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A CONTEMPLATE ON BITOPOLOGICAL SPACES IN PAIR WISE PROPERTIES

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A Contemplate On Bitopological Spaces in Pair Wise Properties

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Abstract – In this paper, we study of bitopological division axioms begun by Kelly. The paper contains about the topological standpoint on graph theory and relaxing the compatibility requirement, Edge Spaces and Separation Axioms

Keywords: Bitopological, Space, Pairs, Topologies, Compact, Cover, Regular, Graph

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INTRODUCTION

In 1963, J.C. Kelly introduced the notion of bitopological spaces. Such spaces equipped with its two (arbitrary) topologies. Furthermore, Kelly was extended some of the standard results of separation axioms in a topological space to a bitopological space. Such extensions are pair wise regular, pair wise Hausdorff and pair wise normal. There are several works dedicated to the investigation of bitopologies, i.e., pairs of topologies on the same set; most of them deal with the theory itself but very few with applications.

Definition-

Definition 1. A bitopological space (X,P,Q) is said to be p-compact if the topological space (X,P) and (X,Q) are both compact. Equivalently, (X, P, Q) is p-compact if every P-open cover of X can be reduced to a finite P-open cover and every Q-open cover of X can be reduced to a finite Q-open cover.

T. Birsan (1969) has given definitions of pair wise compactness which do allow Tychonoff product theorems. According to Birsan, a bitopological space (X, P, Q) is said to be pairwise compact(denote p1-compact) if every P-open cover of X can be reduced to a finite Q-open cover and every Q-open cover of X can be reduced to a finite P-open cover. A bitopological space (X,P,Q) has a particular topological property, without referring specifically to P or Q, and we shall then mean that both (X,P) and (X,Q) have the property.

Bitopological separation axioms

Definition 2 (Kelly). In a space (X, P, Q), P is said to be regular with respect to Q if, for each point x ∈ X, there is a P-neighborhood base of Q-closed sets, or, as is easily seen to be equivalent, if, for each point x ∈ X and each P-closed set P such that x /∈ P , there are a P-open set U and a Q-open set V such that x ∈ U, P ⊆ V, and U ∩ V =∅.(X,P,Q) is, or P and Q are, pair wise regular if P is regular with respect to Q and vice versa.

Theorem 1. In a space (X,P,Q), P is regular with respect to Q if and only if for each point x ∈ X and P-open set H containing x, there exists a P-open set U such that x ∈ U ⊆ Q- cl(U) ⊆ H.

Proof- Suppose P is regular with respect to Q. Let x ∈ X and H is a P-open set containing x. Then G = X−H is a P-closed set which x /∈ G. Since P is regular with respect to Q, then there are P-open set U and Q-open set V such that x ∈ U, G ⊆ V , and U ∩ V =∅. Since U ⊆ X −V , then Q-cl(U) ⊆ Q-cl(X −V) = X − V ⊆ X −G = H. Thus, x ∈ U ⊆ Q-cl(U) ⊆ H as desired.(⇐) Suppose the condition holds. Let x ∈ X and P is a P-closed set such that x /∈ P . Then x ∈ X − P , and by hypothesis there exists a P-open set U such that x ∈ U ⊆ Q-cl(U) ⊆ X−P . It follows that x ∈ U, P ⊆ X−Q-cl(U) and U ∩ (X −Q- cl(U))=∅. This completes the proof.

Definition 3 (Kelly). A bitopological space (X,P,Q) is said to be p-normal if, given a P-closed set A and a Q-closed set B with A ∩ B =∅, there exist a Q-open set U and a P-open set V such that A ⊆ U, B ⊆ V , and U ∩ V =∅.Equivalently, (X,P,Q) is p-normal if, given a Q-closed set C and a P-open set D such that C ⊆ D, there are a P-open set G and Q-closed set F such that C ⊆ G ⊆ F ⊆ D. We shall prove the equivalent definition above in the following theorem.

Theorem 2. A space (X,P,Q) is p-normal if and only if given a Q-closed set C and a P-open set D such that

C ⊆ D, there are a P-open set G and a Q-closed set F such that C ⊆ G ⊆ F ⊆ D.

Proof-Suppose (X, P, Q) is p-normal. Let C be a Q-closed set and D a P-open set such that C ⊆ D. Then K = X−D is a P-closed set with K ∩C =∅. Since (X, P, Q) is p-normal, there exists a Q-open set U and a P-open set V such that K ⊆ U, C ⊆ G, and U ∩G=∅. Hence G ⊆ X − U ⊆ X − K = D. Thus C ⊆ G ⊆ X − U ⊆ D and the result follows by taking X − U = F. (⇐) Suppose the condition holds. Let A be a P-closed set and B a Q-closed set with A∩B =∅. Then D = X−A is a P-open set with B ⊆ D. By hypothesis, there are a P-open set G and a Q-closed set F such that B ⊆ G ⊆ F ⊆ D. It follows that A = X −D ⊆ X −F, B ⊆ G and (X −F) ∩G=∅ where X −F is Q-open set and G is P-open set. Hence proved.

Topological perspective on graph theory

One of the most basic and important building blocks of graph theory is the notion of “connectedness”. The same word also has a very important meaning in the field of general topology; indeed, arguably the latter subject grew precisely out of the efforts of several mathematicians to give the right formalization for concepts like “continuity”, “convergence”, “dimension” and, not least, connectedness. Although formally the two concepts are very different, one depending on finite paths and the other on open sets, the intuition behind the two versions of connectedness is essentially the same, and few will dispute that any link between graph theory and topology should at least reconcile them, if not be entirely dictated by this objective. In fact the usual way of modeling a graph as a topological object does achieve this, albeit in a way which, we feel, is not entirely satisfactory.

Traditionally, a graph is modeled as a one-dimensional cell-complex2, with open arms for edges and points for vertices, the neighbourhoods of a “vertex” being the sets containing the vertex itself and a union of corresponding “tails” of every “edge” (arc) incident with the vertex. If the graph is planar, this is equivalent to taking the subspace topology inherited from the Euclidean plane by an appropriate “drawing” of the graph. If the graph is finite, one can always place the vertices in three-dimensional Euclidean space, and join up pairs of adjacent vertices by pairwise disjoint open arcs (whose accumulation points are the two adjacent vertices) so that the union of the arcs together with the set of vertices inherits the topology of a cell-complex with the above restriction. Also, in the finite case, this concept coincides with that of a graph in continuum theory

Relaxing the compatibility requirement

A class of topological space retains some of the properties of the classical topology of a simple graph.

Definition 1:

A topologized graph is a topological space X such that

- every singleton is open or closed;
- ∀x ∈ X, |∂(x)| ≤ 2.

Note that, in any S1 space, the set E of points which are not closed is open, and therefore its complement, V, is closed. Thus the closure of any subset A of E, in particular any singleton, is of the form E ∪ B for some B ⊆ V, and ∂ (A) = B.

Therefore a “topologized graph” has an underlying combinatorial structure, as well as a topological one.

Definition 2:

A topological space S = (X, T) is compatible with a hyper graph H if X = VH ∪ EH and, for all e ∈ EH, we have that {e} is open and ∂(e) = fH(e); S is strictly compatible if every vertex is also a closed point. We also say that a topology Ton VH ∪EH is (strictly) compatible if (VH ∪ EH, T) is a compatible topological space.

Definition 3: A hyper edge of a topological space is a point which is open but not closed. A hyper edge of a topological space is an edge if its boundary consists of at most two points. An edge of a topological space is a loop if it has precisely one boundary point, a proper edge otherwise. A point in a topological space is classical if the intersection of all its neighborhoods is open. Although a topologized graph can only be compatible with a unique graph, a given graph may be compatible with different topologized graphs.

Edge Spaces and Separation Axioms

A graph consists of vertices and edges. We have mentioned separation axioms along the lines of the standard Hausdorff, regular and normal separation axioms of standard topology that we can meaningfully apply to our spaces. An edge as an open singleton whose boundary consists of the incident vertices, as we do in the case of topologized graphs, in particular the classical topology, and as we shall do in the case of edge spaces.

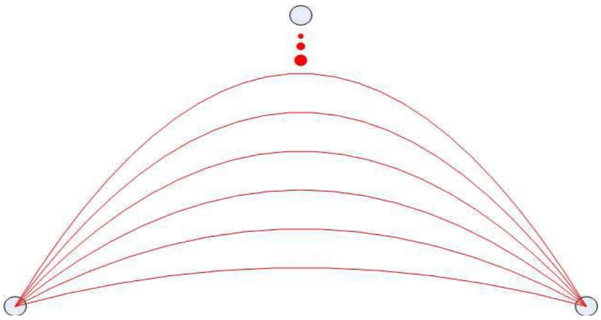
Separation Axioms

Some pathological examples

We begin with two simple examples of two topologized graphs compatible with the sameinfinite graph.

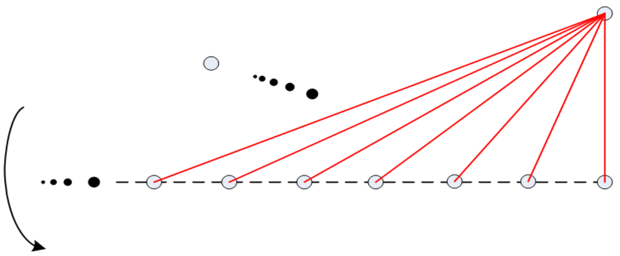
Example 1 (Infinite bond plus regular vertex):

Consider a graph with countable infinitely many edges incident with the same two vertices, and a third vertex of degree zero. The “infinite bond plus regular vertex" is the topologized graph obtained by endowing this graph with the classical topology.



The infinite bond plus irregular vertex

Example 2 (The overcrowded fan):



The space we construct in this example is illustrated in Let M,N, P be pairwise-disjoint sets such that M and N have the cardinality of the continuum, and P has the cardinality of the integers (we could take M,N, P to be subsets of R, but this would obscure the construction). Consider the simple complete bipartite graph with vertex set P ∪M, with every vertex in P adjacent to every vertex in M. Let G be the topologized graph obtained when this graph is endowed with the classical topology. The overcrowded fan

CONCLUSION:

Bitopological spaces equipped with arbitrary topologies. In this paper we study of bitopological separation axioms begun by Kelly. The paper contains about the topological perspective on graph theory. A graph consists of vertices and edges. We have mentioned separation axioms along the lines of the standard Hausdorff, regular and normal separation axioms of standard topology that we can meaningfully apply to our spaces. An edge is an open singleton whose boundary consists of the incident vertices.

REFERENCES:

[1] J.C. Kelly, Bitopological spaces, Proc. London Math. Soc. (3) 13 (1963) 71–89.

[2] J.L. Kelley, General Topology, Springer, New York, 1955.

[3] C.W. Patty, Bitopological spaces, Duke Math. J. 34 (1967) 387–391.

[4] W.J. Pervin, Foundations of General Topology, Academic Press, London, 1964.

[5] M.J. Saegrove, Pairwise complete regularity and compactification in bitopological spaces, J. London Math. Soc. (2) 7 (1973) 286–290.

[6] S. Willard, General Topology, Addison-Wesley, Canada, 1970.

[7] Incidentally, prominent among these were Kuratowski, Menger and Whitney!

[8] A. T. Lundell and S. Weingram. The Topology of CW-Complexes. Van Nostrand Reinhold, New York 1969.