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Quasi B-Normal Spaces

M. C. Sharma[1] Poonam Sharma[2] Raj Singh[3]

[1]Department of Mathematics N. R. E. C. College Khurja-203131 (U.P.) India

[2]Department of Mathematics N. R. E. C. College Khurja-203131 (U.P.) India

[3]Department of Mathematics Govt. College Julana Jind (Haryana) India

Abstract – In this paper, we introduced the concept of quasi b-normal spaces in topological spaces by using b-open sets due to Andrijevic [1] and obtained several properties of such space. We introduced the concepts of gb-closed, πgb-closed, almost b-closed, almost gb-closed, almost πgb- closed, πgb-continuous and almost πgb-continuous functions. Moreover, we obtain some new characterizations and preservation theorems of quasi b-normal spaces.

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1. INTRODUCTION

In this paper, we introduced the concept of quasi b-normal spaces in topological spaces by using b-open sets and obtained several properties of such space. Andrijevic [1] introduced the concept of b -closed sets and discuss some of their basic properties. Zaitsev [10] introduced the concept of quasi - normal space in topological spaces and obtained several properties of such a space. Recently, Sadeq Ali Saad et al. [7] introduced the concept of quasi p-normal spaces by using p-open sets and obtained several properties of such a space.

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Key words and phrases: gb-closed, πgb-closed sets, b-closed, gb -closed, πgb-closed, almost b-closed, almost gb-closed, almost πgb-closed,πgb-continuous ,almost πgb-continuous functions and quasi b- normal spaces

2. PRELIMINARIES

1. Quasi b-Normal Spaces

1.1. Definition. A subset A of a topological space X is said to be

1. b-closed [1] if int(cl(A)) $\cap$ cl(int(A)) $\subset$ A .

2. gb-closed [2] (resp. g*b-closed [9]) if b-cl (A) $\subseteq$ U whenever

A $\subseteq$ U and U is open (resp. g-open) in X.

3. πg-closed [4] (resp. πgb-closed [3], πgp-closed [6], πgsp-closed[3]) if cl(A) $\subset$ U (resp. b-cl(A) $\subset$ U, p-cl(A) $\subset$ U, sp-cl(A) $\subset$ U)whenever A $\subset$ U and U is π-open in X.

The complement of b-closed (resp. gb-closed, g*b-closed, πg - closed, πgb-closed, πgp-closed, πgsp-closed) set is called b-open (resp. gb-open, g*b-open, πg-closed, πgb-open, πgp-open,πgsp-open) set.The intersection of all b-closed sets containing A is called the b- closure of A and denoted by b-cl(A).The union of all b-open subsets of X which are contained in A is called the b-interior of A and denoted by b-int(A). The family of all πgb-closed (resp. πgb-open) subsets of the space X is denoted by πGBC(X) (resp. πGBO(X)).

1.2. Remark [3]. Every b-closed set is πgb-closed.

1.3. Proposition [3]. Every πgp-closed set is πgb-closed.

1.4. Proposition [3]. Every πgb-closed set is πgsp-closed.

1.5. Remark .We have the following implications for the properties of subsets

$$\begin{array}{ccccccc}
\text{closed} & \Rightarrow & \text{b-closed} & & \pi\text{g-closed} & \Rightarrow & \pi\text{gp-closed} \\
 & & \Downarrow & & & & \Downarrow \\
\text{gb*-closed} & \Rightarrow & \text{gb-closed} & \Rightarrow & \pi\text{gb-closed} & \Rightarrow & \pi\text{gsp-closed}
\end{array}$$

where none of the implications is reversible.

1.6. Example. Let X = {a, b, c } and τ = { ϕ, X, {a}}. Let A = {c} is a b- closed set but not a closed in X.

1.7. Example. Let X= {a, b, c}, τ = {X, ϕ, {a}} and A = {a, b}. Then X is the only regular open (π-open) set containing A. Hence A is πgb-closed, but A is not b-closed, since b-cl (A) = X.

1.8. Example. Let X= {a, b, c}, τ = {X, ϕ, {a}, {b}, {a, b}}. Let A= {a}. Then A is b-closed. Hence A is πgb-closed, but A is not πgp-closed, since A is regular open (π-open) and p-cl(A)= {a, c} $\not\subset$ A.

1.9. Definition. A space topological X is said to

1. b-normal if for any two disjoint closed subsets A and B of X, there exist two b-open disjoint subsets U and V of X such that A $\subseteq$ U and B $\subseteq$ V.

2. quasi-normal [10] (resp.quasi b-normal) if for any two disjoint π-subsets A and B of X, there exist two open(resp. b-open) disjoint subsets

U and V of X such that A $\subset$ U and B $\subset$ V.

1.10. Remark. For a topological space, the following implications hold:

normal $\Rightarrow$ quasi-normal

$\Downarrow$ $\Downarrow$

b-normal $\Rightarrow$ quasi b-normal.

1.11. Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {c, d},

{a, c, d}, {b, c, d}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {b}. Also U = {a, c} and V = {b, d} are b-open sets such that A $\subset$ U and B $\subset$ V. Hence X is b-normal.

1.12. Example. Let X = {a, b, c, d} and τ = {ϕ,{c}, {d},{c, d},{a, b}, {a, b, c},{a, b, d}, X }. The pair of disjoint π-closed subsets of X are A = ϕ and B = {d}. Also U = {c} and V = {a, b, d} are open sets such that A $\subset$ U and B $\subset$ V. Hence X is quasi-normal as well as quasi b-normal because every open set is b-open set.

1.13. Corollary [3]. A subset A of a topological space X is πgb-open iff F $\subset$ b-int (A) whenever F $\subset$ A and F is π-closed.

1.14. Theorem. For a topological space X, the following are equivalent:

(a) X is quasi b- normal.

(b) For any disjoint π-closed sets H and K, there exist disjoint gb-open sets U and V such that H $\subset$ U and K $\subset$ V.

(c) For any disjoint π-closed sets H and K, there exist disjoint πgb-open sets U and V such that H $\subset$ U and K $\subset$ V.

(d) For any π-closed set H and any π-open set V containing H, there exists a gb-open set U of X such that H $\subset$ U $\subset$ b-cl(U) $\subset$ V.

(e) For any π-closed set H and any π-open set V containing H, there exists a πgb-open set U of X such that H $\subset$ U $\subset$ b-cl(U) $\subset$ V.

Proof. It is obvious that (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c) and (d) $\Rightarrow$ (e).

(c) $\Rightarrow$ (d). Let H be any π-closed set of X and V be any π-open set containing H. There exist disjoint πgb-open sets U, W such that H $\subset$ U and X – V $\subset$ W. By Corollary 1.13 , we have X – V $\subset$ b-int(W) and U $\cap$ b-int(W) = ϕ.Therefore, we obtain b-cl(U) $\cap$ b-int(W) = ϕ and hence

H $\subset$ U $\subset$ b-cl(U) $\subset$ X – b-int(W) $\subset$ V.

(e) $\Rightarrow$ (a). Let H, K be any two disjoint π-closed set of X. Then H $\subset$ X – K and X – K is π-open and hence there exists a πgb-open set G of X such that H $\subset$ G $\subset$ b-cl(G) $\subset$ X – K. Put U = b-int(G), V = X – b-cl(G). Then U and V are disjoint b-open sets of X such that H $\subset$ U and K $\subset$ V. Therefore, X is quasi b-normal.

1.15. Definition. A topological space X is said to be semi-irreducible [5] if every disjoint family of non - empty open sets of X is finite or equivalently if X has only a finite amount of regularly open sets.

1.16. Theorem. A semi-irreducible, semi-regular space X is normal iff X is quasi b- normal.

Proof. Let X be quasi b-normal and let A and B be disjoint closed sets of X. By semi-regularity of X, A and B are δ-closed. Since X is semi-irreducible, X has only finitely many regularly open sets. Thus, A and B are π-closed. Now, by the quasi b-normality X is normal.

1.17. Definition. A function f: X $\rightarrow$ Y is said to be

1. g-closed [4](resp. πg-closed [4], b-closed, gb-closed, πgb-closed) if f(F) is g-closed,πg-closed, b-closed, gb-closed, πgb-closed in Y for every closed set F of X.

2. almost b-closed(resp. almost gb-closed , almost πgb-closed) if f (F) is b-closed (resp. gb-closed, πgb-closed) in Y for every F $\in$ RC(X).

3. π - continuous [4] (resp. πgb- continuous) if f -1(F) is π-closed (resp. πgb-closed) in X for every closed set F of Y.

4. almost continuous [8](resp. almost π-continuous [4], almost πgb-continuous) if f -1(F) is closed (resp. π-closed,πgb-closed) in X for every regularly closed set F of Y.

From the definitions stated above, we obtain the following diagram:

closed $\Rightarrow$ b-closed $\Rightarrow$ gb-closed $\Rightarrow$ πgb-closed

$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

al. closed $\Rightarrow$ al. b-closed $\Rightarrow$ al. gb-closed $\Rightarrow$ al. πgb-closed

where al. = almost.

1.18. Theorem. A surjection f : X $\rightarrow$ Y is almost πgb-closed if and only if for each subset S of Y and each U $\in$ RO(X) containing f -1(S) there exists a πgb-open set V of Y such that S $\subset$ V and f -1(V) $\subset$ U.

Proof. Necessity. Suppose that f is almost πgb-closed. Let S be a subset of Y and U $\in$ RO(X) containing f –1(S). If V = Y– f (X – U) , then V is a πgb-open set of Y such that S $\subset$ V and f -1(V) $\subset$ U.

Sufficiency. Let F be any regular closed set of X. Then f -1 (Y– f (F)) $\subset$ X – F and X – F $\in$ RO(X). There exists a πgb-open set V of Y such that Y – f (F) $\subset$ V and f -1(V) $\subset$ X – F. Therefore, we have f (F) $\supset$Y–V and F $\subset$ X – f -1(V) $\subset$ f -1(Y – V) . Hence we obtain f (F) = Y – V and f (F) is πgb-closed in Y which shows that f is almost πgb-closed.

1.19. Theorem. If f: X $\rightarrow$ Y is an almost πgb-continuous rc-preserving injection and Y is quasi b-normal, then X is quasi b-normal.

Proof. Let A and B be any disjoint π-closed sets of X. Since f is a rc- preserving injection, f (A) and f (B) are disjoint π-closed sets of Y. By the quasi b-normality of Y, there exist disjoint b-open sets U and V of Y such that f (A) $\subset$ U and f (B) $\subset$ V.

Now if G = int (cl (U)) and H = int (cl (V)), then G and H are disjoint regular open sets such that f (A) $\subset$ G and f (B) $\subset$ H. Since f is almost πgb-continuous, f -1(G) and f –1(H) are disjoint πgb-open sets containing A and B, respectively. It follows from Theorem 1.14 that X is quasi b-normal.

1.20. Theorem. If f: X $\rightarrow$ Y is π-continuous, almost b -closed surjection and X is quasi b - normal space, then Y is b-normal.

Proof. Let A and B be any two disjoint closed sets of Y. Then f -1(A) and f -1(B) are disjoint π-closed sets of X. Since X is quasi b-normal, there exist disjoint b-open sets of U and V such that f -1 (A) $\subset$ U and f -1(B) $\subset$ V. Let G = int(cl(U)) and H = int(cl(V)). Then G and H are disjoint regular open sets of X such that f -1(A) $\subset$ G and f –1(B) $\subset$ H. Set K = Y – f (X – G), L = Y– f (X – H).Then K and L are b-open sets of Y such that A $\subset$ K, B $\subset$ L, f –1(K) $\subset$ G , f –1(L) $\subset$ H. Since G and H are disjoint, K and L are disjoint. Since K and L are b-open and we obtain A $\subset$ b-int(K), B $\subset$ b-int(L) and b-int(K) $\cap$ b-int(L) = ϕ. Therefore Y is b-normal.

1.21. Theorem. Let f: X $\rightarrow$Y be an almost π-continuous and almost πgb-closed surjection. If X is quasi b-normal space then Y is quasi b-normal.

Proof. Let A and B be any disjoint π-closed sets of Y. Since f is almost π - continuous, f –1(A), f –1(B) are disjoint closed subsets of X. Since X is quasi b-normal, there exist disjoint b-open sets U and V of X such that f –1(A) $\subset$ U and f –1(B) $\subset$ V. Put G = int(cl(U)) and H = int(cl(V)).Then G and H are disjoint regular open sets of X such that f -1(A) $\subset$ G and f -1(B) $\subset$ H. By Theorem 1.18, there exist πgb-open sets K and L of Y such that A $\subset$ K, B $\subset$ L, f -1 (K) $\subset$ G and f -1 (L) $\subset$ H. Since G and H are disjoint. So are K and L by Coollary 1.13, A $\subset$ b-int(K), B $\subset$ b-int(L) and b-int(K) $\cap$ b-int(L) = ϕ . Therefore, Y is quasi b-normal.

1.22. Theorem. Let f: X $\rightarrow$ Y be an almost continuous and almost πgb- closed surjection. If X is normal then Y is quasi b-normal. .

Proof. Easy

1.23. Corollary. If f: X $\rightarrow$ Y is an almost continuous and almost closed surjection and X is a normal space, then Y is quasi b-normal.

Proof. Since every almost closed function is almost πgb-closed so Y is quasi b-normal.

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