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Retailer's Optimal Replenishment Policy with Trade Credit under Inflationary and Fuzzy Environment with Different Demand Pattern

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Abstract – In the present paper, inventory models has been developed by incorporating some additional features like impreciseness in costs, inflation, deterioration and credit period offered by supplies to the retailer which can be associated with a number of different types of inventory. By taking impreciseness in cost parameters, decision-makers absorb all the turbulence in cost due to market fluctuation. Inflation permits a proper recognition of the financial implication.-of the opportunity cost in inventory analysis. Generally, supplier offers different price-discount on purchase of item of retailers at different delay periods. Suppliers allow maximum delay period, after which they will not take a risk of getting back money from retailers or any other loss of profit. That is why when delay period M is greater than M_3, then purchasing cost is infinite, i.e., the supplier will not agree to sale items to retailers after the delay period M_3.

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1.1 INTRODUCTION

Traditionally inventory modeler assumed that retailer's capitals are sufficient and must pay for the products as soon as the products are received. However, this is not always true in the actual business world as every businessman strives to increase his profit, his goodwill and his retailer base. In today's business transactions, it is more and more common to see that supplier always offers a specified period (say, 30 days) to the retailer to settle the account. In literature this period has been termed as trade credit period. The trade credit financing produces two benefits to the supplier:

1. It should attract new customers who consider it to be a type of price discount.

2. It should cause a reduction in sales outstanding.

Some established customers pay more promptly in order to take advantage of trade credit more frequently. In India, gas stations adopted a pricing policy that charged less money per gallon to the customer who paid by cash, instead of credit card. Likewise, store owners around the world usually charge a customer 5% more if the customer pays by a credit card, instead of cash. As a result, the customer must decide which alternative to take when the supplier provides not only a cash discount but also a permissible delay. Hence, trade credit can play a major role in inventory control policy for both the retailer and the supplier. This idea caught attention of inventory practitioners and results are that various articles dealing with trade credits have appeared in various inventory journals.

In business world, time is a phenomenon, which affects everything around it. Inflation is a concept closely related to time. Inflation is that state of disequilibrium, in which an expansion of purchasing power tends to cause, or is the effect of an increase in the price level. During inflation, there is too much currency in relation to the physical volume of the business being done. Economists generally agree that high rates of inflation and hyperinflation are caused by an excessive growth of the money supply. Low or moderate inflation may be attributed to fluctuations in real demand for goods and services, or changes in available supplies such as during scarcities, as well as to growth in the money supply. However, the consensus view is that a long sustained period of inflation is caused by money supply growing faster than the rate of economic growth. Due to high inflation and consequent sharp deckle in the purchasing power of money in the developing countries like India, Argentina, Brazil, Bangladesh etc., the financial situation has been completely changed. So, it is sufficient reason in itself to compel the researchers over the world to study the effect of inflation on their analysis.

It was very obvious fact that given some time, every item can create a niche for itself in the customer's mind, hence increasing its demand with the passage of time. Demand of goods may vary with price or even with the instantaneous level of inventory displayed in a supermarket. Demand also depends on demand promotional policies of suppliers such as trade credit. It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more. According to **Levin** *et al.* **(1972),** one of the functions of inventories is that of a motivator, as indicated in the statement that 'At times, the presence of inventory has a motivational effect on the people around it.' In a competitive market, price of the goods plays an important factor to a customer.

Silver and **Peterson (2012)** noted that sales 'at the retail level tend to be proportional to inventory displaced. **Pal** *et al.* **(1993)** observed that sales of some items at the retail level were directly related to the' amount of inventory displayed. Urban and **Baker (1997)** generalized the EOQ model in which the demand is a multivariate function of price, time and level of inventory. **Urban (2005)** conducted a comprehensive review of that literature, distinguishing between type 1 models in which the demand rate of an item was a function of the initial inventory level and type 2 models in which it was dependent on the instantaneous inventory level. **Teng** and **Chang (2005)** extended an Economic Production Quantity (EPQ) model for perishable items, considering the demand rate as the sum of two terms: first term was inversely proportional to the price and second term was directly proportional to the stock-level of inventory displayed. **Sana** and **Chaudhuri (2008)** presented the retailer's profit-maximizing strategy when confronted with supplier's trade offer of credit and price-discount on the purchase of merchandise. They had taken different demand pattern in that analysis. First in literature, credit-linked demand function had been coined by **Jaggi** *et al*. **(2008)** in the analysis of inventory control systems. **Jaggi** and **Kausar (2010)** developed a supply chain model by taking credit-linked demand function and determined the optimal replenishment time and credit period for the retailer. They assumed that items deteriorate when physically present in stocks.

Most of the inventory practitioners while discussing trade credit policy in their models implicitly assumed that the retailer settle the account when whole of its inventory depleted to zero. But, in most business transactions, this assumption is unrealistic. As each retailer wants to settle the accounts as soon as possible in order to pay minimum interest to the supplier. It is also observed that most of the inventory practitioner ignore the fact that credit period offered by the supplier to the retailer has a positive impact on the demand. There are various factors which can stimulate the demand of the product such as by reducing selling price and through the display of stock. It was also observed from literature that most of the researchers take fixed interest rate while developing models with trade credit policy. But this is not realistic as there is always impreciseness present in interest rate. Inventory modelers have not contributed much in that direction. So it is the need of the hour to discuss this in detail. Through this chapter the researcher has tried to pave the path in that direction by developing a retailer inventory model in which retailer settles the account as soon as retailer has sufficient amount and tries to maximize his profit. All analysis has been performed in inflationary and fuzzy environment. In this chapter, two different inventory models have been considered.

From literature it was observed that credit period or selling price have positive impact on demand of the product. In this section to stimulate the demand it is assumed that demand depends on either credit period or selling price, fixed by the retailer. Hence, demand function D(.) can be any one of the following forms:

a. Credit Linked Demand Function: Demand rate is a function of retailer's credit-period offered by the supplier (M). The demand function can be represented as a differential difference equation:

$$D(M+1)-D(M) = r[S-D(M)]$$

Where $D(.) = D(M)$ demand for any M per unit time

S : maximum demand

r : rate of saturation of demand (which can be estimated using the past data) under the assumption that the marginal effect of credit period on sales is proportional to the unrealized potential of the market demand without delay.

The solution of the above difference equation, under the condition that at M=0, D(0) = s(initial demand), keeping other attributes like price, quantity, etc. at constant level, is given by

$$D(.) = s(1-r)^{M}+S(1-(1-r)^{M})$$

$$D(.) = S-(S-s)(1-r)^{M}$$

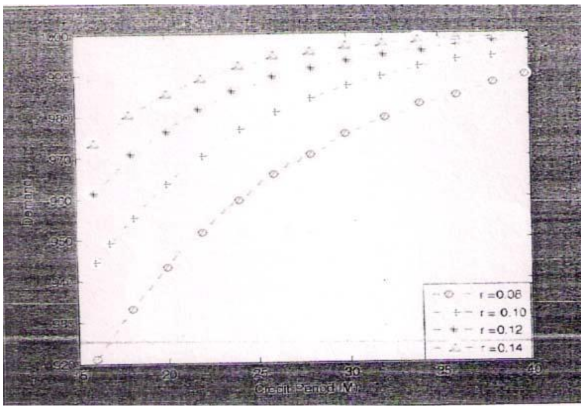

Fig. 1.1: Demand Pattern with Respect to Different Value of 'r'

Fig.1.1 shows the demand function for different values of 'r' and M. It shows that credit period have significant effect on demand and it gradually reaches its saturation level as M increases. Saturation level achieve faster if the value of 'r' is high.

b. **Demand Depends on Selling Price of Retailer:** The market demand rate for the product is a downward sloping function of the selling price. In this case, demand is of the form D(.) = $a(i_1)^{-b}$ where a>0 is a scaling factor and b>l is a price elasticity coefficient.

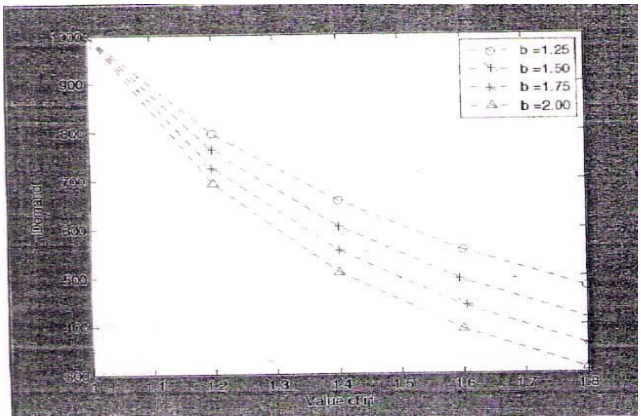

Fig. 1.2: Demand Pattern for Different Value of 'b'

Fig.1.2 shows that demand changes significantly with respect to the markup price (i_1). Demand gradually decreases as the value of 'i_1' increases for fixed value of 'b'.

1.2.1 Assumptions: In this sect' n, mathematical model is developed under the following assumptions.

1. Supplier provides a credit period to the retailer in payment. Supplier offers different discount rate of purchasing price at different credit periods.

2. Demand rate is function of credit period or selling price.

3. The replenishment rate is infinite.

4. Inflation and time value of money is considered.

5. A constant fraction of the on-hand inventory deteriorates per Unit time.

6. Lead-time is negligible.

7. The inventory system involves a single type of items.

8. Time horizon is infinite.

1.2.2 Notations: The following notations are used for the development of mathematical model in this section.

T : Cycle time

D(.) : Demand rate

Q : Initial inventory level

k : Constant representing the difference between discount rate (k_1) and

inflation rate (k_2).

A(t) : Ordering cost for an order placed at time t, i.e., $A(t)=A_0e^{-kt}$, where A_0 is

the ordering cost at time zero

i : Inventory holding cost per unit per year excluding interest charges

θ : A constant fraction of on-hand inventory which deteriorates per unit time

I_e : Annual interest that can be earned per unit

I_p : Annual interest charge payable per unit ($I_p>I_e$)

M : Credit period for settling accounts

C(t) : Unit purchasing cost per item at time t, i.e., $C(t) = C_0e^{-kt}$, where C_0 is the

purchasing cost at time zero and depends on the credit period (M)

P : Payment time for retailer

C_D : Total cost of deterioration per cycle

C_H : Total holding cost per cycle

Dr : Amount of materials deteriorated during a cycle time (T)

S(t) : Selling price per unit item, i.e., $S(t) S_0e^{-kt}$, where S_0 ($=i_1C_0$) is the selling

price at time zero.

1.2.3 FORMULATION OF MATHEMATICAL MODEL:

Inventory level varies with time due to demand and deterioration simultaneously. The deterioration can occur when the materials are physically present in the inventory at time t ($0 \le t \le T$). Let I(t) be the inventory level at any time t. Inventory level, I(t), during the time period ($0 \le t \le T$) is given by [Fig. 5.3]

The purchasing cost at different delay periods are

$$C_0 = \begin{cases} \square_{\square}(1-\square_1), & \square = \square_1, \\ \square_{\square}(1-\square_2), & \square = \square_2, \\ \square_{\square}(1-\square_1), & \square = \square_3, \\ \infty & \square > \square_3, \end{cases}$$

Where C_r = maximum retail price per unit, M_j (i=1,2,3) decision point in settling the account to the supplier at which supplier offers λ_j% discount to the retailer. M_3 is the maximum delay period after which the supplier will not agree to give trade offer of credit and price-discount for sale of the items to the retailer. Consequently, the supplier decides $Co \to \infty$ when $M > M_3$, i.e., retailer never purchases at an infinite cost. $\square_{\square}$ (j=1,2,3) are the constant discount rates decided by the supplier.

Solution of equation (5.1) using the boundary condition is

$$I(t) = \square\square^{-\square\square} + \frac{\square(.)}{\square}\left(\square^{-\square\square} - 1\right) \qquad 0 \le \square \le \square$$
..........(5.2)

Consequently, initial inventory after replenishment becomes

$$\square = +\frac{\square(.)}{\square}\left(\square^{-\square\square} - 1\right)$$
..........(5.3)

Since the total demand during T is D(.)T, the amount of materials which deteriorates during one cycle is

$$D_T = Q - D(.)T = \left(\frac{\square(.)}{\square}\left(\square^{-\square\square} - 1\right) - \square(.)\square\right)$$
...........(5.4)

Now, the various costs associated to inventory are calculated as follows:

Income from selling the product is

$$R_p = \sum_{n=0}^{\infty} S(nT)\int_P^T D(.)dt = i_1\left(\frac{1}{1-e^{-kT}}\right) C_0 D(.)(T-P)$$

Interest earned after P is

$$I_T = \sum_{n=0}^{\infty} \square_e S(nT)\int_P^T D(.)(T-t)dt = i_1\left(\frac{1}{1-e^{-kT}}\right) C_0 I_e D(.)(T-P)^2/2$$

Let A(t) be the ordering cost at time t, then the ordering cost is

C_R = A(0) + A(T) + A(2T) + ...+ A((m-l)T)+....................

$$= A_0\left(\frac{1}{1-e^{-kT}}\right)$$

Total deterioration cost is

$$C_D = \sum_{n=0}^{\infty} C(nT) D_T$$

$$= \sum_{n=0}^{\infty} C(nT)\left(\frac{\square(.)}{\square}\left(\square^{-\square\square}-1\right)-\square(.)\square\right) = \square_0\left(\frac{1}{1-e^{-kT}}\right)\left(\frac{\square(.)}{\square}\left(\square^{-\square\square}-1\right)-\square(.)\square\right)$$

Holding cost is

$$C_H = \square\sum_{n=0}^{\infty} C(nT)\int_P^T \square(nT+t)dt = \sum_{n=0}^{\infty} C(nT)\int_P^T \frac{\square(.)}{\square}\left(\square^{\square(\square-\square)}-1\right)\square\square$$

$$= i\,\square_0\left(\frac{1}{1-e^{-kT}}\right)\left(\frac{\square(.)}{\square}\left(\square^{\square\square}-\square\square-1\right)\right)$$

The interest payable per cycle for the inventory not being sold after the due date is given by

$$P_T\sum_{n=0}^{\infty} I_{\square}\left(\int_0^{\square-\square}\left(\left(\square(\square\square)\square - \square(\square\square)\square(.)\square - \square(\square\square)\square_{\square}\square(.)\square^2/2\right) - \square(\square\square)\square(.)\square\right)\square\square\right)$$

$$= \left(\square_0\square(\square-\square)\square_{\square} - \frac{I_p i_1 C_0 I_e D(.) M^2 (P-M)}{2} - \frac{I_p i_1 C_0 D(.)(P^2-M^2)}{2}\right)\left(\frac{1}{1-e^{-kT}}\right)$$

Theorem-1-1: Initially retailer orders Q quantity and thus owes C_0Q to the supplier. The following two cases may now arise:

Case-1: If $\square(\square\square)\square - \square(\square\square)\square(.)\square - \square(\square\square)\square_{\square}\square(.)\square^2/2 \le 0$ then loan amount will be paid at M=P.

Case-2: If $\square(\square\square)\square - \square(\square\square)\square(.)\square - \square(\square\square)\square_{\square}\square(.)\square^2/2 \le 0$ then loan amount will be off at P such that $\square(\square\square)\square - \square(\square\square)\square(.)\square - \square(\square\square)\square_{\square}\square(.)\square^2/2 = \square(\square\square)\square(.)(\square-\square)$

Thus, total profit of retailer (TP) is defined as

$$TP = R_P + I_T - C_R - C_D - C_H - P_T$$

TP =
$$\left(i_1 C_0 D(.)(T-P) + i_1 C_0 I_e D(.)(T-P)^2/2 - A_0 - C_0\left(\frac{D(.)}{\theta}\left(e^{\theta T}-1\right) - D(.)T\right) - \right.$$

$$i\,C_0\left(\frac{D(.)}{\theta^2}\left(e^{\theta T}-\theta T-1\right)\right)-\left(\square_0\square(\square-\square)\square_\square-\frac{\square_\square\square_l\square_0\square_\square\square(.)\square^2(\square-\square)}{2}-\frac{\square_\square\square_l\square_0\square(.)(\square^2-\square^2)}{2}\right)\right)\left(\frac{1}{1-\square^{-\square\square}}\right)$$ or

$$TP = i_l\,\frac{(\square_0\square(.)(T-P))}{1-\square^{-\square\square}}+i\,\frac{\left(-C_0\left(\frac{D(.)}{\theta^2}\left(e^{\theta T}-\theta T-1\right)\right)\right)}{1-\square^{-\square\square}}+\square_\square\,\frac{(-\square_0\square(.)(P-M))}{1-\square^{-\square\square}}+$$

$$i_l I_e\,\frac{C_0 D(.)(T-P)^2/2}{1-\square^{-\square\square}}+i_l I_p\,\frac{\left(\frac{\square_0\square(.)(\square^2-\square^2)}{2}\right)}{1-\square^{-\square\square}}+\square_p\square_l\square_e\,\frac{\left(\frac{\square_0\square(.)\square^2(\square-\square)}{2}\right)}{1-\square^{-\square\square}}+\frac{\left(-\square_0-\square_0\left(\frac{\square(.)}{\square}\left(e^{\theta T}-1\right)-\square(.)\square\right)\right)}{1-\square^{-\square\square}}$$

...........(5.5)

$TP = i_l F_l + i(-F_2) + I_P(-F_3) + i_l I_e(F_4) + i_l I_P(F_5) + I_P i_l I_e(F_6) + (-F_7)$ ....(5.6)

where

$$F_1=\frac{\square_0\square(.)(T-P)}{1-\square^{-\square\square}}\quad F_2=\frac{\square_0\left(\frac{D(.)}{\theta^2}\left(e^{\theta T}-\theta T-1\right)\right)}{1-\square^{-\square\square}}\quad F_3=\frac{\square_0\square(P-M)}{1-\square^{-\square\square}}$$

$$F_4=\frac{C_0 D(.)(T-P)^2/2}{1-\square^{-\square\square}}\qquad F_5=\frac{\frac{\square_0\square(.)(\square^2-\square^2)}{2}}{1-\square^{-\square\square}}$$

$$F_6=\frac{\frac{\square_0\square(.)\square^2(\square-\square)}{2}}{1-\square^{-\square\square}}\quad F_7=\frac{\square_0+\square_0\left(\frac{\square(.)}{\square}\left(e^{\theta T}-1\right)-\square(.)\square\right)}{1-\square^{-\square\square}}$$

1.2.4 FUZZY INVENTORY MODEL:

In order to find the optimal solution of the proposed inventory model in the presence of uncertainty, opportunity cost, interest earned and interest paid are represented by triangular fuzzy numbers. The triangular fuzzy profit is calculated by using arithmetic operations based on the Function Principle. The Function Principle defines an efficient way to perform fuzzy arithmetic operations on fuzzy numbers **(Chen, 1985)**.

Now, let us consider i, $i_{1,}$ I_e, and I_p as imprecise parameters and expressed by triangular fuzzy numbers, $\tilde{i}, \tilde{i}_l, \tilde{I}_e, \tilde{I}_p$ respectively then

$i = (i^1, i^2, i^3) = (i\text{-}\Delta_1, i, i+\Delta_2)$ $\quad i_l = (i_l^1, i_l^2, i_l^3) = (i_l\text{-}\Delta_3, i_l, i_l+\Delta_4)$

$I_e = (I_e^1, I_e^2, I_e^3) = (I_e\text{-}\Delta_5, I_e, I_e+\Delta_6)$ $\quad I_p = (I_p^1, I_p^2, I_p^3) = (I_p\text{-}\Delta_7, I_p, I_p+\Delta_8)$

A normal triangular fuzzy number, for example, (a-Δ, a, a+Δ) and Δ >0, has the following membership function:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-(a-\Delta)}{\Delta} & a-\Delta<\square<\square,\\ \dfrac{(a+\Delta)-x}{\Delta} & a<\square<\square+\Delta,\\ 0 & \text{otherwise}\end{cases}$$

In this study, **Chen's (1985)** Function Principle for the calculation of the fuzzy profit function is used. According to this, the fuzzy profit function $TP=(TP^1,TP^2,TP^3)$ becomes a triangular fuzzy number where

$TP^1 = (i_l\text{-}\Delta_3)F_1-(i+\Delta_2)(F_2)\text{-}(I_p+\Delta_8)(\square_3)+(i_l-\Delta_3)(I_e-\Delta_5)(\square_4)$

$+ (i_l-\Delta_3)\ (I_p-\Delta_7)\ (\square_5)\ +\ (I_p-\Delta_7)\ (I_e-\Delta_5)\ (i_l-\Delta_3)\ (\square_6)-(\square_7)$

$TP^2 = i_l F_1 - i\ (\square_2)\ \text{-}\ I_p\ (F_3)\ + i_l I_e\ (\square_4)\ + \square_l\ \square_\square\ (\square_5) + \square_\square\square_\square i_l\ (\square_6)-(\square_7)$

$TP^3 = (i_l+\Delta_4)\ F_1 - (i\text{-}\Delta_1)\ (\square_2)\ \text{-}\ (I_\square\text{-}\Delta_7)\ (\square_3)+ (i_l+\Delta_4)\ (I_\square+\Delta_6)\ (\square_4)+ (i_l+\Delta_4)$

$(I_\square+\Delta_8)\ (\square_5)\ +\ (I_\square+\Delta_8)\ (I_\square+\Delta_6)\ (i_l+\Delta_4)\ (\square_6)\ \text{-}\ (\square_7)$, j =1,2,3 and $\tilde{1}$ = (1,1,1)

The membership function of total profit is given by

$$\mu_{T\tilde{P}}(x)=\begin{cases}\dfrac{x-TP^1}{TP^2-TP^1} & \square\square^1<\square<\square P^2\\ \dfrac{TP^3-x}{TP^3-TP^2} & \square\square^2<\square<\square\square^3\\ 0 & \text{otherwise}\end{cases}$$

The α-cut, TP(α), of $T\tilde{P}$ consists of points x such that TP(α) = $\{\square: \mu_{T\tilde{P}}(\square)\geq\alpha\}$. Since the total profit is a triangular fuzzy number, so α-cut of $T\tilde{P}$ is TP(α)= $[TP_L(\alpha), TP_U(\alpha)]$, $\alpha\in[0,1]$, where $TP_L(\alpha) = TP^1+(TP^2-TP^1)\,\alpha$ and $TP_U(\alpha) = TP^3-(TP^3-TP^2)\alpha$. In order to find the optimal value of decision variables, defuzzification of profit expression is performed by signed distance. Using this method, equivalent crisp profit expression is

$TPS \equiv d\,(T\tilde{P}.\tilde{0}) = (TP^1 + 2TP^1 + TP^3)/4$ ....(5.7)

1.3 RETAILER'S ORDERING POLICY WHEN DEMAND DEPENDS ON INITIAL STOCK-LEVEL:

In this case, demand function is considered of the form $D(.) = \alpha + \beta Q^{\gamma}$ where Q is initial stock-level and α, β and γ are positive constant.

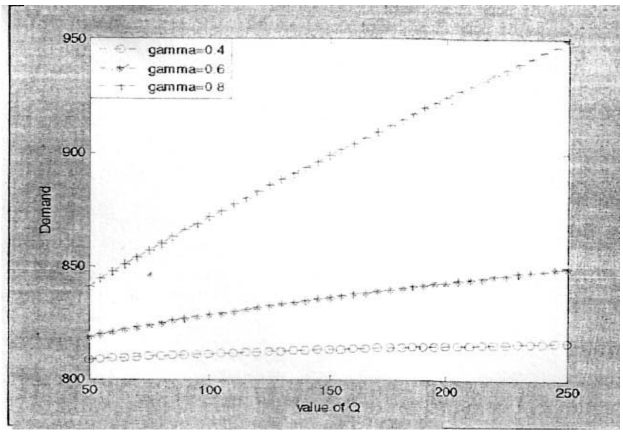

Fig. 1.4: Demand Pattern for Different Value of 'γ'

Fig.1.4 presents the demand pattern with respect to different value of ' γ ' It is observed that as the value of 'γ' increases demand also increases.

1.3.1 FORMULATION OF MATHEMATICAL MODEL:

With the same notations and assumptions proceed up to equation (5.5) and using the relation $D(.)=\alpha + \beta Q^{\gamma}$ profit expression is as follows:

$$TP = i_l \frac{(C_0(\alpha+\beta Q^{\gamma})(T-P))}{1-e^{-kT}} + i\,\frac{\left(-\square_0\left(\frac{\alpha-\beta Q^{\gamma}}{\theta^2}\left(e^{\theta T}-\theta T-1\right)\right)\right)}{1-e^{-kT}}$$

$$+ \square_\square \frac{(C_0 Q(P-M))}{1-e^{-kT}} + i_l\square_\square \frac{(\square_0(\alpha+\beta Q^{\gamma})(T-P)^2/2)}{1-e^{-kT}} + i_l\square_\square \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})(\square^2-\square^2)}{2}\right)}{1-e^{-kT}}$$

$$+ I_p i_l \square_\square \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})\square^2(\square-\square)}{2}\right)}{1-e^{-kT}} + \frac{\left(-\square_0-\square_0\frac{((\alpha+\beta Q^{\gamma}))}{\square}(e^{\theta T}-1)-(\alpha+\beta Q^{\gamma})T\right)}{1-e^{-kT}}$$

or

$$TP = i_l \frac{(C_0(\alpha+\beta Q^{\gamma})(T-P))}{1-e^{-kT}} + i\,\frac{(-C_0(\alpha+\beta Q^{\gamma}))}{1-e^{-kT}}$$

$$+ \square_\square \frac{(-C_0 Q(P-M))}{1-e^{-kT}} + i_l\square_\square \frac{(\square_0(\alpha+\beta Q^{\gamma})(T-P)^2/2)}{1-e^{-kT}} + i_l\square_\square \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})(\square^2-\square^2)}{2}\right)}{1-e^{-kT}} +$$

$$\square_\square\square_\square\square_\square \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})\square^2(\square-\square)}{2}\right)}{1-e^{-kT}} + \frac{\left(-\square_0-\square_0\left((\alpha+\beta Q^{\gamma})\left(\square+\frac{\square\square^2}{2}\right)(\square+\square\square^{\square})\square\right)\right)}{1-e^{-kT}}$$

or

$$\square\square = \square_l \frac{(C_0(\alpha+\beta Q^{\gamma})((R(Q)-P))}{1-e^{-kT}} + i\,\frac{\left(-C_0\left(\frac{(R(Q))^2(\alpha+\beta Q^{\gamma})}{2}\right)\right)}{1-e^{-k(R(Q))}}$$

$$+\square_\square \frac{(C_0 Q(P-M))}{1-e^{-k(R(Q))}} + i_l\square_\square \frac{(\square_0(\alpha+\beta Q^{\gamma})(T-P)^2/2)}{1-e^{-k(R(Q))}}$$

$$+ i_l\square_\square \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})(\square^2-\square^2)}{2}\right)}{1-e^{-k(R(Q))}} + \square_\square i_l I_e \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})\square^2(\square-\square)}{2}\right)}{1-e^{-k(R(Q))}}$$

$$+ \frac{\left(-\square_0-\square_0\left((\alpha+\beta Q^{\gamma})\left((R(Q))+\frac{\square(R(Q))^2}{2}\right)-(\square+\square\square^{\square})(R(Q))\right)\right)}{1-e^{-k(R(Q))}}$$

$$\square\square = \square_l\square_1' i\,(-\square_2') + \square_p\,(-\square_3') + \square_l\square_e\,(\square_4') + \square_l\square_p\,(\square_5') + \square$$ ……(5.8)

Where

$$\square_1' = \frac{(C_0(\alpha+\beta Q^{\gamma})((R(Q)-P)-P))}{1-e^{-k(R(Q))}},\quad \square_2' = \frac{\left(C_0\left(\frac{(R(Q))^2(\alpha+\beta Q^{\gamma})}{2}\right)\right)}{1-e^{-k(R(Q))}}$$

$$\square_3' = \frac{(C_0 Q(P-M))}{1-e^{-k(R(Q))}},\quad \square_4' = \frac{\left(\square_0(\alpha+\beta Q^{\gamma})((R(Q))-P)^2/2\right)}{1-e^{-k(R(Q))}}$$

$$\square_5' = \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})\,(\square^2-\square^2)}{2}\right)}{1-e^{-k(R(Q))}}, \quad \square_6' = \frac{\left(\frac{\square_0(\alpha+\beta Q^{\gamma})\square^2\,(\square-\square)}{2}\right)}{1-e^{-k(R(Q))}}$$

$$\square_7' = \frac{\left(\square_0+\square_0\left((\alpha+\beta Q^{\gamma})\left((R(Q))+\frac{\square(R(Q))^2}{2}\right)-(\square+\square\square^{\square})(R(Q))\right)\right)}{1-e^{-k(R(Q))}}$$

and $\square = \square(\square) = \dfrac{1+\sqrt{1+(2Q\theta/(\alpha+\beta Q^{\gamma}))}}{\theta}$

1.3.2 FUZZY INVENTORY MODEL:

Now, let us consider i, i_1, I_e and I_p, as imprecise parameters and expressed by triangular fuzzy numbers, $\tilde{i}, \tilde{i}_1, \tilde{I}_e, \tilde{I}_p$ respectively. Proceeding as previous section and after defuzzification by signed distance method, profit expression is

TPS = $(TP'^1+2TP'^2+TP'^3)/4$ ....(1.9)

Where

$TP'^1 = (i_l-\Delta_3)\,\square_1'-(i+\Delta_2)(\square_2')-(I_p+\Delta_8)(\square_3')+(i_l-\Delta_3)(I_e-\Delta_5)(\square_4')$

$+ (i_l-\Delta_3)\,(I_p-\Delta_7)\,(\square_5')-(I_p-\Delta_7)\,(I_e-\Delta_5)\,(i_l-\Delta_3)\,(\square_6')-(\square_7')$

$TP'^2 = i_l\square_1'-i\,(\square_2')-I_p\,(\square_3')+i_lI_e\,(\square_4')+i_l\,I_p\,(\square_5')+I_pI_ei_l\,(\square_6')-(\square_7')$

$TP^3 = (i_l+\Delta_4)\,F_1'-(i-\Delta_1)\,(\square_2')-(I_p-\Delta_7)(F_3')+(i_l+\Delta_4)\,(I_{\square}+\Delta_6)\,(\square_4')$

$+ (i_l+\Delta_4)\,(I_p+\Delta_8)\,(\square_5')+(I_p+\Delta_8)\,(I_e+\Delta_6)\,(i_l+\Delta_4)\,(\square_6')-(\square_7')$

1.4 NUMERICAL EXAMPLES:

In this model, the retailer tries to find out the payment time to supplier in order to maximize their profit and minimize interest which has to be paid to the supplier. The proposed model has been illustrated with the help of following data which is taken from the literature in appropriate units:

Example-1: Taking D(.)1000(constant), A_0=200, Cr=20, k=0.2, i = 0.12 per year, i_l = 1.2 per year, I_e=0.13 per year, $I_p = 0.20$, θ=0.2, Δ_1=0.8, Δ_2=0.7, Δ_3= 0.8, Δ_4= 0.7, Δ_5= 0.8, Δ_6 = 0.7, Δ_7 = 0.8, Δ_8 = 0.7, M_1 = 20/365, M_2 = 25/365, M_3 = 30/365, λ_1 = 15%, λ_2= 5%, λ_3 = 0%. Calculation is performed through MATLAB 7.1 and the results thus obtained are listed in Table 1.1 corresponding to M=20, 25, and 30 days.

Table-1.1: Solution Corresponding to Different Value of k and M:

M(days)	k	T*(years)	P*(years)	Q*	Profit
20	0.02	0.1426	0.1234	148	6822.10
	0.03	0.1362	0.1149	138	4429.70
	0.04	0.1309	0.1103	132	3267.10
	0.05	0.1264	0.1065	128	2568.90
25	0.02	0.1304	0.1098	132	8317.70
	0.03	0.1245	0.1047	126	5456.40
	0.04	0.1197	0.1006	121	4026.30
	0.05	0.1156	0.0972	116	3167.90
30	0.02	0.1185	0.0995	119	8977.80
	0.03	0.1132	0.0950	114	5863.90
	0.04	0.1088	0.0912	109	4308.40
	0.05	0.1052	0.0882	106	3380.20

From Table 1.1 the following observations can be made:

- From the previous literature it was found that the k and M have significant decisive effects on inventory cycle time, credit period, order quantity and on profit of retailer. On the analysis of Table 1.1, it appears correct.

- Inflation is the state of a continuous increase in price of goods and service. Hence, as obvious, an increase in the rate of inflation causes the total profit of the system going down. Such a change in the system is very appreciable.

- An increase in the inflation rate reduces the purchasing power of the retailer. So retailer can buy fewer inventories, which ultimately finish off sooner, reducing the initial level of inventory and the payment time P to clear up the account.

- The profit of the retailer shows an increasing trend as the value of M increases from 20 days to 30 days. As the credit period increases, retailer gets an opportunity to earn more interest on the accumulated capital. That is the reason for the change in value of P. Since in this situation retailers have capital to payoff in less time, this result identifies that trade credit is an effective strategy for inventory systems.

- From Table 1.1, it can be observed that T^*, p^*, Q^* decreases whereas profit of retailer increases with an increase in the credit period (M). It shows that the longer the credit period is, shorter the replenishment period, payment time, and order quantity are, whereas the profit will increase. From managerial point of view, if the supplier provides a credit period, retailer can lower their order quantity and increase their frequency of replenishment in order to take the benefits of the credit period more frequently.

Table-1.2: Computational Results with Respect to Different Values of I_p:

I_P	T*	P*	Q*	Profit
0.10	0.1378	0.1162	139.71	18429
0.13	0.1333	0.1124	135.09	18118
0.17	0.1283	0.1081	129.96	17735
0.20	0.1252	0.1055	126.78	17474

- From Table 1.2, it is observed that as the interest paid per unit item increases from 0.10 to 0.20 then inventory cycle, payment time to settle the account, order quantity and profit of the retailer decreases. From managerial point of view it implies that when the interest paid is high then retailer should order less amount of inventory.

Table-1.3: Computational Results with Respect to Different Values of :

I	T*	P*	Q*	Profit
0.1	0.1488	0.1245	149.91	22395
0.2	0.1252	0.1052	126.78	18326
0.3	0.1118	0.0943	113.69	14963
0.4	0.1029	0.0871	105.04	12035

- Through Table 1.3, the effect of deterioration rate on T, P, Q and on profit of the retailer can be analyzed. It is observed that an increase in deterioration rate results in a short cycle length, therefore payment time to settle the account, order quantity and profit of retailer decreases. By decreasing the order quantity retailer tries to decrease the deterioration quantity and increase their profit. Hence, if the retailer can effectively reduce the deterioration rate of items by improving equipment of storehouse, profit earned by the retailer will be increased.

Example-2: To analyze the effect of credit period on demand, we consider D(.) = S – (S-s)(l-r)^{M} i.e., where demand depends on credit period. For this following values are required:

S=1000, s=700, r=0.12 and rest of the value of the parameters are same as used in Example

1. The computational results for various values of M and k are as shown in Table 1.4.

Table-1.4: Computational Results with Respect to Different Values of k and M:

M(days)	k	T*(years)	P*(years)	Q*	Profit
0	0.02	0.1700	0.1441	121	2556.20
	0.03	0.1623	0.1375	115	1635.50
	0.04	0.1599	0.1354	113	1214.00
	0.05	0.1505	0.1273	106	895.50
20	0.02	0.1631	0.1380	116	3086.30
	0.03	0.1557	0.1316	110	1976.50
	0.04	0.1496	0.1264	106	1421.30
	0.05	0.1445	0.1220	102	1089.20
25	0.02	0.1520	0.1283	108	4141.20
	0.03	0.1451	0.1224	103	2671.70
	0.04	0.1395	0.1176	99	1938.40
	0.05	0.1347	0.1135	95	1498.40
30	0.02	0.1427	0.1202	101	4653.40
	0.03	0.1362	0.1146	96	2996.60
	0.04	0.1309	0.1101	93	2170.70
	0.05	0.1265	0.1063	89	1678.10

- From Table 1.4 it is observed that as M increases, there is marginal decrease in cyc1 length, payment period to supplier as well as order quantity but there is significant increase in retailer's profit, which-implies that credit period offered to relation has positive impact on the unrealized demand. Therefore, the retailer must pay attention on the credit policy provided by supplier very carefully to achieve maximum profit as much possible.

Table-1.5: Effect of Variation of 'r' on the Optimal Solution:

M(days)	r	T* (years)	P* (years)	Q*	Profit
20	0.08	0.1631	0.1380	116	3086.30
	0.10	0.1631	0.1380	116	3091.10
	0.12	0.1630	0.1379	116	3094.20
	0.14	0.1630	0.1379	116	3099.20
25	0.08	0.1520	0.1283	108	4141.20
	0.10	0.1520	0.1283	108	4148.10
	0.12	0.1519	0.1282	108	4153.00
	0.14	0.1519	0.1282	108	4160.20
30	0.08	0.1427	0.1202	101	4653.40
	0.10	0.1426	0.1201	101	4659.70
	0.12	0.1425	0.1200	101	4666.20
	0.14	0.1425	0.1200	101	4675.60

- To explore the impact of trade credit and rate of saturation of demand, using the same data as shown in Example-2, optimal solutions for different values of 'r' are listed in Table 1.5. It is observed that due to the escalating values of 'r' profit of the retailer increases. Profit also increases with the increase of trade credit period offered by supplier to the retailer. Profit of the retailer is more sensible for change in credit period in comparison to Y. So, it can be concluded that profit of the retailer increases with the increase of credit period as well as the saturation rate of demand.

Example-3: Now we are going to present the analysis when demand rate is of the form D(.)=a$(i_1)^{-b}$ For this. the following values are required:

a=1 000, b=1.5 and rest of the parameters remains same as used in Example-1.

Table-1.6: Computational Results with Respect to Different Values of i1 and M:

M(days)	i_1	T^* (years)	P^* (years)	Q^*	Profit
20	1.20	0.1583	0.1338	122	3792.50
	1.25	0.1588	0.1289	115	6320.80
	1.30	0.1593	0.1243	109	8551.00
	1.35	0.1598	0.1201	103	10525.00
	1.40	0.1603	0.1162	98	12276.00
25	1.20	0.1470	0.1240	113	4945.70
	1.25	0.1473	0.1193	106	7732.80
	1.30	0.1476	0.1149	101	10190.00
	1.35	0.1479	0.1109	95	12362.00
	1.40	0.1482	0.1071	90	14289.00
30	1.20	0.1372	0.1155	105	5489.10
	1.25	0,1372	0.1108	99	8382.20
	1.30	0.1372	0.1066	93	109.00
	1.35	0.1372	0.1026	88	13180.00
	1.40	0.1372	0.0989	83	15173.00

- As the selling price increases the customer demand decreases so the order quantity is also decreases whereas in this situation retailer has enough money to settle down the account of supplier in lesser time. A higher value of i1 results in a 1argr' optimal replenishment time, a smaller optimal order quantity and larger optimal profit when the supplier offers a cash discount to the retailer or a credit period to the retailer.

Table-1.7: Effect of 'b' on the Optimal Solution $(i_1.2)$:

b	T^*	P^*	Q^*	Profit
1.25	0.1556	0.1315	125	4219.80
1.50	0.1583	0.1338	122	3792.50
1.75	0.1610	0.1362	118	3387.90
2.00	0.1637	0.1385	115	3050.00

- From Table 1.7, it is observed that as the value of 'b' increases the profit of the retailer decreases, whereas replenishment period and payment time increases. Its managerial implication is that the demand of the product is highly affected by the selling price of the product so that profit is also fluctuating due to the change in 'b'.

Example-4: For this example the values of the parameters are as follows: D (.)=...+.... where .....= 800, P= l .8, ...= 0.6 and the rest of the values remain same as used in Example-I.

- From Table 1.8 it is observed that there is a positive impact of y on the optimal value of T, P, Q and on the profit of retailer. This means that demand and profit of the retailer is fluctuating by changing the value of y. So there is a positive correlation between demand and the elasticity of initial inventory level of the product.

Table-1.8: Optimal Solution for Different Values of □:

□	D(.)	Q*	T*	P*	Profit
0.2	805	170	0.2069	0.1758	4748.60
0.4	814	172	0.2070	0.1759	4857.80
0.6	840	178	0.2075	0.1764	5172.40
0.8	928	208	0.2192	0.1865	6175.00

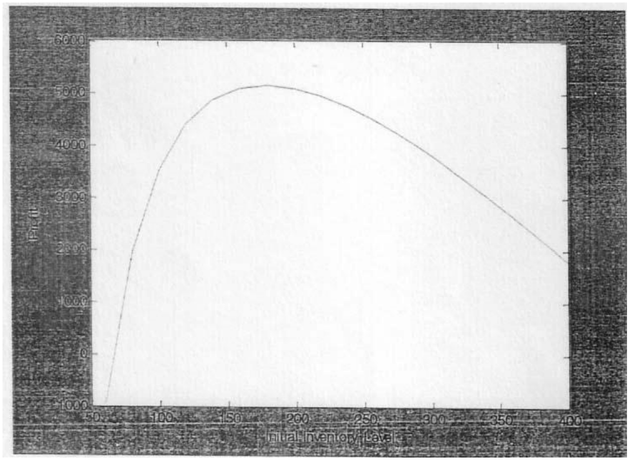

Fig. 1.5: Concavity of Profit for Given Value of Q (□=0.6)

- Fig.1.5 represents the concavity nature of profit with respect to initial inventory level. Profit is maximum when Q=178 at y=0.6. From Table 5.8 it is observed that as the value of y increases from 0.2 to 0.8, the optimal initial inventory level increases from 170 to 208 and due to this variation in profit is about 30%.

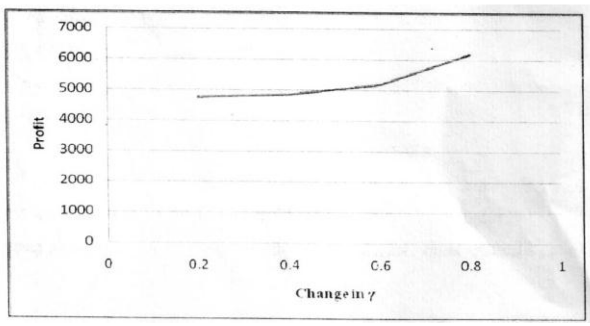

Fig. 1.6 Change in Profit w.r.t to Change in '□'

- From Fig.1.6 it is observed that as the value of '□ 'increases from 0.2 to 0.8 the profit of the retailer increases from 4748.60 to 6175.00.

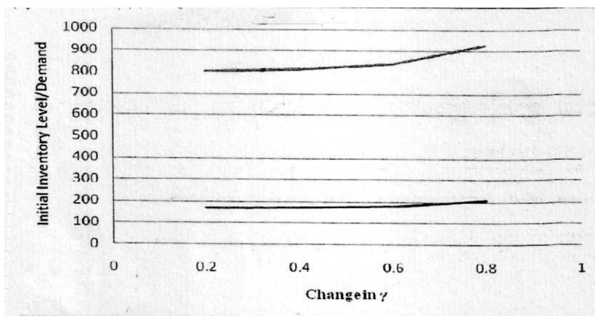

Fig. 1.7 Change in Initial Inventory Level or Demand Rate w.r.t to Change in '□'

• From Fig.1.7 it is observed that as the value of increases from 0.2 to 0.8, the demand of retailer and initial inventory levels both increase. This shows that have positive impact on demand and on initial inventory level.

1.5 SUMMARY AND CONCLUDING REMARKS:

Demand boosting activities have become more and more common in today's business world. For example, Wall-Mart and Costco often try to stimulate demand for specific types of electric equipment by offering price discounts; clothiers Baleno and NET make shelf space for specific clothes items available for longer periods. McDonald's and Burger King often use coupons to attract consumers. Other demand boosting strategies include free goods, advertising and display. The demand boosting policy is very important for the supplier and has a big impact on annual profits.

Our hypothesis is that consumer demand at a retail outlet depends on selling price, trade credit period as well as in-store stock of the product. In particular, demand decreases with selling price but increases with the displayed inventory of the product and credit period. The idea is that a large stock and credit period leads consumers to buy more. This phenomenon is not a general one but may be related to certain product categories, e.g., used automobiles sold from an open yard of a car dealer, or displays of large quantities of soft drinks, detergents, and canned food sometimes found in supermarkets. Keeping this phenomenon in mind, a different form of demand function has been considered in the present chapter under fuzzy environment. The different demand patterns are considered in this chapter to capture more realistic inventory situations and are as follows:

a. Demand depends on credit period offered by supplier to the retailer.

b. Demand is selling price sensitive.

c. Demand is directly proportional to initial inventory level.

In the present chapter, inventory models has been developed by incorporating some additional features like impreciseness in costs, inflation, deterioration and credit period offered by supplies to the retailer which can be associated with a number of different types of inventory. By taking impreciseness in cost parameters, decision-makers absorb all the turbulence in cost due to market fluctuation. Inflation permits a proper recognition of the financial implication.-of the opportunity cost in inventory analysis. Generally, supplier offers different price-discount on purchase of item of retailers at different delay periods. Suppliers allow maximum delay period, after which they will not take a risk of getting back money from retailers or any other loss of profit. That is why when delay period M is greater than M_3, then purchasing cost is infinite, i.e., the supplier will not agree to sale items to retailers after the delay period M_3.

Through numerical example, it is observed that as the inflation rate increases order quantity and profit gradually decreases. Thus as inflation rate increases purchasing power of the retailer decreases. It is also observed that by increasing the value of M, retailer it as discount and orders more frequently and earns more profit. From the analysis some managerial insights are also obtained. The retailer can increase profit by ordering lower quantity when the supplier provides a credit period in payments, improving storage conditions and also takes account of inflation while taking inventory related decisions. Numerically it is found that demand of the product is fairly sensitive of the credit period, selling price and initial inventory level. There is 20% hike in profit when retailer got 20 days as credit period whereas this crops up about 60% when supplier provides credit period of 25 days. So, it is all up to the retailer how they choose credit policy to maximize their profit. When the value of 'i_1' changes from 1.2 to 1.25 then profit of the retailer is escalated by 66% whereas it escalated by 125% when 'i_1' is 1.30. All this shows that profit of the retailer is highly sensitive to 'i_1'. The benefits of dynamic pricing techniques have long been known in the airline, restaurant, hotel, fashion goods and high-tech product industries.

The findings in this paper are important to the real world. The prices of fashion-based goods, for example clothes or computers, will be marked down gradually with the passage of time. These types of products are usually characterized as having a short shelf life. Promotional activities, such as price discount and trade credit, are commonly employed to speed up the movement of the goods. The proposed models here provides an insight for the decision-maker which is useful and practical. The results are also applicable for other types of perishable goods such vegetables/fruit, baked goods, etc. which are similar in nature to the fashionable goods described above.

A future study can further incorporate more realistic assumptions such as imprecise demand, allowable shortages and different deterioration rates etc. In future, this study can be further extended by developing an integrated-supplier and retailer inventory model with two level trade credits.

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