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"Interated Aluthge Transforms: A Review"

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Abstract – The Aluthge transform (defined below) of an operator Ton Hilbert space has been studied extensively, most often in connection with p-hyponormal operators. In the present paper authors initiated a study of various relations between an arbitrary operator T and its associated, and this study was continued in, in which relations between the spectral pictures of T and were obtained.

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INTRODUCTION

Let H be a separable, infinite dimensional, complex Hilbert space, and let L(H) denote the algebra of all bounded linear operators on H. An arbitrary operator T in L(H) has a unique polar decomposition T = U|T|, where |T| = (T ∗T) and U is the appropriate partial isometry (with ker U = ker T and ker U ∗ = ker T ∗). An operator T ∈ L(H) is said to be p-hyponormal if (T ∗T) p − (T T ∗) p ≥ 0, p ∈ (0, ∞) ([6]). If p = 1, T is hyponormal and if p = 1 2 , T is semi-hyponormal ([8]). The L˝owner-Heinz inequality ([7]) implies that p-hyponormal operators are qhyponormal operators for q ≤ p. In particular, T is said to be ∞-hyponormal if T is p-hyponormal for every p > 0 ([9]). It is well known that every qusinormal operators are ∞-hyponormal. Associated with T there is a very useful related operator Ter,t = |T| tU|T| r−t for r ≥ t ≥ 0, called the generalized Aluthge transform of T ([16]). This transform Ter,t is said to be (r, t)-Aluthge transform. Then (1, 1 2)- Aluthge transform is refered as the Aluthge transform which is denoted by Te ([10]). The (1, 1)-Aluthge transform is refered as Duggal transform of T, which is denoted by Tb (i.e., Tb := |T|U) ([11]). In many cases Aluthge and Duggal transforms are useful, and ones concentrate to discuss here their transforms. An operator T is (r, t)-weakly hyponormal if |Ter,t| ≥ |T| ≥ |Te∗ r,t| ([12]). The (1, 1 2)-weakly hyponormal operator is refered as w-hyponormal operator ([13], [14]).

Given an *r* × *r* complex matrix *T*, if $T = U|T|$ is the polar decomposition of*T*, then the Aluthge transform is defined by

$$\Delta(T) = |T|^{1/2}U|T|^{1/2}.$$

Let $\Delta^n(T)$ denote the n-times iterated Aluthge transform of *T*, i.e. $\Delta^0(T) = T$ and $\Delta^n(T) = \Delta(\Delta^{n-1}(T))$, $n \in \mathbb{N}$. In this paper we make a brief survey on the known properties and applications of the Aluthge trasnsorm, particularly the recent proof of the fact that the sequence $\{\Delta^n(T)\}_{n\in\mathbb{N}}$ converges for every *r* × *r* matrix *T*. This result was conjectured by Jung, Ko and Pearcy in 2003.

REVIEW OF LITERATURE:

Let $\mathcal{H}$ be a Hilbert space and T a bounded operator defined on $\mathcal{H}$ whose (left) polar decomposition is $T = U|T|$. The *Aluthge transform* of T is the operator defined by

$$\Delta(T) = |T|^{1/2}U\,|T|^{1/2}.$$

This transform was introduced in [1] by Aluthge, in order to study p-hyponormal and log-hyponormal operators. Roughly speaking, the idea behind the Aluthge transform is to convert an operator into another operator which shares with the first one some spectral properties but it is closer to being a normal operator.

The Aluthge transform has received much attention in recent years. One reason is its connection with the invariant subspace problem. Jung, Ko and Pearcy proved in [2] that T has a nontrivial invariant subspace if an only if $\Delta(T)$ does. On the other hand, Dykema and Schultz proved in [3] that the Brown measure is preserved by the Aluthge transform. Another reason is related with the iterated Aluthge transform. Let $\Delta^0(T) = T$ and $\Delta^n(T) = \Delta(\Delta^{n-1}(T))$ for every $n \in \mathbb{N}$. In [4] Jung, Ko and Peacy raised the following conjecture:

Theorem 1 (Aluthge [1]). Let $T \in L(\mathcal{H})$ be p-hyponormal. Then

- If $p \geq \frac{1}{2}$, then $\Delta(T)$ is hn,
- If $p < \frac{1}{2}$, then $\Delta(T)$ is $(p + \frac{1}{2})$-hn,
- It holds that $\Delta(\Delta(T))$ is hn.

Later on, Jung, Ko and Pearcy proved the next result that allowed to extend Brown's result to p-hyponormal operators:

Theorem 2 (Jung-Ko-Pearcy [5]). If $Lat(T)$ denotes the lattice of invariant subspaces of a given operator $T \in L(\mathcal{H})$, then $Lat(T) \simeq Lat(\Delta(T))$.

This result led to the first version of Jung-Ko-Pearcy conjecture on the iterated Aluthge transform sequence: The sequence of iterates $\{\Delta^n(T)\}_{n\in\mathbb{N}}$ converges to a normal operator for every $T \in L(\mathcal{H})$. As soon as they raised this conjecture, many results supporting this conjecture appeared. The following formula for the spectral radius due to Yamazaki (see also Wang [5]) was one of the most important.

CONCLUSION:

We summarize two works ([16] and [17]) which contain a proof of a positive answer to Conjecture [18] and some results on the regularity of the limit function. In these papers a new approach, based on techniques from dynamical systems, is introduced. The most important result used is the so-called stable manifold theorem for pseudo-hyperbolic systems

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