_r, L_{r-1}]$ and given by C(L)

$$= c_r(L_{r-1} - L) + \sum_{j=1}^{n} (b_j - a_j) \qquad \ldots.(4.1)$$

6. The setup cost K consists of m mutually independents components. The j^{th} component has normal cost e_j and minimum cost d_j, and crashing cost per unit f_j. f_j have arranged such $f_1 < f_2 < \ldots\ldots\ldots < f_m$ for the sake of convenience, since it is clear that the reduction of setup cost should be first on component 1, because it has the minimum unit crashing cost and then on component 2, and so on.

7. Let $K_0 = \sum_{j=1}^{m} e_j$ and K_j be the setup cost with component 1,2,3, ……… j crashed to their minimum cost, then K_j can be expressed as $K_j = \sum_{j=1}^{m} e_j - \sum_{i=1}^{j} (e_i - d_i), j = 1,2 \ldots, m$.

The crashing cost per cycle is C(K) for a given $K_j \epsilon [K_j, K_{r-1}]$ and given by C(K) $\sum_{i=1}^{j} f_i$

……..(4.2)

8. Assuming that a fraction $\beta \ (0 \le \beta < 1)$ of the demand during the stock out period can be backordered so the remaining fraction (1- β) is lost. The backorder ratio β is variable and is in proportion to the price discount π_x offered by the retailer per unit. Thus $\beta = \beta_0 \ (\pi_x / \pi_0)$ where $0 \le \pi_x \le \pi_0$

4.2.2 Notations:

x^+	:	Maximum value of x and 0, i.e., $x^+ = \max\{x, 0\}$
E(.)	:	Mathematical expectation
D	:	Average demand per year
Q	:	Order quantity (decision variable)
K	:	Setup cost per cycle
h	:	Inventory holding cost per unit item per year
π_0	:	Gross marginal profit per unit
π_1	:	Backorder price discount offered by the retailer per unit (decision value)
β	:	Backorder ratio
β_0	:	Upper bound of the backorder ratio
L	:	Length of lead-time(decision variable)
k	:	Safety factor
r	:	Reorder point
X	:	Lead-time demand, having probability density function $f_x(x)$ with finite mean DL and standard deviation σ
EAC	:	Expected annual cost for retailer
EAC^w	:	Least upper bound of expected annual cost

4.3 MATHEMATICAL MODEL FORMULATION:

In the model developed here, the inventory level of retailer is reviewed continuously. The inventory level of retailer declines due to customer demands only and when it is declined to a reorder point r, a lot size of Q is ordered. After lead-time L, order arrives at retailer's place. This process is continued in each cycle. During lead-time there are shortages. With backorder ratio β, the expected number of backorders per cycle is $\beta E(X-r)^+$, the expected demand lost per cycle is (1- β) $E(X-r)^+$. Therefore the annual stock out cost of retailer is $\frac{D}{Q}[\beta\pi_0 + (1-\beta)\pi_0]$ $E(X-r)^+$. The expected net inventory level just before the order arrives is r – DL+(1- β)$E(X-r)^+$ and the expected net inventory level at the beginning of the cycle is Q + r-DL+(1- β)$E(X-r)^+$ Therefore, the expected holding cost per year

$$=h[Q/2+r-DL+(1-\beta)E(X-r)^+]$$

The objective of the problem is to minimize the total expected annual cost of retailer, which is the sum of setup cost, holding cost, stock out cost, lead-time crashing cost, and setup crashing cost.

$$EAC(Q, \pi_x, L) = \frac{KD}{Q} + h\left[\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+\right] + \frac{D}{Q}[\beta\pi_x + (1-\beta)\pi_0]E(X-r)^+ +$$

$$\frac{D}{Q}[C(L) + C(K)]$$

or $EAC(Q, \pi_x, L) = \frac{KD}{Q} + h\left[Q/2 + r - DL + \left(1-\frac{\beta_0 \pi_x}{\pi_0}\right) E(X-r)^+\right] + \frac{D}{Q}\left[\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1-\frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right] E(X-r)^+ + \frac{D}{Q}[C(L) + C(K)]$ ....(4.3)

Here C(L), C(K) are calculated from equation (4.1) and equation (4.2) and $E(X\text{-}r)^+$ is given by

$$E(X-r)^+ = \int_r^\infty (X-r) f(x)\, dx$$

4.3.1 Fuzzy Model and Solution Procedure:

In the real situation, due to various uncertainties, the annual demand may have a fluctuation, especially, in a perfect competitive market. Therefore, it is difficult for the decision-maker to assess the annual demand by a crisp value D but easier to determine it by an interval $[D-\Delta_1, D+\Delta_2]$. Taking a value D* from $[D-\Delta_1, D+\Delta_2]$ then if D* = D, the error relative to D (demand in crisp case) is $|D^*-D| = 0$. Therefore in fuzzy sense, the confidence level is largest. On the other hand if $D^* \in [D-\Delta_1, D) \text{ or } D^* \in (D, D+\Delta_2]$ then the error is greater than 0 and the confidence level will be less than l. When $D^* = D-\Delta_1$ or $D^* = D+\Delta_2$ then the error will attain to the largest and the confidence level be smallest. Therefore, corresponding to the interval $[D-\Delta_1, D+\Delta_2]$, the following triangular fuzzy number is set

$$\tilde{D}\ (D-\Delta_1, D, D+\Delta_2)$$

The membership grade of $\tilde{D}$ is l at D, decreases continuously as the point deviates from D, and reaches to zero at $D-\Delta_1$ and $D+\Delta_2$. If the confidence level is treated as the membership grade, corresponding to the interval $[D-\Delta_1, D+\Delta_2]$, it is easy to set the above triangle fuzzy number $\tilde{D}$. Two different methods of defuzzification are employed: one is signed distance and other is centroid method.

By the signed distance method

$$D^* \equiv d(\tilde{D}, \tilde{0}) = D + \frac{1}{4}(\Delta_2 - \Delta_1) \quad ....(4.4)$$

By the centroid method

$$D^0 \equiv C(\tilde{D}) = D + \frac{1}{3}(\Delta_2 - \Delta_1) \quad ....(4.5)$$

Both D^* and D^0 are regarded as the values of annual demand in the fuzzy sense.

When annual demand of the retailer becomes fuzzy, then fuzzy annual cost of the retailer is

$$EA\tilde{C}(Q, \pi_x, L) = (\tilde{D} \Phi Q) \otimes \left[K \oplus \left\{\frac{\beta_0 \pi_x^2}{\pi_0} \oplus \left(1 \Theta \frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right\} E(X-r)^+ \oplus C(L) \oplus C(K)\right] \oplus h\left[Q/2 \oplus k\sigma\sqrt{L} \oplus \left(1 \Theta \frac{\beta_0 \pi_x}{\pi_0}\right) E(X-r)^+\right]$$

.....(4.6)

Where $\otimes$, Φ, Θ and $\oplus$ are the fuzzy multiplication, division, subtraction, and addition respectively between two fuzzy variable as well as fuzzy with another real variable by Function Principle.

Theorem 4.1: The values of total expected annual cost in the crisp sense are as follow:

(a). Using the signed distance method to defuzzify equation (4.6) results in

$$EACS(Q, \pi_x, L) = EAC(Q, \pi_x, L) + \frac{\Delta_2 - \Delta_1}{4Q}\left[K + \left\{\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1-\frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right\} E(X-r)^+ + C(L) + C(K)\right]$$

..........(4.7)

(b). Using the centroid method to defuzzify equation (4.6) results in

$$EACC(Q, \pi_x, L) = EAC(Q, \pi_x, L) + \frac{\Delta_2 - \Delta_1}{3Q}\left[K + \left\{\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1-\frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right\} E(X-r)^+ + C(L) + C(K)\right]$$

..........(4.8)

Theorem 4.2 : Total annual inventory cost obtained by signed distance is less than centroid method if $\Delta_1 < \Delta_2$ and if $\Delta_1 > \Delta_2$ than total annual cost obtained by centroid method is lesser.

Proof : $EACC(Q, \pi_x, L)$ -$EACS(Q, \pi_x, L)$

$$= \frac{(\Delta_2 - \Delta_1)}{12Q}\left[K + \left\{\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right\} E(X-r)^+ + C(L) + C(K)\right]$$

If $\Delta_1 < \Delta_2$ then $EACC(Q, \pi_x, L) > EACS(Q, \pi_x, L)$

If $\Delta_1 > \Delta_2$ then $EACC(Q, \pi_x, L) < EACS(Q, \pi_x, L)$

Remark- 4.1: If $\Delta_1 < \Delta_2$ then $EACC(Q, \pi_x, L) = EACS(Q, \pi_x, L) = EAC(Q, \pi_x, L)$ i.e., the crisp case is the special case of fuzzy case.

If the demand is more irregular or fluctuating then it is not necessary (as stated in Tersine, 1982) that demand during lead-time to follow normal distribution always. So the case of unknown distribution of demand during lead-time has to discuss. During that case, obviously, shortages will occur more frequently as the nature of demand is more varying. In the lead-time, demand having unknown distribution, solution can be find by two different approaches. On the basis of this, two cases are raised as:

1. Solution by using Minimax-distribution free procedure.

2. Solution by using Chebyshev inequality.

Case-1. When the lead-time demand has unknown distribution then the minimax distribution procedure can be used. Then, the upper bound to the expected shortages during lead-time can be given as:

For any distribution F

$$E(X-r)^+ \leq \frac{1}{2}\left\{\sqrt{\sigma^2 L + (r - DL)^2} - (r - DL)\right\}$$

or $$E(X-r)^+ \leq \frac{\sigma\sqrt{L}}{2}\left\{\sqrt{1 + k^2} - k\right\}$$

Hence, equation (4.7) and equation (4.8) can be reduced as follows to minimize it.

From equation (4.7)

$$EAC_{S_M^W}(Q, \pi_x, L) = \frac{KD}{Q} + h\left[\frac{Q}{2} + \sigma k\sqrt{L}\left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\left(\frac{\sigma\sqrt{L}}{2}\left(\sqrt{1+k^2} - k\right)\right)\right]$$

$$+ \frac{D}{Q}[C(L) + C(K)] + \frac{D}{Q}\left[\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right]\left(\frac{\sigma\sqrt{L}}{2}\left(\sqrt{1+k^2} - k\right)\right)$$

$$+ \frac{(\Delta_2 - \Delta_1)}{12Q}\left[K + \left\{\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right\}\left(\frac{\sigma\sqrt{L}}{2}\left(\sqrt{1+k^2} - k\right)\right) + C(L) + C(K)\right]$$

........(4.9)

From equation (4.8)

$$EAC_{S_M^W}(Q, \pi_x, L) = \frac{KD}{Q} + h\left[Q/2 + \sigma k\sqrt{L}\left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\left(\frac{\sigma\sqrt{L}}{2}\left(\sqrt{1+k^2} - k\right)\right)\right]$$

$$+ \frac{D}{Q}[C(L) + C(K)] + \frac{D}{Q}\left[\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right]\left(\frac{\sigma\sqrt{L}}{2}\left(\sqrt{1+k^2} - k\right)\right)$$

$$+ \frac{(\Delta_2 - \Delta_1)}{3Q}\left[K + \left\{\frac{\beta_0 \pi_x^2}{\pi_0} + \left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\pi_0\right\}\left(\frac{\sigma\sqrt{L}}{2}\left(\sqrt{1+k^2} - k\right)\right) + C(L) + C(K)\right]$$

........(4.10)

Where $EAC_{S_M^W}(Q, \pi_x, L)$ and $EAC_{C_M^W}(Q, \pi_x, L)$ are the least upper bound of $EAC_{S_M^W}(Q, \pi_x, L)$ and $EAC_{C_M^W}(Q, \pi_x, L)$ respectively.

Case-2. When the probability distribution of X is unknown and has only first two moments then it is difficult to get the exact value of $E(Eg-r)^+$ i.e., expected shortages during the lead-time. By using Chebyshev inequality, the real fluctuation in shortages during the lead-time is captured by Providing the better bound. Moreover, the inequality is valid for all distributions for which the standard deviation exists.

Since, it is known that expected stock out quantity during the lead-time is

$$E(X-r)^{+} = \int_{r}^{\infty}(X-r)f(x)\,dx$$

Using Chebyshev inequality

$$E(X-r)^{+} = \int_{\mu+k\sigma}^{\infty}[X-(D-k\sigma)]f(x)\,dx \le \frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)$$

Furthermore, the above inequality holds for any distribution.

Hence, equation (4.7) and equation (4.8) can be reduced as follows to minimize it.

From equation (4.7)

$$EAC^{S_C^W}(Q,\pi_x,L) = \frac{KD}{Q} + h\left[Q/2 + \sigma k\sqrt{L}\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)\left(\frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)\right)\right] + \frac{D}{Q}\left[\frac{\beta_0\,\pi_x^2}{\pi_0}+\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)\pi_0\right]$$

$$\left(\frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)\right)\frac{D}{Q}\,[C(L)+C(K)]$$

$$+\frac{(\Delta_2-\Delta_1)}{4Q}\left[K+\left\{\frac{\beta_0\,\pi_x^2}{\pi_0}+\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)\pi_0\right\}\left(\frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)\right)C(L)+C(K)\right] \quad \ldots\ldots(4.11)$$

From equation (4.8)

$$EAC^{S_M^W}(Q,\pi_x,L) = \frac{KD}{Q} + h\left[Q/2 + \sigma k\sqrt{L}\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)\left(\frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)\right)\right] + \frac{D}{Q}\left[\frac{\beta_0\,\pi_x^2}{\pi_0}+\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)\pi_0\right]$$

$$\left(\frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)\right)\frac{D}{Q}\,[C(L)+C(K)]$$

$$+\frac{(\Delta_2-\Delta_1)}{3Q}\left[K+\left\{\frac{\beta_0\,\pi_x^2}{\pi_0}+\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)\pi_0\right\}\left(\frac{\sigma\sqrt{L}}{2}\left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right)\right)C(L)+C(K)\right] \quad \ldots\ldots(4.12)$$

Where $EAC^{S_C^W}(Q,\pi_x,L)$ and $EAC^{C_C^W}(Q,\pi_x,L)$ and the least upper bound of EACS (Q,π_x,L) and EACS (Q,π_x,L) respectively.

$$\varphi_m(k) = \begin{cases} \sqrt{1+k^2}-k & m=1 \\ \left(\frac{1}{k^2}+\frac{1}{(k+1)^2}+\frac{1}{(k+2)^2}\right) & m=2 \end{cases}$$

and

$$E_1 = K + \frac{\pi_0\,\sigma\sqrt{L}}{2}\varphi_m(k) + C(L) + C(K)$$

$$E_2 = Kk\sigma\sqrt{L} + \frac{h\sigma\sqrt{L}}{2}\varphi_m(k)$$

We define

$$TC_{ml} = \frac{E_1}{Q}\left(D+\frac{\Delta_2-\Delta_1}{l+3}\right)\frac{\pi_x^2}{Q} - \frac{h\beta_0\pi_0\sigma\sqrt{L\varphi_m}(k)}{2\pi_0}\pi_x + \frac{\beta_0\sigma\sqrt{L\varphi_m}(k)}{2\pi_0}$$

$$\left(D+\frac{\Delta_2-\Delta_1}{l+3}\right)\frac{\pi_x^2}{Q} - \frac{\beta_0\sigma\sqrt{L\varphi_m}(k)}{2\pi_0}\left(D+\frac{\Delta_2-\Delta_1}{l-3}\right)\frac{\pi_1}{Q} + E_2 \quad \ldots\ldots(4.13)$$

Where m= 1,2 and l = 0, 1

Where

TC_{10} = Total expected inventory cost for retailer when defuzzification is done by using centroid method and Minimax-distribution free procedure is used for solution. (expression is given by equation 4.10))

TC_{11} = Total expected inventory cost for retailer when defuzzification is done by using signed distance method and Minimax-distribution free procedure is used for solution. (expression is given by equation 4.9))

TC_{20} = Total expected inventory cost for retailer when defuzzification is done by using centroid method and Chebyshev inequality is used for solution. (expression is given by equation 4.12))

TC_{21} = Total expected inventory cost for retailer when defuzzification is done by using signed distance method and Chebyshev inequality is used for solution. (expression is given by equation 4.11))

Theorem 4.3: For fixed (Q, π_x), TC_{ml} is concave in L $\in [L_i, L_{i-1}]$.

Proof:

$$\frac{dTC_{ml}}{dL} = \frac{\frac{\pi_0 \sigma L^{-1/2}}{4}\varphi_m(k) - c_r}{Q}\left(D + \frac{\Delta_2-\Delta_1}{l+3}\right) - \frac{h\beta_0 \sigma L^{-1/2}\varphi_m(k)}{4\pi_0}\pi_x + \frac{\beta_0 \sigma L^{-1/2}\varphi_m(k)}{4\pi_0}$$

$$\left(D + \frac{\Delta_2-\Delta_1}{l+3}\right)\frac{\pi_x^2}{Q} - \frac{\beta_0 \sigma L^{-1/2}\varphi_m(k)}{4}\left(D + \frac{\Delta_2-\Delta_1}{l+3}\right)\frac{\pi_x^2}{Q}$$

$$\frac{hk\sigma L^{-1/2}}{4} + \frac{h\sigma L^{-1/2}}{4}\varphi_m(k)$$

$$\frac{d^2TC_{ml}}{dL^2} = -\left\{\left(\left(\frac{\beta_0 \pi_x^2}{\pi_0} + \pi_0\left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\right)\left(D + \frac{\Delta_2-\Delta_1}{l+3}\right) - \frac{\sigma\varphi_m(k)L^{-3/2}}{8Q}\right.\right.$$

$$\left. + \frac{h\sigma\varphi_m(k)L^{-3/2}}{8}\left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right) + \frac{h\sigma L^{-1/2}}{4}\right\} < 0 \quad \text{Since } \frac{\beta_0 \pi_x}{\pi_0} < 1$$

Theorem 4.4: For a given value of L $\in [L_i, L_{i-1}]$, TC_{ml} is convex in (Q, π_x).

Proof: First, the Hessian matrix H is obtained as follows:

$$H = \begin{bmatrix} \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial Q^2} & \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial Q \partial \pi_x} \\ \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial \pi_x \partial Q} & \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial \pi_x^2} \end{bmatrix}$$

then proceed to evaluate the principle minors of H.

Now, the first principle minor of H is $|H_{11}|$ >0, since

$$\frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial Q^2} = \left[\frac{2}{Q^3}(K + C(L) + C(K)) + \left(\frac{\beta_0 \pi_x^2}{\pi_0} + \pi_0\left(1 - \frac{\beta_0 \pi_x}{\pi_0}\right)\right)\frac{\sigma\varphi_m(k)L^{-3/2}}{Q^3}\right]$$

$$\left(D + \frac{\Delta_2-\Delta_1}{l+3}\right) > 0 \; since \; \frac{\beta_0 \pi_x}{\pi_0} < 1$$

In order to evaluate second principle minor, the following partial derivatives are required.

$$\frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial \pi_x^2} = \frac{\beta_0 \sigma L^{-1/2}\varphi_m(k)}{Q\pi_0}\left(D + \frac{\Delta_2-\Delta_1}{l+3}\right)$$

$$\frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial \pi_x \partial Q} = \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial Q \partial \pi_x} = \frac{\beta_0 \sigma L^{-1/2}\varphi_m(k)}{Q^2}\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)\left(\frac{1}{2} - \frac{\pi_x}{\pi_0}\right)$$

The second principal minor of H is

$$|H_{22}| = \begin{vmatrix} \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial Q^2} & \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial Q \partial \pi_x} \\ \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial \pi_x \partial Q} & \frac{\partial^2 TC_{ml}(Q,\pi_x,L)}{\partial \pi_x^2} \end{vmatrix}$$

$$= \left\{\frac{2\sigma\sqrt{L}\varphi_m(k)\beta_0}{Q^4\pi_0}[K + C(L) + C(K)] + \frac{\sigma^2 L\beta_0\varphi_m^2(k)}{Q^4}\left(1 - \frac{\beta_0}{4}\right)\right\}$$

$$\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)^2 > 0$$

Since $0 \leq \beta_0 \leq 1$

Therefore, it is clear that $TC_{ml}(Q, \pi_x, L)$ is convex function in (Q, π_x)

For fixed L $\in [L_i, L_{i-1}]$, the minimum value of TC_{ml} will occur at the point (Q, π_x) which satisfies $\frac{\partial TC_{ml}(Q,\pi_x,L)}{\partial Q} = 0$ and $\frac{\partial TC_{ml}(Q,\pi_x,L)}{\partial \pi_x} = 0$

On solving above two system of equation

$$\left.\begin{aligned} Q^*_{ml} &= \sqrt{\frac{2\left[E_1 + \frac{\beta_0\sigma\sqrt{L}}{2}\left(\frac{\pi_x^2}{\pi_0} - \pi_x\right)\varphi m(k)\right]\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)}{h}} \\ \pi^*_{mlx} &= \frac{hQ}{2\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)} + \frac{\pi_0}{2} \end{aligned}\right\}$$

.........(4.14)

On putting the value of π_x in the expression of Q and after simplifying

$$\left.\begin{aligned} Q^*_{ml} &= \sqrt{\frac{2\left[E_1 + \frac{\beta_0\sigma\sqrt{L}\,\varphi m(k)}{8}\right]\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)}{h\left(1 - \frac{\beta_0\sigma\sqrt{L}\,\varphi m(k)}{4\pi_0\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)}\right)}} \\ \pi^*_{mlx} &= \frac{hQ}{2\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)} + \frac{\pi_0}{2} \end{aligned}\right\}$$

.........(4.15)

Remark-4.2: Taking m=1, $\Delta_2 = \Delta_1$, C(K) = 0, equation (4.15) reduces to

$$\left.\begin{aligned} Q^*_{ml} &= \sqrt{\frac{2D\left[K + \frac{\pi_0(4-\beta_0)\sigma\sqrt{L}\left(\sqrt{1+k^2}\right)\varphi m(k)}{8} + C(L)\right]}{h\left(1 - \frac{h\beta_0\sigma\sqrt{L}\sqrt{1+k^2}\,k}{4\pi_0}\right)}} \\ \pi^*_x &= \frac{hQ}{2D} + \frac{\pi_0}{2} \end{aligned}\right\}$$

This is same as equation (12) and equation (13) of Lin (2008). So the present model development here is more generalized then Lin (2008).

Now, the value of Q^*, π^*_x can be obtained from the equation (4.15) for the known values of the parameters of the inventory. For each L_j (*j= 0, 1, 2,....... n*), the value of corresponding total expected annual cost can be obtained.

Algorithm is provided to found the optimal ordered quality, lead-time and the backorder discount for given value of safety factor k.

Algorithm:

Step-1. For given value of safety factor k and for each L_j (*j= 0, 1, 2,...... n*), perform step (2)-(4).

Step-2. For given k, find the value of Q from the equation (4.15).

Step-3. Substitute Q into equation (4.15) and computer the relative π_x. Comparing π^*_x with π_0

a. if $\pi_x \leq \pi_0$, then solution is feasible, go to step 4.

b. Otherwise set $\pi_x = \pi_0$ and move to step 4.

Step-4. Computer the corresponding total expected cost from the equation (4.13). Go to step 5.

Step-5. Find $\min_{i=0,1,2,\dots,n} TC_{ml}(Q, \pi_x, L)$ and move to step

Step-6. Say $TC_{ml}(Q^*, \pi^*_x, L^*) = \min_{i=0,1,2,\dots,n} TC_{ml}(Q, \pi_x, L)$

value for Q, π_x, and L.

4.3.2 Analysis on the Basis of Theoretical Results:

1. Notice that if β_0 =1, equation (4.15) reduces to

$$(Q^*_{ml})_{\beta_0=1} = \sqrt{\frac{2\left[E_1 + \frac{\sigma\sqrt{L}}{2}\left(\pi_x - \frac{\pi_x^2}{\pi_0}\right)\varphi m(k)\right]\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)}{h}}$$

When β_0 =0, equation (4.15) reduces to

$$(Q^*_{ml})_{\beta_0=0} = \sqrt{\frac{2E_1\left(D + \frac{(\Delta_2-\Delta_1)}{l+3}\right)}{h}}$$

Hence, for fixed L and π_x, on comparing above two expressions, one gets $(Q^*_{ml})_{\beta_0=0} > (Q^*_{ml})_{\beta_0=1}$ i.e., the order quantity per cycle increases as β_0 decreases. So, order quantity in inversely proportional to upper bound of discount ratio. This result is also varified by Table 4.4.

2. The effect of β_0 on the minimum total expected annual cost be examined. It has minimum value when β_0 = 1 and maximum value when β_0 = 0. Hence for 0 < β_0 <1

$$(TC_{ml})^*_{\beta_0=1} < (TC_{ml})^*_{\beta_0} < (TC_{ml})^*_{\beta_0=0}$$

This result can also be seen through Table 4.4

3. For $\sigma > 0$ and k sufficiently large, we get $\varphi_m(k) \to 0, \Delta_2 = \Delta_1$, and there is no setup crashing cost then equation (4.13) reduces to the result of Ben-Daya and Raouf (1994).

4.4 NUMERICAL EXAMPLE:

In the model, the retailer tries to reduce the lead-time and setup cost by an extra investment with suitable price discount to the customers to convert some portion of lost sale into backorder case. The proposed model has been illustrated with help of following data which is taken is taken from the literature:

D=600 units per year, K = \$200 per order, σ = 7 units per week, π_0 = \$150 per unit, h = \$20 per unit per year. The problem has been solved for different upper bounds of the back order ratio β_0= 0.2, 0.5, 0.8 and 0.95. The lead-time has three components with the respective crashing cost that has been shown in Tale 4.2

Table – 4.2: Lead-time Data:

Lead-time component j	Normal duration (days) b_j	Minimum duration (days) a_j	Crashing cost c_j per days
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

The Setup cost has three components with the respective crashing cost that has been shown in Table 4.3.

Table – 3: Setup Cost Data:

By using the given algorithm, the optimal solution have been obtained by varying the value of backorder ratio and corresponding optimal values are tabulated in Table 4.4

Observations Based on Table 4.4:

With the use of suggested algorithm, the optimal solution has been obtained by varying the value of lead-time and the best possible outputs are tabulated in Table 4.4. Following observations are made from Table 4.4.

1. From the Table 4.4, it is interesting to observe that for fixed value of Δ_1 and Δ_2. If the value of upper bound of the backorder ratio β_0 increases, then the expected annual total cost, the order quantity, and the backorder price discount decreases. This shows that retailer can capture the backorders by increasing the price discount. It observe that if the value of upper bound of the backorder ratio β_0 increases, it results in an increase in the cost saving.

2. If $\Delta_1 < \Delta_2$ i.e., the tolerance level of right is more than left, then the total expected annual cost in the fuzzy sense obtained by using the signed distance is less than that of centroid. It this situation, it is better for the decision-maker to adopt signed distance method for defuzzification to obtain the optimal solution. These numerical results verify Theorem 4.2.

Table 4-4: Optimal Solution in Different Scenarios:

Setup cost component j	Normal cost (days) e_j	Minimum cost (days) d_j	Crashing cost f_j
1	80	20	56
2	80	20	168
3	40	10	350

Given Parameters			By Using Chebyshev Inequality									By Using Minimax distribution free Procedure								
			Centroid Method				Signed Distance method				Best	Centroid Method				Signed Distance method				Best
β_0	Δ_1	Δ_2	L*	Q*	π_x^*	TC_{20}	L*	Q*	π_x^*	TC_{21}		L*	Q*	π_x^*	TC_{10}	L*	Q*	π_x^*	TC_{10}	
0.20	25	50	6	114.61	76.88	2294.2	6	114.42	76.88	2290.2	SD	6	113.28	76.86	2266.8	6	113.08	76.86	2262.9	SD
	35	150	6	117.41	76.83	2294.0	6	116.52	76.85	2332.3	SD	6	116.04	76.81	2322.0	6	115.16	76.83	2304.5	SD
	50	25	6	113.03	76.91	2262.5	6	113.23	76.90	2266.5	CM	6	111.72	76.88	2235.5	6	111.91	76.88	2239.4	CM
	150	35	6	110.13	76.96	2204.5	6	111.07	76.94	2223.2	CM	6	108.85	76.93	2178.1	6	111.07	76.94	2223.1	CM
0.50	25	50	6	114.47	76.88	2291.2	6	114.28	76.88	2287.2	SD	6	113.24	76.86	2266.0	6	113.05	76.86	2262.1	SD
	35	150	6	117.26	76.83	2326.3	6	116.38	76.85	2320.3	SD	6	116.00	76.81	2321.2	6	115.13	76.83	2303.7	SD
	50	25	6	112.89	76.90	2259.6	6	113.09	76.90	2263.6	CM	6	111.68	76.88	2234.8	6	111.88	76.88	2238.7	CM
	150	35	6	110.00	76.95	2201.6	6	110.93	76.94	2220.3	CM	6	108.81	76.93	2177.4	6	109.74	76.92	2195.9	CM
0.80	25	50	6	114.33	76.87	2288.2	6	114.14	76.88	2284.3	SD	6	113.20	76.86	2265.2	6	113.01	76.86	2261.3	SD
	35	150	6	117.12	76.83	2343.9	6	116.23	76.84	2326.3	SD	6	115.96	76.81	2320.4	6	115.09	76.83	2302.9	SD
	50	25	6	112.75	76.90	2256.7	6	112.95	76.90	2260.6	CM	6	111.64	76.88	2234.0	6	111.84	76.88	2237.9	CM
	150	35	6	109.84	76.95	2198.3	6	110.79	76.93	2217.4	CM	6	108.78	76.93	2176.6	6	109.70	76.92	2195.1	CM
0.95	25	50	6	114.26	76.87	2286.7	6	114.07	76.88	2282.8	SD	6	113.19	76.86	2264.8	6	112.99	76.86	2261.0	SD
	35	150	6	117.04	76.83	2342.4	6	116.16	76.84	2324.7	SD	6	115.94	76.81	2320.0	6	115.07	76.83	2302.5	SD
	50	25	6	112.69	76.90	2255.2	6	112.88	76.90	2259.2	CM	6	111.62	76.88	2333.6	6	111.82	76.88	2237.5	CM
	150	35	6	109.79	76.79	2197.3	6	110.72	76.93	2202.8	CM	6	108.76	76.93	2176.3	6	109.68	76.92	2180.9	CM

Table-4.5: Comparison of the Minimum Total Cost per Unit with Backorder Discount:

i	j	L_i	$\sum_{j=1}^{i} C_j$	$\sum_{j=1}^{i} f_j$	Cheng et al. (2004) Model			In our developed model ($\Delta_1 = 25, \Delta_2 = 50$)											
								$\beta_0 = 0.2$			$\beta_0 = 0.5$			$\beta_0 = 0.8$			$\beta_0 = 0.95$		
					Q	ETC	Per Unit Cost	Q	ETC	Per Unit Cost	Q	ETC	Per Unit Cost	Q	ETC	Per Unit Cost	Q	ETC	Per Unit Cost
0	0	8	0	0	167	3702.90	22.173	110.58	2212.9	20.011	110.55	2212.1	20.009	110.51	2211.3	20.009	110.49	2210.9	20.009
	1	8	0	56	156	3459.63	22.177	110.63	2213.7	20.009	110.59	2213.1	20.011	110.55	2212.2	20.010	110.53	2211.8	20.010
	2	8	0	224	144	3710.67	25.768	110.75	2216.3	20.011	110.72	2215.5	20.009	110.68	2214.7	20.009	110.66	2214.3	20.009
	3	8	0	574	138	4562.82	33.063	111.01	2221.5	20.011	110.98	2220.7	20.009	110.94	2219.9	20.009	110.92	2219.5	20.009
1	0	6	5.6	0	162	3545.98	21.888	110.55	2212.2	20.010	110.52	2211.5	20.009	110.48	2210.7	20.009	110.47	2210.4	20.009
	1	6	5.6	56	151	3348.64	22.176	110.60	2213.0	20.009	110.56	2212.3	20.009	110.53	2211.6	20.009	110.51	2211.2	20.009
	2	6	5.6	224	138	3608.19	26.146	110.72	2215.6	20.010	110.69	2214.8	20.009	110.65	2214.1	20.009	110.63	2213.7	20.009
	3	6	5.6	574	132	4481.46	33.950	110.98	2220.8	20.010	110.95	2220.1	20.009	110.01	2219.4	20.174	110.93	2219.0	20.003
2	0	4	22.4	0	157	3393.5	21.614	110.49	2210.6	20.007	110.46	2210.1	20.008	110.43	2209.5	20.008	110.41	2209.2	20.009
	1	4	22.4	56	145	3236.45	22.320	110.53	2211.5	20.008	110.50	2210.9	20.008	110.47	2210.3	20.008	110.46	2210.0	20.007
	2	4	22.4	224	132	3505.21	26.554	110.65	2214.0	20.009	110.63	2213.4	20.007	110.60	2212.8	20.007	110.58	2212.5	20.008
	3	4	22.4	574	125	4400.65	35.205	110.92	2219.3	20.008	110.89	2218.7	20.008	110.86	2218.1	20.008	110.86	2217.8	20.005
3	0	3	57.4	0	159	3396.44	21.361	110.46	2210.1	20.008	110.44	2209.6	20.007	110.41	2209.1	20.008	110.4	2208.8	20.007
	1	3	57.4	56	147	3277.66	22.297	110.51	2210.9	20.006	110.48	2210.4	20.007	110.46	2209.9	20.006	110.44	2209.7	20.008
	2	3	57.4	224	135	3542.96	26.244	110.68	2213.5	19.999	110.61	2212.9	20.006	110.58	2212.4	20.007	110.57	2212.2	20.007
	3	3	57.4	574	128	4430.16	34.610	110.89	2218.7	20.008	110.87	2218.2	20.007	110.85	2217.7	20.006	110.83	2217.4	20.007

4. If $\Delta_1 < \Delta_2$ i.e., the tolerance level of right is more than left, then the total expected annual cost in the fuzzy sense obtained by using the centroid method is less than that of signed distance method. It this situation, it is better for the decision-maker to adopt centroid method for defuzzification to obtain the optimal solution. These numerical results verify Theorem 4.2

OBSERVATION BASED ON TABLE 4.5 AND TABLE 4.6:

The effect of price discounts for backorders by the retailer to the customer on the total expected cost is also analyzed. As Cheng et al. (2004) did not consider backorder price discount, the model developed here is being compared to the model of Cheng et al. (2004)

1. By offering the price discounts a retailer can fetch a large number of backorder.

2. In the model developed here, inventory cost per unit item is less than that of **Cheng** et al. **(2004)**. This is due to providing price discounts of backorder which increases the backorders and finally decreases the total inventory cost which is associated to per unit of item.

3. Table 4.5 shows that the reduction of lead-time accompanies a decrease of ordering quantity.

Table–4.6: Summary of Cost Saving per Unit:

Model	β_0	Minimum Cost per unit	Cost Saving per unit
Cheng et al. (2004) Model		$22.32	
Our Proposed model	0.20	$19.999	$2.321
	0.50	$520.006	$2.314
	0.80	$20.006	$2.314
	0.95	$20.003	$2.317

4.5 SUMMARY AND CONCLUDING REMARKS:

In this chapter, Fuzzy Economic Order Quantity (FEOQ) model is studied to determine the optimal order quantity, discount backordered cost and lead-time. Whole of the study is performed in fuzzy environment. Even today, most of the researchers are ignoring this concept, just for the sake of simplicity of their models. But due to globalization and cut throat competition, it is the need of the hour to study the inventory model in fuzzy environment. So that they can accurately analyze inventory parameters and hence increase the goodwill of the organization in market.

Most of the researchers assumed that Lead-time was zero. Needless to say, this is an unrealistic assumption. Lead-time is controllable which can be controlled by using extra crashing cost. It is considered that lead-time and setup cost are not constant but it can be reduced by an extra crashing cost. Assumption about the form of probability distribution of lead-time demand is relaxed and used the Minimax-distribution free procedure and Chebyshev inequality to solve the problem. The convex nature of expected annual cost functions helps to determine the optimal values of decision variables.

To summarize the discussion, it is observed that by providing suitable discount to customers, the retailer would be able to convert lost sale to backorder case and reduce his expected loss. So it can conclude that backorder ratio depends on backorder discount. It is also observed from Table 5 of Lin (2008) at pp. 124 that as the value of μ decreases from 0 to -1.0, inventory cost per unit quantity increases from 23.51

to 26.41 which is much more higher than developed model as it is about 20 (from Table 4.6). This is due to consideration of crashing cost to reduce setup cost. It is observed that in proposed model there is a significant cost reduction per unit items as compared to the Cheng et al. (2004). The reason behind this is that they did not apply one of the nature phenomena such as backorder price discount to motivate their customers to wait until his/her demand as not fulfilled. So, it can be concluded that decision-makers focused his/her decisions with the capital investment in reducing setup cost and offering backorder price discount to customers, to lower the system cost, and to obtain a significant amount of savings increase the competitive edge in business. The proposed model is solved with the application of Chebyshev inequality, well known for its simplicity and generality. Ben-daya and Raouf (1994) model s deduced as special case, i.e., the developed model is better and has more practical applications in comparison to Ben-Days and Raouf's model. Condition is explored to find the situation, which method of defuzzification (signed distance or centroid method) is better in order to obtain the minimum annual cost theoretically and that result is also verified through numerical example.

In future research on this problem , it would be interesting to analyze the non-linear relationship that exists between crashing cost, lead-time and setup cost and considering impreciseness in different cost.

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