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A Study on Multiple Integrations and Its Significance

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Abstract – The multiple integral is a generalization of the definite integral to functions of two or more than two real variable. Integrals of a function of two variable over a region in R called double integrals, and integrals of a function of three variable over a region of R are called triple integrals, Exam: $\iint_R f(x_1 \;\; x_2 \;\;)dx_1dx_2, \; \iiint_R f(x_1x_2x_3)dx_1dx_2dx_3$ ***variable represents the area of the region between the graph of the function and the x-axis, the double integral of a positive function of two variable represents the volume of the region between the surface defined by the function.(on the three-dimensional Cartesian coordinate where z=f(x,y)] and the plane which contains its domain. The same volume can be obtained by the triple integral. If there are more variables, a multiple integral will be hyper volumes of multidimensional functions.***

Keywords: - Multiple Integral, Functions

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INTRODUCTION

Multiple integration of a function in an variables f(x1,x2….xn) over a domain d is most commonly represented by integral sign in the reverse order of operation (the leftmost integral is operated last) if the function and integrand are in proper order. Just as the definite integral of a positive of one variable represent the area of the region between the graph of the function and x-axis, the double integral of a positive function of two variables represents the volume of the region between the surface defined by the function (on the three dimensional Cartesian plane where z=f(x,y) and the plane which contain its domain, the same volume can be obtained by the triple integral. The integral of a function in three variables of constant function f(x,y,z)=1Over the above maintain region between the surface and the plane. If there are more variable a multiple integral will be hyper volumes of multidimensional functions. Multiple integration of a function in n variables say f (x1,x2…..xn) or a domain D is most commonly represent by integral sign. In the reverse order of operation.

$$\iiint \ldots \ldots \ldots \int_D f(x_1 \;\; , x_2 \;\; \ldots \ldots xn)dx_1 \;\; \ldots \ldots dxn$$

Since the concept of ant derivative is only defined for function of a single real variable the usual definition of the indefinite integral does not immediately extend to the multiple integral.

PROPERTIES:

Multiple integrals have many properties common to those of integrals of a function of one variable. One major property of multiple integral is that the value of an integral is intendment of the order of

Integrand under specific condition.

PARTICULER CASE:

In the case of T c R, the integral

if $l = \iint_T \;\; f(x,y)dxdy$

Is the double integral of the T,

If $l = \iiint_T \;\; f(x,y,z)dxdydz$

Is called triple integral of f on T.

MATHODS OF INTEGRAION:

The solution of the problem with multiple integral consists in most of the cases for finding a way to reduce the multiple integral to an iterated integral, a series of integral of one variable is being directly solvable if function is continues than Fubini's theorem can be used sometime it is possible to obtained the result by direct integration without any calculation.

Following are the simple integration methods.

INTEGRATING CONSTANT METHOD:

When the integrand is a constant function say c, the integral is equal to the product of c and mea sue of the domain of integration. If c=1 and the domain is a sub region of R the integral gives the area of the region while if the domain is a sub region of R integral give the volume of the region.

Exam: f(x,y)=2, D=((x,y)2≤x≤4, 3≤y≤6)

f(x,y)=z

$$\int_3^0 \int_2^4 2dxdy = 12$$

When the domain of integration is symmetric about the origin with respect to last variable of integration and the integrand is odd with respect to this variable, the integral is equal to zero as the integral over the two half of the domain have the same absolute value but opposite sign when integrand is even with respect to this variable the integral is equal to twice the integral over one half on the domain. As, the integral, over the two half of the domain are same.

Example: consider a function f (x,y)=2 sinx-3y³ +5 integrated over the domain

$$T=\{(x,y)\in R^2, x^2+y^2\leq 1\}$$

A having radius 1 and center as origin with bounded domain.

Using the linerarity property the integral can be dividing in three parts

$$\iint_T (2\sin x - 3y^3 + 5)dxdy = \iint_T \sin x dxdy - \iint_T 3y^3 dxdy + \iint_T 5dxdy$$

The function 2sinx is odd function in the variable x and symmetric with respect to y-axis, so the value of the fast integral is 0 similarly the function 3y³ is an odd function and symmetric above x-axis so it will be contribution to the final result is that the third integral. Therefore the original integral is equal to the area of the disc times 5, or 5π.

X-AXIS:

If the domain D is normal with respect to the x-axis, and f: D → R is a continuous function; then α(x) and β(x) (defined on the interval [a, b]) are the two functions that determine D. Then:

$$\iint_D f(x,y)dxdy = \int_a^b dx \int_{\alpha(x)}^{\beta(x)} f(x,y)dy$$

Y-AXIS:

If D is normal with respect to the y-axis and f: D → R is a continuous function; then α(y) and β(y) (defined on the interval [a, b]) are the two functions th at determine D.

Then $\iint_D f(x,y)dxdy = \int_\alpha^b dy \int_{\alpha(y)}^{\beta(y)} f(x,y)dx$

Example:

Consider the region

$$D=\{(x,y)\in R^2 \quad x\geq 0\ y\leq 1\ y\geq x^2\}$$

Calculate $\iint_D (x+y)dxdy$

It is now possible to apply the formula:

$$\iint (x+y)dxdy = \int_0^1 dx \int_{x^2}^1 (x+y)dy = \int_0^1 dx[xy + \tfrac{y^2}{2}] = \tfrac{13}{20}$$

If we choose normality with respect to the y-axis we could calculate $\int_0^1 dy \int_0^{\sqrt{y}} (x+y)dx$ and obtain the same value.

Normal domain on R³

The extension of these formulae to triple integrals sho uld be applied: if T is a domain that is normal with respect to the xy-plane and determined by the functions α(x, y) and β(x, y), then

$$\iiint_T f(x,y,z)dxdydz = \iint_D \int_{\alpha(x,y)}^{\beta(x,)} f(x,y,z)dxdydz$$

POLAR COORDINATES:

In R if the domain has a circular symmetry and the function has some particular characteristics you can apply the transformation to polar coordinates (see the example in the picture) which means that the generic points P(x, y) in Cartesian coordinates change to their respective points in polar coordinates. That allows one to change the shape of the domain and simplify the operations.

$$F(x,y)\to f(r\cos\theta,$$

Example : the function is $f(x,y)=x^2+y^2$

Transformation one obtains

$$f(r\cos\theta) = r\cos^2\theta + r\sin^2\theta$$

$$f(r\cos\theta) = r^2$$

Transformation from Cartesian to polar coordinates.

Example : The domain is $D=\{x^2+y^2 \leq 4\}$,

That is a circumference of radius 2; it's evident that the covered angle is the circle angle, so θ varies from 0 to 2π, while the crown radius varies from 0 to 2(the crown with inside radius null is just a circle). $T=\{0 \leq r \leq 2, \quad 0 < \theta \leq 2\pi\}$

CYLINDRICAL COORDINATES:

$$f(x,y,z) \rightarrow f(r\cos\theta, r\sin\theta, z)$$

In R the integration on domains with a circular base c an be made by the passage to cylindricalcoordinates; t he transformation of the function is made by the followi ng relation:

Cylindrical coordinates are useful in connection with object and phenomena that have some rotational symmetric.

The domain transformation can be graphically attained, because only the shape of the base varies while the height follows the shape of the starting region.

Example:

The region is $D=\{x^2+y^2 \leq 9, x^2+y^2 \geq 4, 0 \leq z \leq 5\}$

Whose base is circle crown and whose height is 5;

$$T=\{2<r\leq 3, 0\leq \theta \leq 2\pi, 0 \leq z \leq 5\}$$

Z component is unchanged during the transformation, the dx dy dz differentials becomes r, dr, dθ, dz.

And

$$\iiint_D f(x,y,z)dxdydz = \iiint_T f(r\cos\theta, r\sin\theta, z) r dr d\theta dz$$

SPHERICAL COORDINATE:

In R some domains have a spherical symmetry, so it's possible to specify the coordinates of every point of the integration region by two angles and one distance. It's possible to use therefore the passage to spherical coordinates; the function is transformed by this relation:

$$f(x,y,z) \rightarrow f(r\sin\theta\cos\emptyset, r\sin\theta\sin\emptyset, r\cos\theta)$$

three dimensional coordinate that specify point positions by the distance from a chosen reference axis, the distance from the axis relative to a chosen reference direction and the distance from a chosen reference plane perpendicular to the axis.

Points on the z axis do not have a precise characterization in spherical coordinates, so θ varies from 0 to π can vary between 0 and 2π.

Example : The domain is $D = x^2+y^2+z^2 \leq 16$

(sphere with radius 4 and center at origin); applying the transformation you get the region,

$$T=\{0 \leq r \leq 4, 0 \leq \theta \leq \pi, 0 \leq \emptyset \leq 2\pi\}$$

Where

$dxdydz = r^2 \sin\theta dr d\theta d\emptyset, \iiint_D f(x,y,z)dxdydz = \iiint_T f(rsi$

Example: Let

$A=\{(x,y) \in R^2, 11 \leq x \leq 14, 7 \leq y \leq 10\}$

$$f(x,y) = x^2 + 4y$$

Let us assume that we wish to integrate a multivariable function f over a region A:

$$\int_7^{10} \int_{11}^{14} (x^2 + 4y) dxdy$$

From this we formulate the double integral

The inner integral is performed first, integrating with respect to x and taking y as a constant, as it is not the variable of integration. The result of this integral, which is a function depending only on y, is then integrated with respect to y.

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