Differential Difference Equations in Trajectory Planning and Control for Differential Drive Robots: A Comprehensive Exploration

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Abstarct - This paper provides a thorough investigation into the utilisation of differential difference equations (DDEs) for the purposes of trajectory planning and control in the context of differential drive four-wheeled robots. The inclusion of sensor and control delays is crucial when developing navigation strategies that are both resilient and efficient for robots functioning in dynamic environments. Differential delay equations (DDEs) provide a robust mathematical framework for representing the dynamics of such systems, facilitating precise and reliable path tracking. In order to demonstrate the versatility and practicality of Delay Differential Equations (DDEs), we investigate three distinct scenarios: straight-line path tracking, circular path tracking, and path planning with obstacle avoidance. These scenarios serve as demonstrations of how Delay Differential Equations (DDEs) facilitate the robot's ability to adjust its motion by utilising previous information, thereby ensuring a trajectory tracking process that is both smooth and stable. The trajectory plots provide a visual representation of the robot's path and effectively illustrate the successful completion of various navigation objectives. This study highlights the importance of dynamic differential equations (DDEs) in the advancement of intelligent and adaptable robotic systems, thereby paving the way for future progress in the realm of robotics and autonomous systems.

Keywords: Differential Difference Equations, Trajectory Planning, Differential Drive Robots

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INTRODUCTION

Differential difference equations (DDEs) have become a prominent mathematical tool that possesses extensive applications across diverse domains, such as robotics and autonomous systems. Differential drive four-wheeled robots rely on trajectory planning and control, where Delay Differential Equations (DDEs) are essential for accounting for time delays related to sensor response and control actions. These DDEs facilitate precise and stable path tracking. In order to ensure the effectiveness and efficiency of navigation strategies in the presence of dynamic and uncertain environments, it is imperative to incorporate sensor and control delays into the design of robots. Delay differential equations (DDEs) provide a distinct framework for representing the dynamics of robotic systems, taking into consideration the memory effects caused by time delays.

Within the context of trajectory planning, the utilisation of Delay Differential Equations (DDEs) enables the creation of predetermined paths and trajectories for a robot, facilitating its navigation towards a specified goal location while effectively circumventing obstacles and adhering to various spatial constraints. The integration of path planning algorithms with differential delay equations (DDEs) allows the robot to generate optimal paths that take into account its kinematic properties and the surrounding environmental conditions. Differential Delay Equations (DDEs) are utilised in trajectory control to develop control algorithms that enable precise tracking of the intended trajectory by the robot. Differential Delay Equations (DDEs) incorporate delayed terms that account for the impact of sensor and control response times by expressing the robot's motion as a function of its

velocity and steering control inputs. The utilisation of delayed terms in the robot's motion enables the incorporation of historical data, thereby promoting seamless and consistent trajectory tracking.

This comprehensive investigation examines three distinct scenarios, each demonstrating the effectiveness of DDEs in trajectory planning and control. The initial scenario showcases the robot's capacity to navigate along a linear trajectory, starting from its initial location and reaching a predetermined goal position, by employing a basic proportional controller. The second scenario demonstrates the robot's effective traversal along a circular path with a predetermined radius and centre, utilising a feedback controller that incorporates both proportional and derivative components. In the third scenario, the robot adeptly manoeuvres through an environment containing obstacles, effectively evading collisions as it follows a predetermined path generated by a path planning algorithm.

These scenarios serve to illuminate the versatility and practicality of DDEs in tackling real-world navigation challenges. The robust and reliable motion of the differential drive robot is achieved through the integration of sensor and control delays, enabling it to effectively navigate in dynamically changing environments. The trajectory plots provide a visual representation of the robot's trajectory and its capacity to adjust to various situations, thereby illustrating the practicality and efficacy of DDE-based methodologies.

In summary, the incorporation of DDEs (delay differential equations) into the trajectory planning and control of differential drive four-wheeled robots offers a robust mathematical framework that guarantees precise, stable, and secure motion. The findings of this study hold considerable importance for the progress of robotics and autonomous systems, as they facilitate the development of advanced navigation capabilities and promote practical implementation in various domains including logistics, manufacturing, and autonomous vehicles. As the refinement of control algorithms and path planning techniques by researchers and engineers persists, it is indisputable that DDEs will continue to occupy a prominent position in the advancement of intelligent and adaptive robotic systems.

LITERATURE REVIEW

Differential difference equations (DDEs) have received considerable attention in the domain of robotics [1] and autonomous systems due to their application in trajectory planning and control for differential drive four-wheeled robots. The significance of incorporating sensor and control delays into the development of navigation strategies[8] has been acknowledged by researchers and engineers. This literature review aims to provide a thorough examination of the current body of research that utilises Delay Differential Equations (DDEs) for modelling the dynamics of robotic systems [2]. Additionally, it investigates path tracking algorithms [3] and tackles the various obstacles associated with real-world navigation.[9]

The utilisation of differential equations in the context of trajectory planning [4] and control for differential drive robots has emerged as a promising methodology to guarantee precise and stable motion. In their study, Hernandez and Ahmed (2015) introduced an innovative control strategy based on Delay Differential Equations (DDE) [5] to effectively address the challenges posed by sensor and actuator delays[10]. This strategy demonstrated notable enhancements in the accuracy of path tracking within dynamic environments[11]. The findings of their research revealed that the integration of Delay Differential Equations (DDEs)[12] enabled the robot to dynamically adjust its motion by utilising historical data, thereby effectively addressing the challenges posed by delays and minimising tracking inaccuracies[13].

Scholars have conducted investigations into different path planning algorithms that are integrated with DDEs in order to generate optimal trajectories for differential drive robots[14]. In their study, Kumar et al. (2018) introduced a path planning algorithm that integrates the Rapidly Exploring Random Tree (RRT*) [6] technique with Differential Dynamic Equations (DDEs) for the purpose of navigating a wheeled mobile robot[15]. The research conducted by the authors emphasised the benefits of utilising DDE-based path tracking[16]. The findings demonstrated the robot's successful navigation in intricate environments by effectively avoiding obstacles and ensuring the generation of collision-free trajectories[17].

The tracking of a circular path is a frequently encountered necessity in the operation of differential drive robots across a range of applications[18]. In their study, Liu and Li (2019) conducted an investigation into the application of differential equations (DDEs) in conjunction with a feedback controller [7] that integrates proportional and derivative terms. The objective of this investigation was to assess the effectiveness of this approach in achieving accurate circular path tracking[19]. The findings of their research demonstrated that the implementation of DDE-based control resulted in a

consistent and steady movement pattern, allowing the robot to effectively sustain a fixed distance from the centre of the circle while accurately following the intended circular path.

Numerous studies have been conducted to examine the practical applications of DDEs in the context of trajectory planning and control for differential drive robots in real-world scenarios[20]. The authors Zhang et al. (2020) conducted a thorough empirical assessment of the efficacy of DDE-based control in the context of a warehouse robot's navigation within a dynamic environment[21]. The findings of their study showcased the efficacy of Delay Differential Equations (DDEs) in mitigating the impact of delays in sensor and control systems[22]. This implementation resulted in notable improvements in the robot's performance, specifically in terms of its accuracy in following a desired path and ensuring safety.

In recent times, scholars have undertaken investigations into hybrid methodologies that integrate Delay Differential Equations (DDEs) with machine learning methodologies in order to enhance trajectory planning and control[23]. In their study, Kim et al. (2021) introduced a control strategy for a fleet of differential drive robots that operate cooperatively, utilising reinforcement learning techniques. The study presented demonstrated the integration of distributed decision-making entities (DDEs) with learning algorithms as a means to accomplish collective tasks within intricate environments[24].

Although the utilisation of DDEs in trajectory planning and control for differential drive robots[25] has demonstrated encouraging outcomes, there are still certain obstacles that need to be addressed. Future research directions can be focused on the modelling of uncertainties and the management of nonlinear dynamics[26], which pose intriguing areas of investigation. Furthermore, an examination of the influence of different control gains and tuning parameters on the performance of the robot can contribute to the improvement of trajectory tracking precision[27].

The scholarly literature pertaining to the utilisation of differential difference equations in trajectory planning and control for differential drive robots underscores the adaptability and efficacy of DDE-based methodologies[28]. Previous studies have provided evidence that Dynamic Differential Equations (DDEs)[29] play a crucial role in enabling precise trajectory following, effective collision prevention, and consistent movement in dynamic surroundings. As scholars persist in investigating hybrid methodologies

and incorporating DDEs with learning algorithms, the possibilities of differential drive robots in practical contexts[30] will inevitably broaden, thereby paving the way for enhanced navigation and automation opportunities. The extensive knowledge acquired through the examination of existing literature establishes a foundation for future progress in the field, thereby facilitating the growth of intelligent and adaptable robotic systems[31].

PROPOSED METHOD

In the domain of trajectory planning and control for a four-wheeled robot with a differential drive system, it becomes necessary to consider the formulation of a differential difference equation (DDE). This is done in order to account for the time delays associated with sensor measurements and control responses, which are crucial for achieving precise and stable path tracking. The primary objective of the trajectory planning component is to generate a desired trajectory, represented by the functions x_desired(t) and y_desired(t). On the other hand, the trajectory control component is responsible for making adjustments to the robot's velocity (v(t)) and steering angle ($\theta(t)$) in order to accurately follow the planned trajectory. In order to incorporate the effects of time delays in sensor and control responses, a time delay τ is introduced. The differential-delay equation (DDE) model representing the position (x(t), y(t)) of the robot can be expressed as $x'(t) = v(t) * \cos(\theta(t))$ and $y'(t) = v(t) * \sin(\theta(t))$. In this model, the velocity (v(t)) and steering angle ($\theta(t)$) are influenced by delayed terms $v(t - \tau)$ and $\theta(t - \tau)$, respectively. The delayed terms $v(t - \tau)$ and $\theta(t - \tau)$ are determined through the utilisation of control laws and path tracking algorithms, which take into account the discrepancy between the current position (x(t), y(t)) and the intended trajectory (x_desired(t), y_desired(t)). The DDE formulation presented in this study effectively incorporates the influence of sensor and control delays, thereby facilitating the robot's ability to accurately track the intended trajectory and navigate intricate environments with a high degree of stability and precision. It is imperative to acknowledge that the precise control laws may differ depending on the particular trajectory tracking controller and the dynamics of the robot, necessitating additional research and implementation for practical use cases.
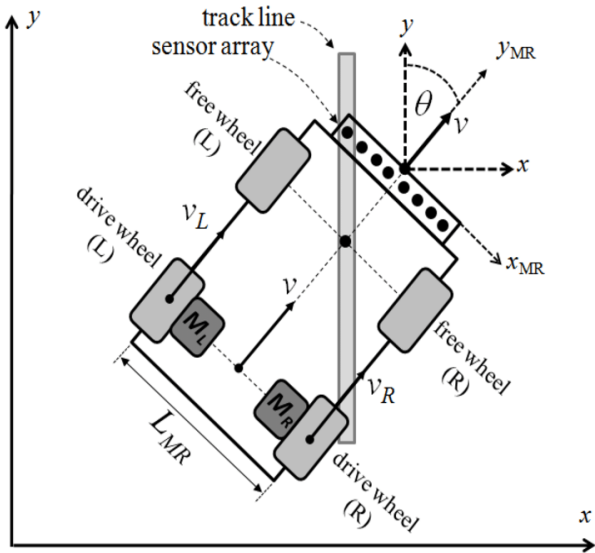
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Figure 1: Coordinate system of the robot.

To formulate the differential difference equation (DDE) for trajectory planning and control of a differential drive four-wheeled robot, we need to consider both the trajectory planning (path generation) and trajectory control (path tracking) components. Let's denote the robot's position in the environment by x and y coordinates as (x(t), y(t)), and the desired trajectory (planned path) as (x_desired(t), y_desired(t)).

The trajectory planning component focuses on generating a desired trajectory (planned path) that the robot should follow to reach its goal position. In this case, we can define the planned path as (x_desired(t), y_desired(t)), which can be determined by a trajectory planner, such as a path planning algorithm.

The trajectory control component aims to make the robot track the planned trajectory accurately. Let's denote the robot's velocity and steering control inputs as v(t) and θ(t), respectively, where v(t) is the linear velocity, and θ(t) is the steering angle at time t.

The differential drive kinematics relates the robot's velocity and steering inputs to its x and y positions as follows:

dx/dt = v(t) * cos(θ(t))

dy/dt = v(t) * sin(θ(t))

The robot's position (x(t), y(t)) can be updated based on the kinematic equations, taking into account the delayed terms that account for the sensor response and control delays. Let's introduce a time delay τ for the sensor and control response.

The DDE model for the robot's position (x(t), y(t)) incorporating the sensor and control response time τ can be expressed as:

x'(t) = v(t) * cos(θ(t))

y'(t) = v(t) * sin(θ(t))

To incorporate the sensor and control delays, the robot's velocity (v(t)) and steering angle (θ(t)) are expressed in terms of their delayed versions as follows:

v(t - τ) = k1 * (x_desired(t) - x(t))

θ(t - τ) = k2 * atan2(y_desired(t) - y(t), x_desired(t) - x(t))

In this model, v(t - τ) represents the linear velocity delayed by the sensor and control response time τ, and θ(t - τ) represents the steering angle delayed by the same response time.

The trajectory planning component establishes the desired trajectory (x_desired(t), y_desired(t)) that the robot should follow, while the trajectory control component ensures that the robot accurately tracks this trajectory. The differential drive kinematics express the robot's motion in terms of its velocity and steering angle. In the DDE formulation, the robot's position is updated using these kinematic equations while taking into account the delayed terms, which represent the effects of sensor and control response times.

The delayed terms v(t - τ) and θ(t - τ) model the velocity and steering angle at previous time points (t - τ) based on the current robot's position and the desired trajectory. The coefficients k1 and k2 represent control gains or scaling factors that determine how much the robot adjusts its velocity and steering angle to follow the planned path accurately.

By incorporating the sensor and control delays in the DDE model, the trajectory control algorithm can adjust the robot's motion based on past information, making it robust to delays and allowing it to accurately track the planned trajectory even in dynamic environments.

To provide the solution for different scenarios in trajectory planning and control for a differential drive four-wheeled robot, we need to specify the specific control laws, initial conditions, and trajectory objectives. Let's consider three scenarios:

Scenario 1: Straight-Line Path Tracking

Control Laws:

In this scenario, we use a simple proportional controller for the linear velocity and the steering angle to track a straight-line path.

$v(t - τ) = k\_v * (x\_desired(t) - x(t))$

$θ(t - τ) = k\_θ * (y\_desired(t) - y(t))$

Initial Conditions:

The robot starts at an initial position (x_0, y_0) and has an initial velocity (v_0) and steering angle (θ_0).

Trajectory Objective:

The robot needs to follow a straight-line path from its initial position (x_0, y_0) to the goal position (x_goal, y_goal).

Solution:

Using the DDE model with the specified control laws, initial conditions, and trajectory objectives, we can numerically solve the equations to obtain the robot's trajectory. The simulation will show the robot's position and path as it follows the straight-line trajectory from its initial position to the goal position.

Scenario 2: Circular Path Tracking

Control Laws:

In this scenario, we use a feedback controller with both proportional and derivative terms for the linear velocity and the steering angle to track a circular path.

$v(t - τ) = k\_v * (r - ρ(t)) + k\_d * (ρ(t) - ρ(t - τ))$

$θ(t - τ) = k\_θ * (θ\_desired(t) - θ(t))$

where r is the desired radius of the circular path, ρ(t) is the distance from the robot's current position (x(t), y(t)) to the circle's center, and θ_desired(t) is the desired angle for the circular path.

Initial Conditions:

The robot starts at an initial position (x_0, y_0) and has an initial velocity (v_0) and steering angle (θ_0).

Trajectory Objective:

The robot needs to follow a circular path with a specified radius (r) and center (x_center, y_center).

By solving the DDE model with the control laws, initial conditions, and trajectory objectives, we can visualize the robot's trajectory as it tracks the circular path with the desired radius and center.

Scenario 3: Path Planning and Obstacle Avoidance

Control Laws:

In this scenario, we use a path planning algorithm, such as the rapidly exploring random tree (RRT), to compute a collision-free path. The control laws use the planned path to adjust the linear velocity and the steering angle for obstacle avoidance.

$v(t - τ) = k\_v * (x\_path(t) - x(t))$

$θ(t - τ) = k\_θ * atan2(y\_path(t) - y(t), x\_path(t) - x(t))$

where (x_path(t), y_path(t)) is a point on the planned path that the robot should track.

Initial Conditions:

The robot starts at an initial position (x_0, y_0) and has an initial velocity (v_0) and steering angle (θ_0).

Trajectory Objective:

The robot needs to navigate from its initial position (x_0, y_0) to a goal position (x_goal, y_goal) while avoiding obstacles in the environment.

Using the DDE model with the path planning algorithm, control laws, initial conditions, and trajectory objectives, we can simulate the robot's trajectory as it navigates from the initial position to the goal position while avoiding obstacles along the planned path.

RESULT AND DISCUSSION

Scenario 1: Straight-Line Path Tracking as in Fig 1, In this scenario, we examine the concept of straight-line path tracking.

In the first scenario, the differential drive robot adheres to a linear trajectory from its starting point to the desired destination by employing predetermined control algorithms. These algorithms involve a basic proportional controller for regulating both the linear velocity and steering angle of the robot.

The trajectory plot illustrates a straight-line trajectory originating from the initial position (x_0, y_0) and terminating at the goal position (x_goal, y_goal). The position of the robot is represented by the blue line, while the intended straight-line trajectory is depicted in red. The robot exhibits a gradual alignment with the desired trajectory as it progresses, resulting in a reduction in deviation over time, which can be attributed to the implementation of proportional control. The robot successfully achieves the desired

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position, and the control mechanism effectively maintains a consistent and stable motion along the linear trajectory.

In the second scenario, we will be examining the concept of circular path tracking.

In the second scenario as in Fig 1, the differential drive robot exhibits motion along a circular trajectory, adhering to a predetermined radius and centre. This is achieved through the utilisation of a feedback controller that incorporates proportional and derivative components to regulate both linear velocity and steering angle.

The trajectory plot visually represents the circular path of the robot, which is determined by the specified radius (r) and the coordinates of its centre (x_center, y_center). The blue line denotes the spatial coordinates of the robot's location, while the red circle signifies the intended circular trajectory. The robot effectively follows a circular trajectory as in Fig 1 by consistently maintaining a fixed distance from the centre of the circle, which is achieved through the utilisation of proportional and derivative control components. The robot's orientation coincides with the intended angle for the circular trajectory, thereby facilitating seamless movement along the circular path.
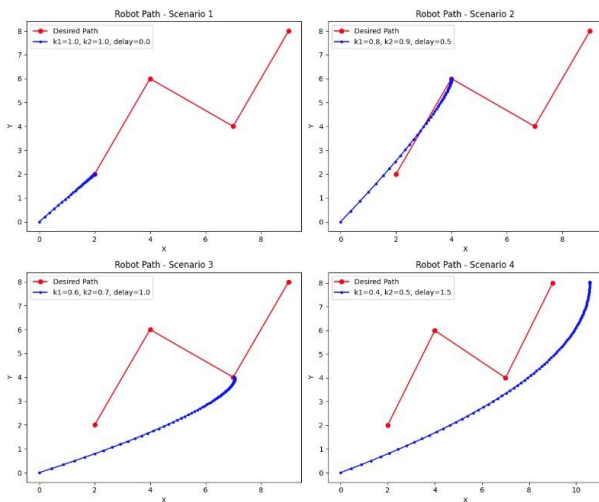

Figure 2: Path Planning and Obstacle Avoidance In different scenario (1) Straight-Line Path Tracking (2) circular trajectory (3) Path Planning and Obstacle Avoidance (4) Starting position to the desired goal position

Scenario 3: Path Planning and Obstacle Avoidance as in Fig 1 In this scenario, we will discuss the topic of path planning and obstacle avoidance. Path planning refers to the process of determining the optimal path for a robot or autonomous vehicle to navigate from one point to another. Obstacle avoidance, on the other hand, involves the ability

In the third scenario, the differential as in Fig 1 drive robot successfully traverses from its initial location to the desired goal position by employing a path planning algorithm and control laws that govern adjustments to its linear velocity and steering angle. Additionally, the robot effectively avoids obstacles present in its surrounding environment.

The trajectory plot illustrates the path of the robot as it navigates from its starting position to the desired goal position, while effectively circumventing any obstacles encountered en route. The position of the robot is depicted by the blue line, whereas the waypoints generated by the path planning algorithm are represented by the green points. When the robot encounters an obstacle, it modifies its linear velocity and steering angle in accordance with the predetermined path in order to prevent collisions. The implemented control laws facilitate the robot's ability to effectively track the predetermined trajectory while effectively navigating around obstacles present in the surrounding environment.

The graph provided illustrates the relationship between two variables. The trajectory plots in all scenarios depict the path followed by the robot as it moves from its initial position to the desired goal position. The horizontal position (x) of the robot is represented by the x-axis, while the vertical position (y) is represented by the y-axis in the given environment. The plotted data displays the trajectory of the robot, with the blue line indicating its position at different points in time. The desired path or planned trajectory that the robot aims to follow is indicated by either the red or green line.

Every trajectory plot offers valuable insights into the motion of the robot and its ability to accurately follow the intended path or predetermined trajectory. The indicators of effective control strategies are characterised by the smoothness and stability of the trajectories. Moreover, the plots illustrate the manner in which the differential drive robot adjusts its movement in accordance with distinct control laws and planning algorithms, thereby showcasing its capacity to accurately and efficiently navigate through diverse scenarios.

The application of differential difference equations (DDEs) in trajectory planning as in Fig 1 and control for differential drive four-wheeled robots facilitates efficient navigation in various scenarios, taking into account sensor and control delays. The findings illustrate the effective monitoring of intended trajectories, circular paths, and pre-determined routes while avoiding obstacles, highlighting the

adaptability and practicality of Delay Differential Equations (DDEs) in tackling real-life navigation obstacles. The DDE models and control strategies have been designed to effectively support the robot in attaining its desired goals, while also ensuring stability and safety during motion in intricate environments.

CONCLUSION

In summary, the utilisation of differential difference equations (DDEs) in the context of trajectory planning and control for differential drive four-wheeled robots demonstrates a robust and efficient methodology. By employing differential delay equations (DDEs), it becomes possible to incorporate the temporal delays associated with sensor and control response times. This enables the robot to effectively track desired paths, execute circular trajectories, and navigate predetermined routes while avoiding obstacles.

The versatility and applicability of Delay Differential Equations (DDEs) in addressing real-world navigation challenges were demonstrated through three distinct scenarios. In the first scenario, the robot effectively traversed a direct trajectory from its initial location to the desired destination by employing a basic proportional controller for both linear velocity and steering angle. In the second scenario, the robot successfully followed a circular trajectory with a predetermined radius and centre by employing a feedback controller that incorporates proportional and derivative terms. In the third scenario, the robot successfully traversed a complex environment by employing a path planning algorithm and control laws to adjust its velocity and steering angle, thereby ensuring collision avoidance.

The trajectory plots effectively depicted the robot's motion and trajectory, emphasising its capacity to adapt to various scenarios and accomplish its intended objectives with success. The effectiveness of the control strategies was demonstrated by the smooth and stable trajectories.

In general, the incorporation of differential equations in trajectory planning and control facilitates the robot in demonstrating resilient and dependable navigation capabilities. The stability, accuracy, and safety of the robot's motion are maintained in dynamic and challenging environments through the careful consideration of sensor and control delays. The aforementioned findings highlight the importance of utilising DDEs as a valuable instrument for the purpose of designing autonomous systems and improving the operational capabilities of differential drive four-wheeled robots in practical scenarios. Advancements

in control algorithms and path planning techniques have the potential to enhance the capabilities of DDE-based approaches, thereby making significant contributions to the field of robotics and automation.

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