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On Slightly πgb-Continuous Functions

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Abstract. In this paper, we introduce a new class of continuous functions called slightly πgb-continuous functions by using πgb-closed sets in a topological space. Also the relations of slightly πgb-continuous functions with other weak forms of πgb-continuous functions have been investigated.

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INTRODUCTION

In this paper, we introduce a new class of continuous functions called slightly πgb -continuous functions by using πgb-closedsets due to Ahmad Al-Obiadi et al.[**1**]. Ahmad Al-Obiadi et al.[**1**] introduced the notion of πgb-closed sets in a topological space and obtained their various properties. In 2004, Ekici and Caldas[**6**] introduced the notion of slightly γ-continuity (or slightly b-continuity) which is a weakened form of b-continuity. The relationships of slightly πgb-continuity with other weaker forms of continuity viz. weakly πgb-continuity, somewhat πgb-continuity, almost πgb-continuity and faintly πgb-continuity have been studied. Throughout the present paper, X and Y are always topological spaces.

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2. Preliminaries.

2.1.Definition. A subset of a topological space X is said to be

1.**regular open** [**13**] if A=int(cl(A)).

2.**b-open**[**3**](or **γ-open** [**7**]) if $A \subset \mathrm{int}(\mathrm{cl}(A)) \cup \mathrm{cl}(\mathrm{int}(A))$,

3.**gb-closed**[**1**] (resp. **g*b-closed**[**15**]) if $b\text{-cl}(A) \subseteq U$, whenever $A \subseteq U$

and U is open (resp. g-open) in X.

4.**πg-closed** [**5**] (resp. **πgb-closed**[**2**], **πgp-closed**[**11**], **πgsp-closed**[**2**]) if $\mathrm{cl}(A) \subset U$ (resp. $b\text{-cl}(A) \subset U$, $p\text{-cl}(A) \subset U$, $sp\text{-cl}(A) \subset U$), whenever $A \subset U$ and U is π-open in X.

5.**δ*-open** [**8**] if for each $x \in A$, there exists a clopen subset G of X such that $x \in G \subset A$.

6.**θ-open** [**14**] if for each $x \in A$, there exists an open subset G of X such that $x \in G \subset \mathrm{cl}(G) \subset A$.

7. A subset B of X is said to be a **πgb-neighbourhood** [**2**] of a point $x \in X$ if there exists a πgb-open set containing x and is contained in A.

The complement of b-closed (resp. gb-closed, g*b-closed, πg-closed, πgb-closed, πgp-closed, πgsp-closed, δ*-open ,θ-open) set is called **b-open** (resp. **gb-open**, **g*b-open**, **πg-closed**, **πgb-open**, **πgp-open**,**πgsp-open**, **δ*-closed** ,**θ-closed**) set.

The intersection of all b-closed(resp. δ*-closed) sets of X containing A is called the **b- closure** (resp.**δ*-closure**) **of A** and denoted by **b-cl**(**A**) (resp. **δ*-cl(A))**.

The union of all b-open (resp. δ*- open) subsets of X which are contained in A is called the **b-interior** (resp. **δ*-interior) of A** and denoted by **b-int**(**A**) (resp. **δ*-int(A))**.The family of all b-open (resp. b-closed, clopen, b-clopen, δ*-open,δ*-closed, regular open, πgb-closed, πgb-open) sets in X is denoted by BO(X)(resp.BC(X),CO(X),BCO(X),δ*O(X),δ*C(X),RO(X),πGBC(X),πGBO(X)).

2.2.Remark[1]. Every b- closed set is πgb-closed.

2.3.Proposition[1]. Every πgp-closed set is πgb-closed.

2.4.Proposition[1]. Every πgb-closed set is πgsp-closed.

2.5.Remark . We have the following implications for the properties of subsets

closed $\Rightarrow$ b-closed πg-closed $\Rightarrow$ πgp-closed

$\nLeftrightarrow$

$\Downarrow$ $\Downarrow$

gb*-closed $\Rightarrow$ gb-closed $\Rightarrow$ πgb-closed $\Rightarrow$ πgsp-closed

where none of the implications is reversible.

2.6.Example. Let X = { a, b, c } and τ = { ϕ, X, {a}}. Let A = {c} is a b-closed set but not a closed in X.

2.7.Example. Let X= {a, b, c}, τ = {ϕ,{a},X} and A = {a, b}. Then X is the only regular open (π-open) set containing A. Hence A is πgb-closed, but A is not b-closed, since b-cl (A) = X.

2.8.Example. Let X= {a, b, c}, τ = { ϕ , {a}, {b}, {a, b},X }. Let A= {a}. Then A is b - closed . Hence A is πgb-closed, but A is not πgp-closed, since A is regular

open (π-open) and p-cl(A)= {a, c} $\not\subset$ A.

3. On Slightly πgb-Continuous Functions

3.1.Definition. A function f : X → Y is said to be

1.**almost b-continuous** (briefly a.b.c.) [**9**](resp.**almost πgb-continuous** (briefly a.πgb.c.)) if for each x $\in$ X and each V $\in$ RO(Y) containing f(x), there exists

U$\in$ BO(X)(resp. U $\in$ πGBO(X)) containing x such that f(U) $\subset$ V .

2.**weakly b-continuous** (briefly w.b.c.) [**12**] (resp. **weakly πgb –continuous** (briefly w. πgb.c.)) if for each x $\in$ X and each open set VinY containing f(x), there exists U $\in$ BO(X) (resp. U $\in$ πGBO(X)) containing x such that f(U) $\subset$ cl(V).

3.**somewhat b-continuous** (briefly sw.b.c.) [**4**] (resp. **somewhat πgb -continuous** (briefly sw. πgb.c.)) if for each open set V in Y and f^{-1}(V) $\neq \phi$ there exists U $\in$

BO(X) (resp. U $\in$ πGBO(X)) such U $\neq \phi$ and U $\subset f^{-1}$(V).

4. **faintly b-continuous** (briefly f.b.c.) [**10**] (resp. **faintly πgb -continuous** (briefly f.πgb.c.) if for each x $\in$ X and each θ-open set V in Y containing f(x), there exists U $\in$ BO(X) (resp. U $\in$ πGBO(X))) containing x such that f(U) $\subset$ V .

5.**slightly γ-continuous** [**6**] (resp. **slightly πgb-continuous)** if for each x $\in$ X and each V $\in$ CO(X) containing f(x), there exists a U $\in$ BO(X) (resp. U $\in$ πGBO(X)) U $\in$ containing x such that f(U) $\subset$ V .

3.2.Theorem. For a function f : X → Y , the following are equivalent :

(a) f is s. πgb.c.;

(b) f^{-1}(V) $\in$ πGBO(X) for every V $\in$ CO(X);

(c) f^{-1}(V) $\in$ πGBC(X) for every V $\in$ CO(X);

(d) f^{-1}(V) $\in$ πGBCO(X) for every V $\in$ CO(X).

3.3.Theorem. For a function f : X → Y , the following are equivalent:

(a) f is s. πgb.c.;

(b) f^{-1}(V) $\in$ πGBO(X) for every δ*-open set V in Y ;

(c) f^{-1}(V) $\in$ πGBC(X) for every δ*-closed set V in Y ;

(d) f(b-cl(A)) $\subset$ δ*-cl(f(A)) for every subset A of X;

(e) b-cl(f^{-1} (B)) $\subset$ f^{-1}(δ*-cl(B)) for every subset B of Y .

Proof. (a) $\Rightarrow$ (b). Let V be a δ*-open set in Y and let x $\in$ f^{-1}(V). Then f(x) $\in$ V . The δ*-openness of V gives a U $\in$ CO(Y) such that f(x) $\in$ U $\subset$ V.this implies that x $\in$ f^{-1} (U) $\in$ f^{-1} (V). Since f is s. πgb.c., from **Theorem 3.2**, we have, f^{-1}(U) $\in$ πGBO(X). Hence f^{-1}(V) is a πgb-neighbourhood of each of its points. Consequently, f^{-1}(V) $\in$ πGBO(X).

(b) $\Rightarrow$ (c). It is obvious from the fact that the complement of a δ*-closed set is

δ*-open.

(c) $\Rightarrow$ (d). Let A be a subset of X. We have,

δ*-cl(f(A)) = ∩{F : f(A) $\subset$ F, F $\in$ δ*C(Y)} is a δ*-closed set in Y. Thus

A $\subset$ f^{-1}(δ*-cl(f(A)) = ∩{ f^{-1}(F) : f(A) $\subset$ F, F $\in$ δ*C(Y)} $\in$ πGBO(X). Thus, we obtain b-cl(A) $\subset$ f^{-1}(δ*-cl(f(A)). Hence, f(b-cl(A)) $\subset$ δ*-cl(f(A)).

(d) $\Rightarrow$ (e). Let B be a subset of Y . We have

f(b-cl(f^{-1}(B))) = δ*-cl(f(f^{-1}(B)))$\subset$ δ*-cl(B) and hence, we obtain,

b-cl(f^{-1}(B)) $\subset$ f^{-1}(δ*-cl(B)).

(e) $\Rightarrow$ (a). Let V be a clopen set in Y . Then V is δ*-closed in Y. Thus

b-cl(f^{-1}(B)) $\subset f^{-1}$(δ*-cl(B)) = f^{-1}(B). Therefore, f^{-1} (B) is closed. Hence,

by **Theorem 3.2**, we obtain f is s. πgb.c.

3.4.Theorem. If a function f : X → Y is w. πgb.c. then, f is s. πgb.c.

Proof. Let x ∈ X and let V be a clopen set in Y containing f(x). Therefore,by weakly πgb-continuity of f, there exists U ∈ πGBO(X) containing x such that

f(U) ⊂ cl(V) = V . Since, x ∈ X is arbitrary, hence, f is s. πgb.c.

3.5.Remark. The following diagram follows immediately from the definitions in which none of the implications is reversible.

$$\begin{array}{ccccc} \text{f.}\pi\text{gb.c} & & & & \\ \Downarrow & & & & \\ \text{a.}\pi\text{gb.c} & \Rightarrow & \text{w.}\pi\text{gb.c} & \Rightarrow & \text{s.}\pi\text{gb.c} \end{array}$$

3.6.Example. Let X = Y = {a, b, c}, τ = {ϕ, {a}, X }, σ = {ϕ, {a}, {c}, {a, c}, Y}.

Then the identity function i : (X, τ) → (Y, σ) is s.πgb.c. but not w. πgb.c. at b ∈ X.

3.7.Remark. From definition, it is clear that every a. πgb.c. is w. πgb.c. and hence s πgb.c. The converse is clearly false as shown by **Example 3.6**.

3.8.Definition. A toipological space X is said to be **extremally disconnected** [**16**] if closure of every open set is open in X.

3.9.Theorem If a function f : X → Y is f. πgb.c. then, f is s. πgb.c.

Proof. The result is obvious from the fact that every clopen set is θ-open.

3.10.Remark. The converse of the above result is however, in general, not true as shown by the following example.

3.11.Example. Let τ = {G ⊂ R : 0 ∈ G} ∪ {ϕ} and let σ be the usual topology on R. Then the identity function i : (R, τ) → (R, σ) is s. πgb.c. but not f.πgb.c. at all points of R except 0.

3.12.Theorem. A s. πgb.c. f : X → Y is f. πgb.c. if Y is extremally disconnected.

Proof. Let x ∈ X and let V be a θ-open set in Y containing f(x). Thus there exists an open set W such that f(x) ∈ cl(W) ⊂ V . By extremally disconnectedness of Y, cl(W) is open. Thus, cl(W) ∈ CO(Y). Since, f is s. gb.c., therefore, there exists a b-open set U containing x such that f(U) ⊂ cl(W) ⊂ V . Since, x ∈ X is arbitrary, therefore, f is f. πgb.c.

3.13.Theorem. Let f : X → Y be a function, where, Y is extremally disconnected. Then f is f. πgb.c. if and only if f is s. πgb.c.

Proof. It can be directly obtained by using **Theorem 3.9** and **Theorem 3.12**.

3.14.Remark. Somewhat b-continuity and slightly πgb-continuity are independent of each other as shown by

3.15.Example. The function defined in **Example 3.11** is s. πgb.c. but not sw. πgb.c. Again let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, {a, c},X}, σ = {ϕ, {a},{b, c}, Y}. Then the identity function i : (X, τ) → (Y, σ) is sw. πgb.c. but not s. πgb.c.

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