Journal of Advances in Science and Technology

Vol. IV, No. VII, November-2012, ISSN 2230-9659

A RESEARCH OF STAR PRODUCTS AND ALSO QUANTUM TIME MAPS

A Research of Star Products and Also Quantum Time Maps

Rajesh

Research Scholar, Monad University, Hapur (U.P.)

Abstract – We describe a characteristic class of star products: those which are given by an arrangement of bidifferential drivers which at request k in the twisting parameter have at generally k subsidiaries in every contention. We indicate that any such star item on a symplectic complex describes an one of a kind symplectic association. We parametrise such star products, study their invariance and give indispensible and sufficient conditions for them to yield a quantum minute map.

We demonstrate that Kravchenko's sufficient condition for a minute guide for a Fedosov star item is likewise fundamental.

---------------------------♦-----------------------------

INTRODUCTION

The connection between a star product on a symplectic complex and a symplectic association on that complex shows up in numerous connections. Specifically, when one studies lands of invariance of star products, outcomes are much simpler when there is an invariant association. We demonstrate here that there is a characteristic class of star products which characterize a novel symplectic association. We consider the invariance of such products and the conditions for them to have a minute guide.

ASSOCIATIONS

The connection between the thought of star product on a symplectic complex and symplectic associations recently shows up in the fundamental paper of Bayen, Flato, Fronsdal, Lichnerowicz furthermore Sternheimer, and was further improved by Lichnerowicz who demonstrated that any purported Vey star product (i.e. a star product described by bidifferential drivers whose vital images at every request harmonize with those of the Moyal star product) confirms a novel symplectic association. Fedosov gave a development of Vey star products starting from a symplectic association and an arrangement of shut two structures on the complex. It was demonstrated that any star product is proportionate to a Fedosov star product.

By the by, numerous star products which show up in regular settings (cotangent bunches, Kaehler manifolds. . .) are not Vey star products (however are regular in the sense described above). The point of this area is to generalise the consequence of Lichnerowicz and to show that on any symplectic complex, a regular star product confirms a novel symplectic association.

Given any torsion free linear association ∇ on (M, P), the term of order 1 of a natural star product can be written

$$C_1 - \{\,,\,\} - \partial E_1 - \{\,,\,\} + (\mathrm{ad}\, E_1)\, m$$

where $E_1 \in \mathscr{D}_2^0$

and the term of order 2 can be written in a chart

$$\begin{aligned} C_2(u,v) \;-\; & \tfrac{1}{2}((\mathrm{ad}\,E_1)^2 m)(u,v) + ((\mathrm{ad}\,E_1)\,\{\,,\,\})(u,v) \\ & + \tfrac{1}{2} P^{ij} P^{i'j'}\, \nabla^2_{ii'} u\, \nabla^2_{jj'} v \\ & + \tfrac{1}{6}(P^{rk}\nabla_r P^{jl} + P^{rl}\nabla_r P^{jk})(\nabla^2_{kl} u\, \nabla_j v + \nabla_j u\, \nabla^2_{kl} v) \\ & - \partial E_2(u,v) \mid c_2(u,v) \end{aligned}$$

where $E_2 \in \mathscr{D}_3^0$ and where $c_2 \in \mathscr{D}_1^1$ is skewsymmetric.

Remark that E is not uniquely defined; two choices differ by an element $X \in \mathscr{D}_1^0$.

Observe that the first lines in the definition of C_2 for two such different choices only differ by an element in $\mathscr{D}_1^{\mathbf{1}}$. Indeed

$$\begin{aligned} \left(-\tfrac{1}{2} P^{rk} P^{sl} S^j_{rs} \mid \tfrac{1}{3}(P^{rk} S^j_{rs} P^{sl} \mid P^{rk} S^l_{rs} P^{js})\right)(\nabla^2_{kl} u\, \nabla_j v \mid \nabla_j u\, \nabla^2_{kl} v) \;=\; \\ - \bigoplus_{jkl} \tfrac{1}{6} P^{rk} P^{sl} S^j_{rs} (\nabla^2_{kl} u\, \nabla_j v + \nabla_j u\, \nabla^2_{kl} v) \end{aligned}$$

$$\tfrac{1}{2}\left(\operatorname{ad}(E_1 + X)^2 m\right)(u,v) + ((\operatorname{ad} E_1 + X)\{\,,\,\})(u,v) = \tfrac{1}{2}((\operatorname{ad} E_1)^2 m)(u,v) + ((\operatorname{ad} E_1)\{\,,\,\})(u,v) + \tfrac{1}{2}((\operatorname{ad} X)\circ(\operatorname{ad} E_1)\, m)(u,v) + ((\operatorname{ad} X)\{\,,\,\})(u,v)$$

and (adEi) m, (adX) { , } are in $\mathscr{D}^1_1$, so also is ((adX) o (ad Ei) m).

Changing the torsion free linear connection gives a modification of the terms of the second line of C_2; writing $\nabla' = \nabla + S$, this modification involves terms of order 2 in one argument and 1 in the other given by as well as terms of order 1 in each argument, where $\circlearrowleft$ denotes a cyclic sum over the indicated variables.

Notice that the terms above coincide with the terms of the same order in the cobound- ary of the operator $E' = \frac{1}{6} \bigoplus_{jkl} P^{rk} P^{sl} S^j_{rs} \nabla^3_{jkl}$.

If the Poisson tensor is invertible (i.e. in the symplectic situation), the symbol of any differential operator of order 3 can be written in this form E', hence we have:

Recommendation - A star product $* = \sum_{r\geq 0} \nu^r C_r$ on a symplectic complex (M, ω) , so that Ci is a bidifferential driver of request 1 in every contention and C2 of request at generally 2 in every contention, confirms an extraordinary symplectic association ∇ such that

$$C_1 = \{\,,\,\} = \partial E_1 \qquad C_2 = \tfrac{1}{2}(\operatorname{ad} E_1)^2 m + ((\operatorname{ad} E_1)\{\,,\,\}) + \tfrac{1}{2}P^2(\nabla^2 u, \nabla^2 v) + A_2$$

where $A_2 \in \mathscr{D}^1_1$ *and* $P^2(\nabla^2 u, \nabla^2 v)$ *denotes the bidifferential operator which is given by* $P^{ij} P^{i'j'} \nabla^2_{ii'} u \nabla^2_{jj'} v$ *in a chart.*

Comments - This shows, in particular, that any natural star product $* = \sum_{r\geq 0} \nu^r C_r$ on a symplectic manifold (M, ω) determines a unique symplectic connection ∇.

QUANTUM TIME MAPS

A derivation $D \in \operatorname{Der}(M, *)$ is said to be essentially inner or Hamiltonian if *D* $= \frac{1}{\nu}$ad* *u* for some $u \in C^\infty(M)[\![\nu]\!]$. We denote by Inn(M, *) the essentially inner deriv

ations of *. It is a linear subspace of Der(M, *) and is the quantum analogue of the Hamiltonian vector fields.

By analogy with the classical case, we call an action of a Lie group almost *- Hamiltonian if each D_ξ is essentially inner, and call a linear choice of functions u_ξ satisfying $D_\xi = \frac{1}{\nu} \operatorname{ad}_* u_\xi, \quad \xi \in \mathfrak{g}$ a (quantum) Hamiltonian. We say the action is *-Hamiltonian if can be chosen to make $\xi \mapsto u_\xi : \mathfrak{g} \to C^\infty(M)[\![\nu]\!]$ a homomorphism of Lie algebras. When $D_\xi = \tilde{\xi}$, this map is called a quantum time map.

Considering the map $a : C^\infty(M)[\![\nu]\!] \to \operatorname{Der}(M, *)$ given by

$$a(u)(v) = \tfrac{1}{\nu} \operatorname{ad}_* u(v) = \tfrac{1}{\nu}(u * v - v * u).$$

and defining a bracket on $C^\infty(M)[\![\nu]\!]$ by

$$[u, v]_* = \tfrac{1}{\nu}(u * v - v * u)$$

then, by associativity of the star product, *a* is a homomorphism of Lie algebras whose image is Inn(M, *). Since $D \circ a(u) - a(u) \circ D = a(Du)$, Inn(M, *) is an ideal in Der(M, *) and so there is an induced Lie bracket on the quotient Der(M, *)/Inn(M, *).

Lemma - *If * is a star product on a symplectic manifold* (M, ω) *then the space of derivations modulo inner derivations,* Der(M, *)/Inn(M, *), *can be identified with* $H^1(M, \mathbb{R})[\![\nu]\!]$ *and the induced bracket is zero.*

proof - The first part is well known; let us recall that locally any derivation $D \in$ Der(M, *) is inner, and that the ambiguity in the choice of a corresponding function u is locally constant so that the exact 1-forms *du* agree on overlaps and yield a globally defined (formal) closed 1-form a_D. The map $\operatorname{Der}(M, *) \to Z^1(M)[\![\nu]\!]$ defined by $D \mapsto \alpha_D$ if $D|_U = \frac{1}{\nu} \operatorname{ad}_* u$ and $\alpha_D|_U = du$ is a linear isomorphism with the space of formal series of closed 1-forms and maps essentially inner derivations to exact 1-forms inducing a bijection $\operatorname{Der}(M, *)/\operatorname{Inn}(M, *) \to Z^1(M)[\![\nu]\!]/d(C^\infty(M)[\![\nu]\!]) \quad H$.

Let D_i and D_2 be two derivations of $*$. $(D_1 \circ D_2 - D_2 \circ D_1)|_U = a([u_1, u_2]_*)$. But $[u_i, u_2]_*$ does not change if we add a local constant to either function, so is the restriction to U of a globally defined function which depends only on D_i and D_2.

We denote this function by *b(D$_i$, D$_2$)* and have the identity

$$D_1 \circ D_2 - D_2 \circ D_1 - \frac{1}{\nu} \mathrm{ad}_* b(D_1, D_2).$$

This shows that $[\mathrm{Der}(M,*), \mathrm{Der}(M,*)] \subset \mathrm{Inn}(M,*)$ and hence that the induced bracket on $H^1(M,\mathbb{R})[\nu]$ is zero. □

The kernel of *a* consists of the locally constant formal functions $H^0(M,\mathbb{R})[\nu]$ and hence:

Remark - If * is a differential star product on a symplectic manifold (M,ω) then

there is an exact sequence of Lie algebras

$$0 \to H^0(M,\mathbb{R})[\nu] \hookrightarrow C^\infty(M)[\nu] \xrightarrow{a} \mathrm{Der}(M,*) \xrightarrow{c} H^1(M,\mathbb{R})[\nu] \to 0$$

where $c(D) - [\alpha_D]$.

Result -Let G be a Lie assembly of symmetries of a star product * on (M,ω) and $d\sigma\colon \mathfrak{g} \to \mathrm{Der}(M,*)$ the prompted minuscule movement. Assuming that $H^1(M,\mathbb{R}) - 0$ or $[\mathfrak{g},\mathfrak{g}] \subset \mathfrak{g}$ at that point the activity is essentially *-Hamiltonian.

Surely, by definition, the movement is practically *-Hamiltonian if $d\sigma(\mathfrak{g}) \subset \mathrm{Inn}(M,*)$. This is the situation under either of the two conditions.

TIME MAPS FOR AN INVARIANT STAR PRODUCT WITH AN INVARIANT ASSOCIATION

Let (M,ω) be endowed with a differential star product *,

$$u * v = uv + \sum_{r\geq 1} \nu^r C_r(u,v).$$

Recognize a polynomial math 0 of vector fields on M comprising of inferences of * and accept that there is a symplectic companionship ∇ which is invariant under $\mathfrak{g}$ (i.e. $\mathscr{L}_X \nabla \quad 0$, $\forall X \in \mathfrak{g}$).

This is obviously immediately correct if the star product is regular and invariant. It was demonstrated in that * is identical, through an equivariant proportionality

$$T - \mathrm{Id} + \sum_{r\geq 1} \nu^r T_r$$

(i.e. $\mathscr{L}_X T - 0$), to a Fedosov star product constructed from ∇ and an arrangement of invariant shut 2-structures Ω which give an agent of the trademark class of *. Watch that

$$X(u) - \frac{1}{\nu}(\mathrm{ad}_* \mu_X)(u)$$

for any $X \in \mathfrak{g}$ if and only if

$$X(u) = T \circ X \circ T^{-1}(u) = T(\frac{1}{\nu}(\mathrm{ad}_* T\mu_X)(T^{-1}u)) = \frac{1}{\nu}(\mathrm{a}$$

Henceforth the Lie variable based math $\mathfrak{g}$ comprises of inward inductions for * if and just if this is correct for the Fedosov star product $*_{\nabla,\Omega}$ and this correct if and just if there exists an arrangement of capacities λ_X such that $i(X)\omega - i(X)\Omega - d\lambda_X$.

Thus

$$X(u) - \frac{1}{\nu}(\mathrm{ad}_* \mu_X)(u) \quad \text{with} \quad \mu_X - T\lambda_X.$$

Specifically, this yields

Hypothesis - Let G be a reduced Lie gathering of symplectomorphisms of (M,ω) and $\mathfrak{g}$ the comparing Lie variable based math of symplectic vector fields on M. Recognize a star product

* on M which is invariant under G. The Lie algebra $\mathfrak{g}$ comprises of inward inferences for

* if and just if there exists an arrangement of functions λ_X and a representative $\frac{1}{\nu}(\omega - \Omega)$ of the trademark class of * such that

$$i(X)\omega - i(X)\Omega = d\lambda_X.$$

REFERENCES

• D. Arnal, J.-C. Cortet, *-products in the method of orbits for nilpotent groups. *J. Geom. Phys.* 2 (1985) 83-116.

- F. Bayen, M. Flato, C. Fronsdal, A. Lichnerowicz and D. Sternheimer, Deformation theory and quantization, *Ann. Phys.* 111 (1978) 61-110.

- M. Bertelson, P. Bieliavsky and S. Gutt. Parametrizing equivalence classes of invariant star products, *Lett. Math. Phys.* 46 (1998) 339-345.

- M. Bertelson, M. Cahen and S. Gutt, Equivalence of star products. *Class. Quan. Grav.* 14 (1997) A93-A107.

- M. Bordemann, N. Neumaier, S. Waldmann, Homogeneous Fedosov star products on cotangent bundles. I. Weyl and standard ordering with differential operator representation. *Comm. Math. Phys.* 198 (1998) 363-396.

- M. Cahen, S. Gutt, Regular * representations of Lie algebras. *Lett. Math. Phys.* 6 (1982) 395-404.

- M. De Wilde, P. B. A. Lecomte, Existence of star-products and of formal deformations of the Poisson Lie algebra of arbitrary sympletic manifolds. *Lett. Math. Phys.* 7 (1983) 487-496.

- B.V. Fedosov, A simple geometrical construction of deformation quantization, *J. Diff. Geom.* 40 (1994) 213-238.

- B.V. Fedosov, *Deformation quantization and index theory.* Mathematical Topics Vol. 9, Akademie Verlag, Berlin, 1996

- M. Gerstenhaber, On the deformation of rings and algebras. *Ann. Math.* 79 (1964) 59-103.

- K. Hamachi, *Differentiability of quantum moment maps,* math.QA/0210044.

- Karabegov, Cohomological classification of deformation quantisations with separation of variables. *Lett. Math. Phys.* 43 (1998) 347-357.

- M. Kontsevich, *Deformation quantization of Poisson manifolds, I.* IHES preprint q-alg/9709040, 1997.

- O. Kravchenko, Deformation quantization of symplectic fibrations. *Compositio Math.,* 123 (2000) 131-165.